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**MULTILAYER MODELLING OF LUBRICATED CONTACTS - A NEW APPROACH
BASED ON A POTENTIAL FIELD DESCRIPTION**

¹Scholle M., ²Gaskell P.H., ^{1,2}Marner F.

¹Heilbronn University, Germany

²Durham University, UK

A first integral approach [1,2], derived in an analogous fashion to Maxwell's use of potential fields [3], is applied for the investigation and minimisation of friction in fluid flow between parallel aligned plates as a model for a lubricated joint, whether naturally occurring or engineered replacements. An idealised system of two-dimensional steady Couette flow is considered as the physical model as shown in Fig. 1: the upper plate translates with speed U while the lower, corrugated, one remains stationary. The region separating the plates is taken to be filled with one or more contiguously contacting, immiscible liquid layers having different dynamic viscosity η ; the case illustrated is for a two-layer system, mimicking the more general case of a joint in which the synovia meets a protective layer exhibiting visco-elastic properties. In the work reported here the associated results are restricted to the simpler case of two Newtonian liquids.

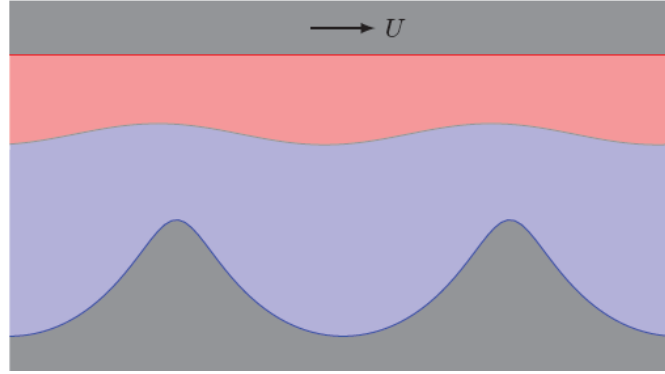


Fig. 1. Schematic of the shear-driven flow generated between two plates, one flat and moving steadily the other stationary, separated by two immiscible liquid layers.

A mathematical model of the resulting steady flow is formulated in terms of a scalar potential Φ , an auxiliary unknown, allowing integration of the governing equations of motion, leading to the following two field equations [2]:

$$\frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial x^2} + 2\eta \frac{\partial^2 \Psi}{\partial x \partial y} = \frac{\rho}{2}(u^2 - v^2), \quad \eta \frac{\partial^2 \Psi}{\partial y^2} - \eta \frac{\partial^2 \Psi}{\partial x^2} - 2 \frac{\partial^2 \Phi}{\partial x \partial y} = \rho uv. \quad (1)$$

These are supplemented by the usual accompanying boundary conditions: no-slip/no-penetration conditions for the velocities u and v at the lower and the upper plate together with periodic boundary conditions at inflow and outflow region to the left and right, respectively. For a two-layer system, additional jump conditions have to be satisfied at the sharp interface $y = f(x)$, between the two, namely:

$$\left[\frac{\partial \Phi}{\partial x} \right] + \frac{\sigma f'(x)}{2\sqrt{1 + f'(x)^2}} = 0, \quad \left[\frac{\partial \Phi}{\partial y} \right] - \frac{\sigma}{2\sqrt{1 + f'(x)^2}} = 0, \quad (2)$$

where σ is the interfacial tension and the double square brackets denote the discontinuity of the associated term. The flow is explored by varying the geometric parameters of the lower contoured plate by considering three models of increasing complexity: a lubrication approximation, Stokes flow and solution of the full Navier-Stokes equations (in first integral form). The latter enables Reynolds number effects to be investigated and, more generally, the validity of the two simpler models to be assessed. In terms of the adopted first integral approach, the standard mathematical form of the jump conditions (2) are advantageous, as is the fact that the resulting friction coefficient can be calculated conveniently from the auxiliary potential field Φ based on the first integral FE formulation used without the need to

approximate velocity derivatives in a post-processing step as would be case if a primitive variable formulation had been adopted.

In the case of a single fluid system, the degenerate case of the two-layer model when both layers have the same properties, is that one possible way to reduce hydrodynamic friction in a system consisting of lubricant liquid confined between two rigid surfaces translating relative to each other is surface contouring allowing for the controlled generation of eddies which act like hydrodynamic roller bearing [3]. In order to illustrate this, the impact of sinusoidal contouring of the lower plate on the friction developed is shown in Fig. 2.

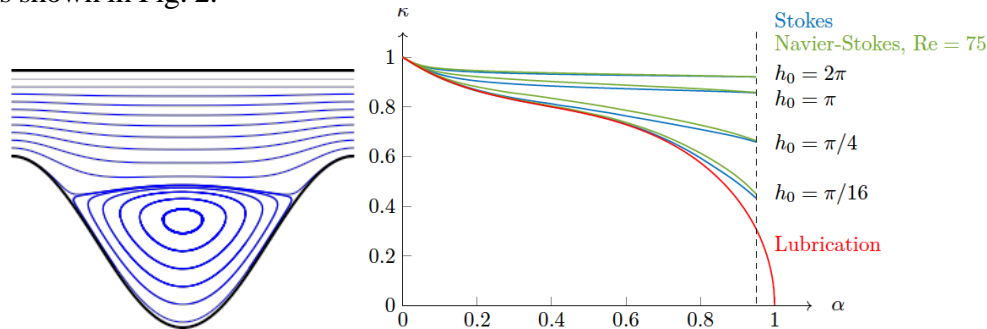


Fig. 2. Single-layer system: geometrically induced eddy (left); resulting friction coefficient κ relative to the same for Couette flow between two flat plates (right). The flow is from left to right and α represents the ratio of the corrugation amplitude to the mean plate separation.

It is found that the lubrication approximation underestimates the friction significantly for large mean plate separation, h_0 , while, for $Re < 100$, the Stokes solution produces a good approximation.

Given that the minimum gap, the distance between the upper moving flat plate and the peak value of the lower corrugated one, is a technical constraint, a comparison of different surface contours – resulting in the same fluid volume between the plates, which is ensured by variation of the amplitude – can be used to reveal a theoretical optimum for a particular castellated shape.

For the case of two layers it will be shown how the friction coefficient is influenced by the mean thickness of the two layers and the respective densities and viscosities of the individual liquid layers. This will include basic considerations about the stability of the flow. Perspectives will be provided as to how the method may be extended and applied to the more challenging case of employing viscoelastic liquids.

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