## PHYSICS OF MAGNETIC PHENOMENA

# ELECTROMAGNETIC WAVE SCATTERING ON THE STRUCTURE CONSISTING OF A MAGNETODIELECTRIC BODY AND THIN CONDUCTORS 

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UDC 537.874.6


#### Abstract

The method of auxiliary sources is used to solve the problem of electromagnetic wave scattering on the structure consisting of a magnetodielectric body and several thin conductors with finite lengths located close to it. The paper presents the results of numerical computations characterizing the effect of one or two thin conductors on the scattering cross-section of a magnetodielectric body.


Keywords: method of auxiliary sources, electromagnetic scattering, magnetodielectric body, thin conductor, scattering cross-section.

## INTRODUCTION

Defect detection, radio direction finding, radiolocation, and antenna technologies benefit from research on the scattering of electromagnetic waves in the resonance frequency domain on the structures consisting of a threedimensional magnetodielectric body and several thin conductors located in close proximity to that body. Analysis of available literature shows that there are many Russian and international publications on the scattering of electromagnetic waves on the individual magnetodielectric (dielectric) bodies. One can mention publications [1-4] as an example. There are also many publications, for instance [5, 6], that examine the scattering of electromagnetic waves on one or several thin conductors. In addition to that, some publications, such as [7-9], focus on the electromagnetic scattering on the perfectly conducting structures consisting of closely located solid bodies and thin conductors.

Relative scarcity of publications on the scattering of electromagnetic waves on the structures consisting of closely located bodies (the distance between which is much smaller than the wavelength) is explained by the fact that correct organization of such studies requires solving the boundary problems of the scattering theory on the systems of three-dimensional interacting (in electromagnetic sense) bodies. As a rule, such problems can be solved only by numerical methods. The latter can rely both on the use of Maxwell's equations in the differential form and integral relations of the field theory. However, the corresponding computational algorithms turn out to be extremely resource and time consuming. For the finite methods this is due to the need to extend the computations to the entire examined spatial region, and for the integral equation methods this is due to the need to calculate a large number of surface or volume integrals.

Over the recent years, researchers started using the method of auxiliary sources to solve the problems of electromagnetic scattering on the systems of interacting bodies [9-11]. For instance, in publication [9] the method of auxiliary sources is used to solve the problems of scattering on the structures consisting of solid perfectly conducting bodies and thin conductors; in publication [10] it is used to solve the problems of scattering on the structures consisting

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Fig. 1. Problem geometry.
of a finite number of three-dimensional perfectly conducting bodies, and in publication [11] - to solve the problems of scattering on the structures consisting of a finite number of three-dimensional impedance bodies.

In this paper, the method of auxiliary sources is used to solve the problem of electromagnetic wave scattering on the structure consisting of a magnetodielectric body and several thin conductors with finite lengths located close to it. It presents the results of numerical computations characterizing the effect of one or two thin conductors on the scattering cross-sections of a magnetodielectric body.

## 1. FORMULATION OF THE PROBLEM AND SOLUTION METHOD

The problem geometry is shown in Fig. 1. Let us consider the stationary problem of electromagnetic field scattering $\left\{\boldsymbol{E}_{0}, \boldsymbol{H}_{0}\right\}$ on the structure consisting of a solid dielectric body $D_{i}$ bounded by the surface $S$, with dielectric $\varepsilon_{i}$ and magnetic $\mu_{i}$ permeabilities, and $U$ thin conductors bounded by the surfaces $S_{u}^{\prime}(u=1,2, \ldots, U)$ and located in arbitrary positions in relation to the body $D_{i}$ (dependence on time was chosen in the form of $\exp (-i \omega t)$ ). Let us understand a solid body as a body, the maximum and minimum cross dimensions of which are comparable, and a thin conductor as a perfect conductor with a circular cross-section, the diameter of which is finite, but small compared to the conductor length and the wavelength. This structure is placed in the homogenous infinite medium $D_{e}$ with dielectric and magnetic permeabilities $\varepsilon_{e}, \mu_{e}$ in the Cartesian coordinates with a center selected inside the dielectric body. One needs to find the scattered field $\left\{\boldsymbol{E}_{e}, \boldsymbol{H}_{e}\right\}$ in $D_{e}$.

The mathematical formulation of the problem looks as follows:

$$
\begin{gather*}
\nabla \times \boldsymbol{E}_{e}=i \omega \mu_{e} \boldsymbol{H}_{e}  \tag{1}\\
\nabla \times \boldsymbol{H}_{e}=-\left.i \omega \varepsilon_{e} \boldsymbol{E}_{e}\right|_{D_{e}}, \quad \nabla \times \boldsymbol{E}_{i}=i \omega \mu_{i} \boldsymbol{H}_{i}  \tag{2}\\
\nabla \times \boldsymbol{H}_{i}=-\left.i \omega \varepsilon_{i} \boldsymbol{E}_{i}\right|_{D_{i}},  \tag{3}\\
\boldsymbol{n} \times\left(\boldsymbol{E}_{i}-\boldsymbol{E}_{e}\right)=\boldsymbol{n} \times\left.\boldsymbol{E}_{0}\right|_{n}, \boldsymbol{n}_{u} \times \boldsymbol{E}_{e}=-\boldsymbol{n}_{u} \times \boldsymbol{E}_{0} \text { at } S_{u}^{\prime}, u=1,2, \ldots, U, \\
\boldsymbol{n} \times\left(\boldsymbol{H}_{i}-\boldsymbol{H}_{e}\right)=\boldsymbol{n} \times\left.\boldsymbol{H}_{0}\right|_{S} \\
\left\{\sqrt{\varepsilon_{e}} \boldsymbol{E}_{e} ; \sqrt{\mu_{e}} \boldsymbol{H}_{e}\right\} \times \boldsymbol{R} / R+\left\{\sqrt{\mu_{e}} \boldsymbol{H}_{e} ;-\sqrt{\varepsilon_{e}} \boldsymbol{E}_{e}\right\}=O\left(R^{-1}\right), R \rightarrow \infty,
\end{gather*}
$$

where $\boldsymbol{E}_{e}, \boldsymbol{H}_{e}$ and $\boldsymbol{E}_{i}, \boldsymbol{H}_{i}$ are the fields in the regions $D_{e}$ and $D_{i}, \boldsymbol{n}$ is the unit normal vector to the surface $S$, $\boldsymbol{n}_{u}(u=1,2, \ldots, U)$ are the unit normal vectors to the surfaces $S_{u}^{\prime}$ of the thin conductors, $R=\left(x^{2}+y^{2}+z^{2}\right)^{1 / 2}$, $\boldsymbol{a} \times \boldsymbol{b}$ is the vector product.

The idea of the suggested method is as follows. In a similar way to what is done in [4], let us introduce (see Fig. 1) two auxiliary surfaces $S_{i}$ and $S_{e}$, similar to the dielectric body surface $S$ in terms of homothety with a center in point $O$ located inside the body and acting as the origin of coordinates. If the surface $S$ is the central one, then the center of homothety is chosen in such a way that it coincides with the surface center. Surface $S_{e}=K_{e} S$ is located inside the dielectric body and is characterized by the coefficient of homothety (similarity coefficient) $K_{e}$ that is smaller than unity. Surface $S_{i}=K_{i} S$ is located outside the body and characterized by similarity coefficient $K_{i}$ that is larger than unity. If $K_{e}=K_{i}=1$, auxiliary surfaces $S_{e}$ and $S_{i}$ coincide with $S$.

Let us select the finite set of points $\left\{M_{n, e}\right\}_{n=1}^{N_{e}}$ on the auxiliary surface $S_{e}$, placing in each of them a pair of independent elementary electric dipoles with moments $\boldsymbol{p}_{\tau_{1}}^{n, e}=p_{\tau_{1}}^{n, e} \boldsymbol{e}_{\tau_{1}}^{n, e}$ and $\boldsymbol{p}_{\tau_{2}}^{n, e}=p_{\tau_{2}}^{n, e} \boldsymbol{e}_{\tau_{2}}^{n, e}$, and on the auxiliary surface $S_{i}$ - the finite set of points $\left\{M_{n, i}\right\}_{n=1}^{N_{i}}$, placing in each of them a pair of independent auxiliary electric dipoles with moments $\boldsymbol{p}_{\tau_{1}}^{n, i}=p_{\tau_{1}}^{n, i} \boldsymbol{e}_{\tau_{1}}^{n, i}$ and $\boldsymbol{p}_{\tau_{2}}^{n, i}=p_{\tau_{2}}^{n, i} \boldsymbol{e}_{\tau_{2}}^{n, i}$. Unit vectors $\boldsymbol{e}_{\tau_{1}}^{n, e}, \boldsymbol{e}_{\tau_{2}}^{n, e}$ are chosen in the plane tangential to $S_{e}$ in point $M_{n, e}$, and unit vectors $\boldsymbol{e}_{\tau_{1}}^{n, i}, \boldsymbol{e}_{\tau_{2}}^{n, i}$ - in the plane tangential to $S_{i}$ in point $M_{n, i}$. It is assumed that dipoles that are placed on $S_{e}$ radiate into the homogenous medium with parameters $\varepsilon_{e}, \mu_{e}$, while dipoles that are placed on $S_{i}$ - into the homogenous medium with parameters $\varepsilon_{i}, \mu_{i}$.

Let us place continuously distributed auxiliary current $\boldsymbol{J}_{u}$ on its axis inside each of the thin conductors, as it was done in [9].

Let us express an unknown scattered field $\left\{\boldsymbol{E}_{e}, \boldsymbol{H}_{e}\right\}$ in $D_{e}$ as a sum of the fields of auxiliary dipoles located on the auxiliary surface $S_{e}$ and auxiliary currents:

$$
\begin{gather*}
\boldsymbol{E}_{e}(M)=\frac{i \omega}{k_{e}^{2}}\left\{\sum_{n=1}^{N_{e}} \nabla \times\left(\nabla \times \Pi_{n, e}\right)+\sum_{u=1}^{U} \nabla \times\left(\nabla \times \boldsymbol{\Pi}_{u}\right)\right\}, \\
\boldsymbol{H}_{e}(M)=\frac{1}{\mu_{e}}\left\{\sum_{n=1}^{N_{e}} \nabla \times \Pi_{n, e}+\sum_{u=1}^{U} \nabla \times \Pi_{u}\right\},  \tag{4}\\
\Pi_{n, e}=\Psi_{e}\left(M, M_{n, e}\right) \boldsymbol{p}_{\tau}^{n, e}, \boldsymbol{p}_{\tau}^{n, e}=p_{\tau_{1}}^{n, e} \boldsymbol{e}_{\tau_{1}}^{n, e}+p_{\tau_{2}}^{n, e} \boldsymbol{e}_{\tau_{2}}^{n, e},
\end{gather*}
$$

$$
\Pi_{u}=\int_{l_{u}} \Psi_{e}\left(M, M_{l, u}\right) \boldsymbol{J}_{u} d l, M \in D_{e},
$$

and let us express field $\boldsymbol{E}_{i}, \boldsymbol{H}_{i}$ in $D_{i}$ as a sum of the fields of auxiliary dipoles located on the auxiliary surface $S_{i}$ :

$$
\begin{gather*}
\boldsymbol{E}_{i}(M)=\frac{i \omega}{k_{i}^{2}} \sum_{n=1}^{N_{i}} \nabla \times\left(\nabla \times \Pi_{n, i}\right), \\
\boldsymbol{H}_{i}(M)=\frac{1}{\mu_{i}} \sum_{n=1}^{N_{i}} \nabla \times \Pi_{n, i},  \tag{5}\\
\Pi_{n, i}=\Psi_{i}\left(M, M_{n, i}\right) \boldsymbol{p}_{\tau}^{n, i}, \boldsymbol{p}_{\tau}^{n, i}=p_{\tau_{1}}^{n, i} \boldsymbol{e}_{\tau_{1}}^{n, i}+p_{\tau_{2}}^{n, i} \boldsymbol{e}_{\tau_{2}}^{n, i}, M \in D_{i} .
\end{gather*}
$$

In representations (4), (5) $\quad k_{e, i}=\omega \sqrt{\varepsilon_{e, i} \mu_{e, i}}, \quad \Psi_{e}\left(M, M_{n, e}\right)=\exp \left(i k_{e} R_{M M_{n, e}}\right) / 4 \pi R_{M M_{n, e}}, \quad \Psi_{e}\left(M, M_{l, u}\right)$ $=\exp \left(i k_{e} R_{M M_{l, u}}\right) / 4 \pi R_{M M_{l, u}}, \Psi_{i}\left(M, M_{n, i}\right)=\exp \left(i k_{i} R_{M M_{n, i}}\right) / 4 \pi R_{M M_{n, i}}, R_{M M_{n, e}}$ and $R_{M M_{l, u}}$ are the distances from point $M_{n, e}$ on the auxiliary surface $S_{e}$ and point $M_{l, u}$ on the axis of the conductor with a number $u$ to the observation point $M$ in $D_{e}, R_{M M_{n, i}}$ is the distance from point $M_{n, i}$ on the auxiliary surface $S_{i}$ to point $M$ in $D_{i}$, $p_{\tau_{1}}^{n, e}, p_{\tau_{2}}^{n, e}\left(n=1,2, \ldots, N_{e}\right)$ and $p_{\tau_{1}}^{n, i}, p_{\tau_{2}}^{n, i}\left(n=1,2, \ldots, N_{i}\right)$ are the unknown dipole moments, $N_{e}$ and $N_{i}$ are the numbers of points of dipole location on the auxiliary surfaces $S_{e}$ and $S_{i}, \boldsymbol{J}_{u}(u=1,2, \ldots, U)$ are the unknown axial auxiliary currents; integration in (4) is done along conductor axes $l_{u}$.

Representations (4), (5) satisfy the Maxwell's equations (1) and radiation conditions (3). In order to satisfy the boundary conditions (2), it is necessary to appropriately select the values of dipole moments $p_{\tau_{1}}^{n, e}, p_{\tau_{2}}^{n, e}$ $\left(n=1,2, \ldots, N_{e}\right)$ and $p_{\tau_{1}}^{n, i}, p_{\tau_{2}}^{n, i}\left(n=1,2, \ldots, N_{i}\right)$ and axial current distribution $\boldsymbol{J}_{u}(u=1,2, \ldots, U)$.

Let us introduce the piecewise constant approximation of axial currents. Let us split line $l_{u}$ of each current $\boldsymbol{J}_{u}$ into $N_{u}$ small segments, within each of which the current can be considered constant. Then the expression for $\Pi_{u}$ in (4) can be approximately written as

$$
\begin{equation*}
\Pi_{u}=\sum_{i=1}^{N_{u}} J_{u, i} \boldsymbol{e}_{u, i} \int_{l_{i-1, u}}^{l_{i, u}} \Psi_{e}\left(M, M_{l, u}\right) d l \tag{6}
\end{equation*}
$$

where $J_{u, i}$ is the value of current on the $i$-th segment of the conductor with a number $u, \boldsymbol{e}_{u, i}$ is the unit vector the direction of which coincides with the direction of the tangent in the midpoint of the examined segment. In the case of this approach, finding the unknown axial current distributions is reduced to finding the values of $\sum_{u=1}^{U} N_{u}$ current elements.

In order to determine the values of dipole moments and current elements, let us use the boundary conditions (2), satisfying them in accordance with the collocation method. Let $M_{j}(j=1,2, \ldots, L)$ be the collocation points on the dielectric body surface $S$, and let $M_{j}^{\prime}\left(j=1,2, \ldots, L_{u}\right)$ be the collocation points on the surface of conductors $S_{u}^{\prime}$. Let $L$ be the number of collocation points on $S$, and let $L_{u}$ be the number of collocation points on $S_{u}^{\prime}$. In view of assumption about the small conductor diameter compared to conductor length and wavelength, let us assume that one
can neglect the contribution of azimuthal current components on the surfaces of thin conductors into the scattered field. Then to find the unknown $p_{\tau_{1}}^{n, e}, p_{\tau_{2}}^{n, e}\left(n=1,2, \ldots, N_{e}\right), p_{\tau_{1}}^{n, i}, p_{\tau_{2}}^{n, i}\left(n=1,2, \ldots, N_{i}\right)$ and $J_{u, i}(u=1,2, \ldots, U$, $i=1,2, \ldots, N_{u}$ ) we shall produce the following system of linear algebraic equations with a complex $\left(4 L+\sum_{u=1}^{U} L_{u}\right) \times\left(2 N_{e}+2 N_{i}+\sum_{u=1}^{U} N_{u}\right)$ matrix:

$$
\begin{gather*}
\boldsymbol{n}^{j} \times\left(\boldsymbol{E}_{i}^{j}-\boldsymbol{E}_{e}^{j}\right)=\boldsymbol{n}^{j} \times \boldsymbol{E}_{0}^{j}, \boldsymbol{n}^{j} \times\left(\boldsymbol{H}_{i}^{j}-\boldsymbol{H}_{e}^{j}\right)=\boldsymbol{n}^{j} \times \boldsymbol{H}_{0}^{j}, j=1,2, \ldots, L  \tag{7}\\
E_{e, u, l}^{j}=-E_{0, u, l}^{j}, u=1.2, \ldots, U, j=1,2, \ldots, L_{u}
\end{gather*}
$$

where $\boldsymbol{n}^{j}$ is the value of the unit normal vector in point $M_{j}$ on the dielectric surface body; $\boldsymbol{E}_{e}^{j}, \boldsymbol{H}_{e}^{j}$ and $\boldsymbol{E}_{i}^{j}, \boldsymbol{H}_{i}^{j}$ are the values of the internal and external field components in point $M_{j} ; \boldsymbol{E}_{0}^{j}, \boldsymbol{H}_{0}^{j}$ are the values of the excitation field components in the same point; $E_{e, u, l}^{j}$ and $E_{0, u, l}^{j}$ are the values of the scattered and excitation field components along the axis of the conductor with a number $u$ in the collocation points on its surface.

Solution of system (7) is determined by minimizing the functional

$$
\begin{gather*}
\Phi=\sum_{j=1}^{L}\left\{\left|\boldsymbol{n}^{j} \times\left(\boldsymbol{E}_{i}^{j}-\boldsymbol{E}_{e}^{j}\right)-\boldsymbol{n}^{j} \times \boldsymbol{E}_{0}^{j}\right|^{2}+\frac{\mu_{e}}{\varepsilon_{e}}\left|\boldsymbol{n}^{j} \times\left(\boldsymbol{H}_{i}^{j}-\boldsymbol{H}_{e}^{j}\right)-\boldsymbol{n}^{j} \times \boldsymbol{H}_{0}^{j}\right|^{2}\right\}  \tag{8}\\
\\
+\sum_{u=1}^{U} \sum_{j=1}^{L_{u}}\left|E_{e, u, l}^{j}+E_{0, u, l}^{j}\right|^{2}
\end{gather*}
$$

After solving the problem of minimization (determining the unknown dipole moments $p_{\tau_{1}}^{n, e}, p_{\tau_{2}}^{n, e}$ $\left(n=1,2, \ldots, N_{e}\right), p_{\tau_{1}}^{n, i}, p_{\tau_{2}}^{n, i}\left(n=1,2, \ldots, N_{i}\right)$ and current elements $\left.J_{u, i}\left(u=1,2, \ldots, U, i=1,2, \ldots, N_{u}\right)\right)$, the necessary scattered field characteristics are determined from (4).

Accuracy control of solving (4), (5) is done by calculating the relative value of the functional (8) on the grid of points intermediate in relation to the collocation points and selected both on the dielectric body surface $S$ and on the surfaces $S_{u}^{\prime}$ of all conductors that are part of the structure:

$$
\begin{equation*}
\Delta=\left(\Phi^{\prime} / \Phi_{0}\right)^{1 / 2}, \Phi_{0}=\sum_{j=1}^{L^{\prime}}\left\{\left|\boldsymbol{n}^{j} \times \boldsymbol{E}_{0}^{j}\right|^{2}+\frac{\mu_{e}}{\varepsilon_{e}}\left|\boldsymbol{n}^{j} \times \boldsymbol{H}_{0}^{j}\right|^{2}\right\}+\sum_{u=1}^{U} \sum_{j=1}^{L_{u}^{\prime}}\left|E_{0, u, l}^{j}\right|^{2} \tag{9}
\end{equation*}
$$

where $\Phi^{\prime}$ is the value of the functional (8) on the set of points indicated above, $\Phi_{0}$ is the value of the corresponding incident field norm on the same set of points, $L^{\prime}$ is the number of intermediate points on the dielectric body surface, $L_{u}^{\prime}$ is the number of intermediate points on the surface of conductor with a number $u$.

The applicability domain of the method chosen for solving the problem is determined by the applicability domain of representations of the fields scattered by thin conductors in the form of fields of axial electric currents. Publication [9] shows that it is advisable to use such representation if the thin conductor radius $k_{e} r_{0}$ does not exceed $0.2\left(r_{0}<0.03 \lambda\right)$. Restrictions on geometrical dimensions of the dielectric body and conductor lengths are determined by the capabilities of the available computer.


Fig. 2. Bistatic scattering cross-sections of the ellipsoid with parameters $k_{e} a=k_{e} b=3$, $k_{e} b=4, \varepsilon_{i} / \varepsilon_{e}=8, \mu_{i} / \mu_{e}=1$ and of the identical ellipsoid with a $k_{e} l=5.65$ long conductor located on the incident field side, curve 1 - single ellipsoid, curve 2 - ellipsoid with a conductor, $\Delta l=0.01 \lambda$, curve $3-$ ellipsoid with a conductor, $\Delta l=0.1 \lambda$.

## 2. NUMERICAL RESULTS

The method described above was implemented in the form of software for calculation of scattered field components and accuracy control of the produced solution. The dielectric body can be either a three-axial ellipsoid or a finite cylinder with elliptical cross-section. The cylinder ends are meant to be the rounded halves of three-axial ellipsoids. The maximum number of thin conductors in the structure envisaged by the software is three. It is assumed that all conductors are straight. The position of conductors in relation to the dielectric body and their lengths are determined by the set coordinates of the initial and final conductor points.

Besides the type of dielectric body geometry and position of conductors, the input values are the geometrical parameters of the dielectric body (in wavelengths), the values of relative dielectric $\varepsilon_{i} / \varepsilon_{e}$ and relative magnetic $\mu_{i} / \mu_{e}$ permeabilities, the excitation field $\left\{\boldsymbol{E}_{0}, \boldsymbol{H}_{0}\right\}$, the similarity parameters $K_{e}$ and $K_{i}$, the number of dipole placement points $N_{e}$ and $N_{i}$, the number of breakdown segments $N_{u}$ of axial currents, as well as the number of collocation points $L$ and $L_{u}$ on the surfaces of the dielectric body and thin conductors. Let us note that the collocation points are placed only on the cylindrical part of the thin conductor; collocation points are not placed on the conductor ends. This means that we neglect the effect of ends on the scattered field. Comparison of the calculation results of bistatic scattering cross-sections produced when placing additional collocation points on the ends to the results produced when placing collocation points only on the cylindrical part of the conductor showed that one can indeed neglect the effect of the ends on this characteristic of the scattered field, if the conductor length exceeds its diameter 10 times and more.

Minimization of the functional (8) is done by the conjugate gradient method. The iteration process is stopped, if the relative change of the functional (8) on each of the ten latest iterations does not exceed $10^{-4}$. This software was used to carry out a series of computational experiments aimed at studying the effect of thin conductors on bistatic scattering cross-section and backscattering cross-section of dielectric bodies. Some results are presented below.

Fig. 2. and 3 characterize the effect of the closely located conductor on bistatic scattering cross-section of the ellipsoid with parameters $k_{e} a=k_{e} b=3, k_{e} c=4, \varepsilon_{i} / \varepsilon_{e}=8, \mu_{i} / \mu_{e}=1$. Conductor length $l=0.9 \lambda \quad\left(k_{e} l=5.65\right)$,


Fig. 3. The same as in Fig. 2, for the case when the conductor is located in the shadow region.
conductor radius $r_{0}=0.02 \lambda\left(k_{e} r_{0}=0.125\right)$. The ellipsoid semi-axes $k_{e} a, k_{e} b, k_{e} c$ are oriented along the axes $x$, $y, z$ respectively; the conductor is oriented along axis $x$ and symmetrically located in relation to the ellipsoid surface. The ellipsoid and the conductor are excited by the plane wave falling along axis $z$, with vector $\boldsymbol{E}_{0}$ oriented along axis $x$. Fig. 2 concerns the case when the conductor is located on the side of the incident excitation wave (before the ellipsoid), and Fig. 3 concerns the case when the conductor is located on the opposite side (in the shadow region). Curves 1 on these figures are the bistatic scattering cross-section of the single ellipsoid; curves 2 are the bistatic scattering cross-sections of the ellipsoid and conductor, when the latter is located at the distance $\Delta l$, equal to $0.01 \lambda$, from the ellipsoid; curves 3 are the same characteristics when the conductor is located at the distance $\Delta l$, equal to $0.1 \lambda$, from the ellipsoid. Bistatic scattering cross-sections are presented in the $E$-plane (plane of vectors $\boldsymbol{k}_{e}$ and $\boldsymbol{E}_{0}$ ); the diagram of scattering in the $E$-plane is symmetrical in relation to axis $z$, that is why the scattering cross-section is presented only in the semi-plane $\varphi=0$. The values of angle $\theta$ are marked on the horizontal axis in Fig. 2 and 3, and the scattering cross-section values normalized over the square wavelength and expressed in dB are marked on the vertical axis.

In accordance with the used method, to present the scattered field one uses auxiliary sources in the form of pairs of tangentially oriented dipoles located on the auxiliary surfaces $S_{e}$ and $S_{i}$ that are also ellipsoids, as well as in the form of current that is continuously distributed along the conductor axis. Positions of auxiliary surfaces are characterized by the following values of similarity parameters: $K_{e}=0.6, K_{i}=4$. Numbers of dipole placement points on the inner and outer auxiliary surfaces are chosen to be the same: $N_{e}=N_{i}=484$. These points are distributed as follows. 22 dipole placement points are selected uniformly over angle $\theta$ in each of the 22 half-sections $\varphi=$ const that are spaced apart at angular distance $\Delta \varphi=16.36^{\circ}$. The number of collocation points $L$ on the ellipsoid surface is chosen equal to 968 . The algorithm of their placement over angle $\theta$ is chosen the same as for the dipole placement points, but these points are located both in half-sections $\varphi=$ const of the dipole placement points and in half-sections drawn in the middle between them. The line of current inside the conductor is split into 35 segments: $N_{u}=35$. The number of cross-sections $x=$ const , in which the collocation points are located on the conductor surface, was also chosen as equal to 35 . In each section four collocation points are placed uniformly over azimuthal angle; $L_{u}=140$.

The results presented in Fig. 2 and 3 allow making the following conclusions. The presence of a thin conductor to the maximum extent affects the scattering cross-sections in the directions related to the back half-space $\left(90^{\circ}<\theta<180^{\circ}\right)$. The conductor located in the shadow region affects the bistatic scattering cross-section of the


Fig. 4. Bistatic scattering cross-sections of the ellipsoid with parameters $k_{e} a=k_{e} b=3$, $k_{e} b=4, \varepsilon_{i} / \varepsilon_{e}=8, \mu_{i} / \mu_{e}=1$ and the same ellipsoid with two conductors located on the incident field side, curve 1 - single ellipsoid, curve 2 - ellipsoid with conductors, $\Delta l=0.01 \lambda$, curve 3 - ellipsoid with conductors, $\Delta l=0.1 \lambda$.
dielectric body to a lesser extent than the identical conductor located at the same distance on the side of the incident excitation wave. This is explained by the fact that in the latter case the currents induced on the conductor by the incident wave are larger than the currents induced on the conductor by the close scattered field of the ellipsoid, when the conductor is in the shadow region. Finally, as comparison of curves $1-3$ shows, the value of the scattering cross-section to a large extent depends on the distance between the ellipsoid surface and the conductor. For instance, (see Fig. 2) when the conductor is located at the distance of $0.01 \lambda$ from the ellipsoid surface, the scattering cross-section in direction $\theta=140^{\circ}$ equals -9 dB , and when this conductor is located at the distance $0.1 \lambda$, it equals -19 dB (decreases by 10 dB$)$.

Fig. 4 and 5 characterize the effect of two closely located conductors on the bistatic scattering cross-section of the same ellipsoid. The lengths and radii of the conductors are chosen as identical and equal, as in the previous case, to $0.9 \lambda$ and $0.02 \lambda$ respectively. The distance between conductors $\Delta l_{1}$ equals $0.1 \lambda$. As in the previous case, conductors are oriented along the axis $x$ and symmetrically located in relation to the ellipsoid surface. The structure is excited by the plane wave falling along axis $z$, with vector $\boldsymbol{E}_{0}$ oriented along axis $x$. Fig. 4 refers to the case when the conductors are located on the incident excitation wave side, and Fig. 5 concerns the case when the conductors are located in the shadow region. Curves 1 on these figures are the bistatic scattering cross-section of the single ellipsoid, curves 2 are the bistatic scattering cross-sections of the ellipsoid and conductors at $\Delta l$ equal to $0.01 \lambda$, curves 3 are the same characteristics at $\Delta l=0.1 \lambda$. In all cases the distance $\Delta l_{1}$ between the conductors was preserved unchanged and equal to $0.1 \lambda$.

As in Fig. 2 and 3, bistatic scattering cross-sections are presented in the $E$-plane. The same similarity parameters of auxiliary surfaces are chosen as in the case of one conductor: $K_{e}=0.6, K_{i}=4$. In order to get the same value of closing error as in the previous case, one had to increase the number of points of dipole placement on the auxiliary surfaces $S_{e}$ and $S_{i}$ and the number of collocation points on the ellipsoid surface: the numbers of dipole placement points $N_{e}$ and $N_{i}$ on $S_{e}$ and $S_{i}$ were chosen as equal to 676, and the number of collocation points $L$ on the ellipsoid surface as equal to 1352 . Points of dipole placement on auxiliary surfaces are distributed as follows.


Fig. 5. The same as in Fig. 4, for the case when the conductors are located in the shadow region.

26 points of dipole placement are selected uniformly over angle $\theta$ in each of the 26 half-sections $\varphi=$ const spaced apart at angular distance $\Delta \varphi=13,85^{\circ}$. The same algorithm of collocation point placement in relation to the dipole placement points was chosen as in the previous case. The number of segments into which the current lines were split inside the conductors, the number and coordinates of sections of the conductors in which the collocation points are located, as well as the number of collocation points in each section were preserved the same as in the previous case.

Analysis of results presented in Fig. 4 and 5 leads to the conclusions that are analogous to those made when considering the effect of one conductor.

## CONCLUSIONS

To sum up, in this paper the method of auxiliary sources is used to solve the problem of electromagnetic wave scattering on the structure consisting of a magnetodielectric body and several thin conductors with finite lengths located close to it. The software implementing this solution is briefly described. The results of numerical computations are presented, characterizing the effect of one or two thin conductors on the scattering cross-section of a magnetodielectric body. It was established that the presence of thin conductors to the maximum extent affects the scattering cross-sections in the directions that belong to the back half-space $\left(90^{\circ}<\theta<180^{\circ}\right)$. It is also established that the conductors located in the shadow region affect the bistatic scattering cross-section of a dielectric body to a smaller extent than identical conductors located at the same distance on the side of the incident excitation wave. The method used to solve the problem can be easily generalized to the structures containing one or more dielectric bodies.

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