

Role of QCD compositeness in the production of scalar and tensor mesons through single-photon annihilation $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$

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We study the exclusive production of scalar $S = 0^{++}$ and tensor $T = 2^{++}$ mesons through single-photon annihilation $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$. Using QCD compositeness of the involved hadrons considered as quark-antiquark systems, the prediction for the scaling of the differential cross sections of these processes is $d\sigma/dt \sim 1/s^3$ at large s . We further derive the scaling of the $\gamma^* \rightarrow \gamma S$ and $\gamma^* \rightarrow \gamma T$ transition form factors: $F_{\gamma^*\gamma S}(s) \sim 1/s$ and $F_{\gamma^*\gamma T}(s) \sim 1/s^2$. Results for the respective cross sections of the scalar and tensor meson production are presented. Note, when scalar and tensor mesons are considered as tetraquark systems of two tightly bound color diquarks, corresponding to them transition form factors and differential cross sections have the same falloffs as in case of quark-antiquark picture. For other tetraquark or two-hadron molecule configurations the transition form factors $F_{\gamma^*\gamma S(T)}(s)$ and the differential cross section $d\sigma/dt$ have additional $1/s$ and $1/s^2$ falloffs, respectively.

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I. INTRODUCTION

In this paper we present a study of the exclusive production of scalar $S = 0^{++}$ and tensor $T = 2^{++}$ mesons through single-photon annihilation $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$, which is described by the diagram in Fig. 1. Here p_1, p_2, q, q_1 , and q_2 are the momenta of the initial electron, positron, intermediate photon, final photon, and scalar (tensor) meson, respectively.

The main idea of the paper is to make predictions for the integral cross section of the production of scalar and tensor mesons in the reaction $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$ in whole region of variable $s = q^2$ (i.e., without restriction to small or high values of s). Our strategy is the following. Using the QCD prediction for power scaling of differential cross section of $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$ we constrain the power scaling of the $\gamma^* \rightarrow \gamma S(T)$ transition form factors (denoted by big black vertex in Fig. 1). Explicit form of the form factors can be finally fixed using available data or available results from phenomenological approaches. Finally, we make the numerical analysis of the integral cross sections of $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$ processes. Our predictions are valid in whole region of s and will be useful for planning experiments at electron-positron colliders.

The scaling results for the exclusive cross section $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$ must be consistent with the leading-twist quark fixed-angle counting rules [1–3]: $\frac{d\sigma}{dt}(A + B \rightarrow C + D) \propto F(\theta_{CM})/s^{N-2}$, where $N = N_A + N_B + N_C + N_D$ is the total twist or number of elementary constituents and $F(\theta_{CM})$ is the square of the fixed θ_{CM} -angle amplitude. In our case

when scalar and tensor mesons are considered as $q\bar{q}$ systems we have $N - 2 = 3$ resulting in the scaling behavior $\frac{d\sigma}{dt} \sim 1/s^3$. Alternatively, the scalar and tensor mesons could have tetraquark or two-hadron molecule structure. Application of the QCD compositeness for the tetraquarks has been done in Refs. [4–6]. It was shown that when tetraquarks are systems of two tightly bound color diquarks, the differential cross section has the same falloff as in the case of quark-antiquark picture. For other tetraquark configurations and hadronic molecules the differential cross section $d\sigma/dt$ have additional $1/s^2$ falloff for each exotic state in the final state.

Note that recently, in Ref. [6], QCD compositeness was successfully applied to the study of production of vector mesons—of single and double vector meson production in e^+e^- annihilation. It was shown that both the differential and integral cross sections scale as $1/s$. The reason for this behavior is that the corresponding amplitudes are dominated by the spin $-\frac{1}{2}$ electron exchange in the t and u channels, which gives an extra factor of $s^{\frac{1}{2}}$. The results for the production of vector mesons have been generalized in

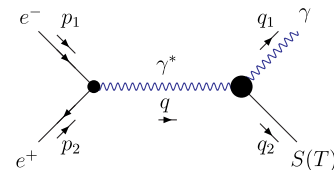


FIG. 1. The single-photon annihilation $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$.

Ref. [6] to the exclusive double-electroweak vector-boson annihilation processes, accompanied by the forward production of hadrons, such as $e^+e^- \rightarrow Z^0V^0$ and $e^+e^- \rightarrow W^-\rho^+$, and the exclusive production of exotic hadrons–tetraquarks. It motivates to continue study of production of other meson states using principle of the QCD compositeness. In particular, in this paper we focus on production of scalar and tensor mesons.

In the case of $\gamma S(T)$ production the dominant diagram is the s channel process $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$. Such processes have been searched for experimentally by the SND Collaboration at the VEPP-2M e^+e^- collider in Ref. [7], and studied in Refs. [8,9]. In particular, in Refs. [8,9] the production of scalar mesons has been studied using the vector dominance model (VDM) and the kaon loop model, while in the case of tensor mesons only the VDM was used in the analysis. It was found that the kaon loop model gives a good description of data up to $\sqrt{s} \simeq 2$ GeV.

We stress that the claim presented in Refs. [8,9] which states that the integral cross section for γS and γT productions should scale as $1/s$ does not appear to be correct since it contradicts the results of QCD compositeness (quark counting rules). The integrated cross section should scale as $1/s^2$, while the differential cross section should scale as $1/s^3$. Our observation is consistent with Ref. [6], where the falloff of the differential cross section for the production of photon and vector meson is $1/(st^2)$, i.e., a generic Mandelstam variable with power -3 . The difference of γV and γS production is clear: in the first case the falloff of the differential cross section is $1/(st^2)$, while in the second case it is $1/s^3$.

As we mentioned above, using the specific falloff of the differential cross section for $\gamma S(T)$ production we can constrain the falloff of the $\gamma^* \rightarrow \gamma S$ and $\gamma^* \rightarrow \gamma T$ transition form factors at large s as $F_{\gamma^*\gamma S}(s) \sim 1/s$ and $F_{\gamma^*\gamma T}(s) \sim 1/s^2$, respectively. Note that the $1/s$ and $1/s^2$ scalings of the $F_{\gamma^*\gamma S}(s)$ and $F_{\gamma^*\gamma T}(s)$ form factors are to be expected, because in the Euclidean region the $\gamma^*(Q^2)\gamma \rightarrow f_0(980)$ and $\gamma^*(Q^2)\gamma \rightarrow f_2(1270)$, $a_2(1320)$ transition form factors have similar power scaling, $1/Q^2$ [10] and $1/Q^4$ [11] at large Q^2 , respectively.

Note that when the scalar mesons $f_0(980)$ and $a_0(980)$ are considered as tetraquarks, there are two scenarios for the quark configuration–hadronic molecular (HM) and color diquark-antidiquark (CD) configuration. In Refs. [4,5] it was shown that for the a system of two tightly bound diquark in the CD scenario the corresponding form factors, involving these states, have the same falloff as form factors involving quark-antiquark systems, otherwise each tetraquark state costs an additional $1/s$ falloff. This model-independent feature was recently confirmed in Ref. [6], where electron-positron annihilation into single and double tetraquarks was considered in a soft-wall AdS/QCD approach. It means that in case of the $f_0(980)$ and $a_0(980)$ states two possible

scenarios for their inner structure–quark-antiquark configuration and tetraquark system with two tightly bound color diquarks we have $1/s$ scaling of the corresponding $\gamma^* \rightarrow \gamma S$ transition form factor at large s , which is consistent with results of Ref. [10]. Therefore, in this paper we consider these scenarios for the $f_0(980)$ and $a_0(980)$ states.

In the following we proceed to set up the effective formalism with the aim of describing the $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S(T)$ annihilation reactions constrained by the scaling behavior for large s . We introduce form factors in these transitions which are minimally parametrized to reproduce the available cross section data and results of other phenomenological approaches. We also give predictions for e^+e^- annihilation reactions of a variety of scalar and tensor mesons.

II. FORMALISM AND RESULTS

Our starting point is the effective Lagrangian for the couplings of scalar (S) and tensor (T) mesons to two photons. In Table I we specify quantum numbers, masses and their two-photon decay widths, which will be used in numerical analysis.

The off-shell behavior of one photon annihilating into γS and γT is parametrized by the form factors $F_{\gamma^*\gamma S}$ and $F_{\gamma^*\gamma T}$ included in the Lagrangians:

$$\begin{aligned} \mathcal{L}_{\gamma^*\gamma S}(x) &= \frac{e^2}{4} g_{S\gamma\gamma} S(x) F_{\mu\nu}(x) \int d^4y \mathcal{F}_{\gamma^*\gamma S}(x-y) F^{\mu\nu}(y), \\ \mathcal{L}_{\gamma^*\gamma T}(x) &= \frac{e^2}{2} g_{T\gamma\gamma} T_{\mu\nu}(x) F^\mu_\alpha(x) \int d^4y \mathcal{F}_{\gamma^*\gamma T}(x-y) F^{\alpha\nu}(y), \\ \mathcal{F}_{\gamma^*\gamma S(T)}(x-y) &= 2F_{\gamma^*\gamma S(T)}(x-y) - \delta^4(x-y), \end{aligned} \quad (1)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the stress tensor of the electromagnetic field, S is the $J^{PC} = 0^{++}$ scalar field, and $T_{\mu\nu}$ is the $J^{PC} = 2^{++}$ tensor field, which is a symmetric ($T_{\mu\nu} = T_{\nu\mu}$), traceless ($T^\mu_\mu = 0$) rank-2 tensor obeying the transversity condition ($\partial^\mu T_{\mu\nu} = 0$) [12]. Here e is the elementary electric charge. The on-shell couplings $g_{S\gamma\gamma}$ and $g_{T\gamma\gamma}$ define the two-photon decay widths of the S and T mesons:

TABLE I. Scalar and tensor mesons.

Name	$I^G(J^{PC})$	Mass (MeV)	$\Gamma(H \rightarrow \gamma\gamma)$ (keV)
$f_0(980)$	$0^+(0^{++})$	990 ± 20	$0.31^{+0.05}_{-0.04}$
$f_0(1370)$	$0^+(0^{++})$	1370^{+130}_{-170}	
$f_0(1500)$	$0^+(0^{++})$	1504 ± 6	
$f_0(1710)$	$0^+(0^{++})$	1723^{+6}_-5	
$a_0(980)$	$1^-(0^{++})$	980 ± 20	0.30 ± 0.10
$a_0(1450)$	$1^-(0^{++})$	1474 ± 19	
$a_0(1950)$	$1^-(0^{++})$	$1931 \pm 14 \pm 22$	
$f_2(1270)$	$0^+(2^{++})$	1275.5 ± 0.8	2.6 ± 0.5
$a_2(1320)$	$1^-(2^{++})$	$1318.3^{+0.5}_{-0.6}$	1.00 ± 0.06

$$\Gamma(S \rightarrow \gamma\gamma) = \frac{\pi}{4} \alpha^2 g_{S\gamma\gamma}^2 M_S^3, \quad \Gamma(T \rightarrow \gamma\gamma) = \frac{\pi}{20} \alpha^2 g_{T\gamma\gamma}^2 M_T^3, \quad (2)$$

where M_S and M_T are the respective meson masses. The relativistic form factors which take into account the off-shellness of the virtual γ^* photon are normalized

$$M(e^+e^- \rightarrow \gamma^* \rightarrow \gamma S) = -e^3 g_{S\gamma\gamma} \bar{v}(p_2) \gamma_\mu u(p_1) \epsilon_\nu^{*\lambda}(q_1) (g^{\mu\nu} q_1 - q^\nu q_1^\mu) \frac{F_{\gamma^*\gamma S}(s)}{s},$$

$$M(e^+e^- \rightarrow \gamma^* \rightarrow \gamma T) = -e^3 g_{T\gamma\gamma} \bar{v}(p_2) \gamma_\alpha u(p_1) \epsilon_\sigma^{*\lambda T}(q_1) \epsilon_{\mu\nu}^{*\lambda T}(q_2) (g_\rho^\alpha q^\mu - g^{\mu\alpha} q_\rho) (g^{\rho\sigma} q_1^\nu - g^{\sigma\nu} q_1^\rho) \frac{F_{\gamma^*\gamma T}(s)}{s}, \quad (3)$$

where $\epsilon_\nu^{*\lambda}$ and $\epsilon_{\mu\nu}^{*\lambda T}$ are the polarization vectors of final photon and tensor meson, respectively. The corresponding differential cross sections are derived as

$$\frac{d\sigma}{dt}(e^+e^- \rightarrow \gamma S) = \frac{4\pi\alpha\Gamma(S \rightarrow \gamma\gamma)}{M_S^3} \frac{|F_{\gamma^*\gamma S}(s)|^2}{s} \left[1 + \frac{2t}{s} - \frac{2M_S^2}{s} + \frac{2t^2}{s^2} - \frac{2M_S^2 t}{s^2} + \frac{M_S^4}{s^2} \right],$$

$$\frac{d\sigma}{dt}(e^+e^- \rightarrow \gamma T) = \frac{10\pi\alpha\Gamma(T \rightarrow \gamma\gamma)}{3M_T^3} \frac{|F_{\gamma^*\gamma T}(s)|^2 s}{M_T^4} \left[1 + \frac{2t}{s} - \frac{2M_T^2}{s} + \frac{2t^2}{s^2} + \frac{7M_T^4}{s^2} - \frac{14tM_T^2}{s^2} - \frac{12t^2M_T^2}{s^3} + \frac{24tM_T^4}{s^3} - \frac{12M_T^6}{s^3} + \frac{12t^2M_T^4}{s^4} - \frac{12tM_T^6}{s^4} + \frac{6M_T^8}{s^4} \right]. \quad (4)$$

As dictated by the QCD compositeness, the $1/s^3$ scaling of the differential cross sections for the production of scalar and tensor mesons as quark-antiquark systems requires the following scaling of the $F_{\gamma^*\gamma S}(s)$ and $F_{\gamma^*\gamma T}(s)$ transition form factors at large s

$$F_{\gamma^*\gamma S}(s) \sim \frac{1}{s}, \quad F_{\gamma^*\gamma T}(s) \sim \frac{1}{s^2}. \quad (5)$$

These results are hold when scalar and tensor mesons are systems of two tightly bound diquark in a possible CD tetraquark scenario, while for other tetraquarks we get extra falloffs $1/s$ and $1/s^2$ for $F_{\gamma^*\gamma S}(s)$ and $F_{\gamma^*\gamma T}(s)$, respectively. The integral cross sections are in agreement with Refs. [8,9] and are given by

$$\sigma(e^+e^- \rightarrow \gamma S) = \frac{8\pi\alpha\Gamma(S \rightarrow \gamma\gamma)}{3M_S^3} |F_{\gamma^*\gamma S}(s)|^2 \left(1 - \frac{M_S^2}{s} \right)^3,$$

$$\sigma(e^+e^- \rightarrow \gamma T) = \frac{20\pi\alpha\Gamma(T \rightarrow \gamma\gamma)}{9M_T^3} \left| \frac{s}{M_T^2} F_{\gamma^*\gamma T}(s) \right|^2 \times \left(1 - \frac{M_T^2}{s} \right)^3 \left[1 + \frac{3M_T^2}{s} + \frac{6M_T^4}{s^2} \right]. \quad (6)$$

The total cross sections scale as $1/s^2$ when the corresponding form factors scale according to (5). In particular, the asymptotic expressions for integral cross sections at leading order in $1/s$ expansion for scalar and tensor mesons read

to 1 at $s = 0$. Note that the idea to introduce a form factor for the $\gamma^* \rightarrow \gamma T$ transition was originally proposed in Ref. [9].

The invariant matrix elements describing the $e^+e^- \rightarrow \gamma^* \rightarrow \gamma S$ and $e^+e^- \rightarrow \gamma^* \rightarrow \gamma T$ annihilation processes are given by

$$\sigma_{\text{Asymp}}(e^+e^- \rightarrow \gamma S) = \frac{8\pi\alpha\Gamma(S \rightarrow \gamma\gamma)}{3M_S^3} \frac{\Lambda_S^4}{s^2}, \quad (7)$$

$$\sigma_{\text{Asymp}}(e^+e^- \rightarrow \gamma T) = \frac{20\pi\alpha\Gamma(T \rightarrow \gamma\gamma)}{9M_T^3} \frac{\Lambda_T^4}{s^2}. \quad (8)$$

First, we focus on extraction of transition form factors for scalar and tensor mesons. In case of scalar mesons, using production data on $f_0(980)$ and $a_0(980)$ we deduce the behavior of the $F_{\gamma^*\gamma f_0(980)}(s)$ and $F_{\gamma^*\gamma a_0(980)}(s)$ form factors depending on the variable s (total energy squared). For two scenarios—scalar mesons $f_0(980)$ and $a_0(980)$ are quark-antiquark systems and tetraquark as a system of two tightly bound color diquarks we use the double-pole expression for these form factors, which has 3 free parameters—scale parameter Λ and two dimensionless parameters a and b , fixed from a fit to the data:

$$F_{\gamma^*\gamma S}(s) = \frac{1 + a\hat{s}}{1 - b\hat{s} + a\hat{s}^2}, \quad \hat{s} = \frac{s}{\Lambda^2}. \quad (9)$$

The $F_{\gamma^*\gamma S}(s)$ form factor is normalized to 1 at $s = 0$ and displays the $1/s$ scaling for large s , as required by the QCD compositeness, with

$$F_{\gamma^*\gamma S}(s) \sim \frac{\Lambda^2}{s}. \quad (10)$$

Therefore, the value of the scale parameter Λ must be fixed from the asymptotic behavior of the cross sections for γf_0 and γa_0 production. Using the available data from Ref. [8], we fix the values of Λ for γf_0 and γa_0 production as

$$\Lambda_{f_0} = 350 \pm 50 \text{ MeV}, \quad \Lambda_{a_0} = 250 \pm 50 \text{ MeV}. \quad (11)$$

In this case the parameters a and b are fixed as

$$a = 0.011_{-0.005}^{+0.008}, \quad b = 0.190_{-0.052}^{+0.063} \quad (12)$$

for the case of the f_0 form factor, while we have

$$a = 0.003_{-0.002}^{+0.003}, \quad b = 0.090_{-0.033}^{+0.042} \quad (13)$$

for the case of the a_0 . For the ratio of the $e^+e^- \rightarrow \gamma f_0(980)$ and $e^+e^- \rightarrow \gamma a_0(980)$ cross sections at large s we get

$$\begin{aligned} \frac{\sigma(e^+e^- \rightarrow \gamma f_0(980))}{\sigma(e^+e^- \rightarrow \gamma a_0(980))} &= \left(\frac{\Lambda_{f_0}}{\Lambda_{a_0}}\right)^4 \left(\frac{M_{a_0}}{M_{f_0}}\right)^3 \frac{\Gamma(f_0 \rightarrow \gamma\gamma)}{\Gamma(a_0 \rightarrow \gamma\gamma)} \\ &\simeq \left(\frac{\Lambda_{f_0}}{\Lambda_{a_0}}\right)^4. \end{aligned} \quad (14)$$

Plots of the form factors $F_{\gamma^*\gamma f_0(980)}(s)$ and $F_{\gamma^*\gamma a_0(980)}(s)$ are shown in Fig. 2. In case of tensor mesons we have only results of phenomenological consideration in Ref. [8]. We use the predictions of Ref. [8] for integral cross sections of production of tensor mesons to constraint the $\gamma^* \rightarrow \gamma T$ transition form factors. Taking into account the $1/s^2$ scaling of these form factors at large s we use for them the following parametrization

$$F_{\gamma^*\gamma T}(s) = \frac{1}{1 + \hat{s}} \frac{1 + a\hat{s}}{1 - b\hat{s} + a\hat{s}^2}, \quad \hat{s} = \frac{s}{\Lambda^2}. \quad (15)$$

In case of the specific tensor meson states $f_2(1270)$ and $a_2(1320)$ the parameters a, b and Λ are fixed as

$$a = 0.241, \quad b = 0.771, \quad \Lambda = 950 \text{ MeV} \quad (16)$$

for the case of the $f_2(1270)$ form factor, while we have

$$a = 0.310, \quad b = 0.964, \quad \Lambda = 1000 \text{ MeV} \quad (17)$$

for the case of the $a_2(1320)$. The plots of the $\gamma^* \rightarrow \gamma f_2(1270)$ and $\gamma^* \rightarrow \gamma a_2(1320)$ transition form factors are shown in Fig. 3.

In Fig. 4 we show our results for the integral cross sections $\sigma(e^+e^- \rightarrow \gamma f_0(980))$ and $\sigma(e^+e^- \rightarrow \gamma a_0(980))$ and compare them with data points of the OLYA [13] and DM2 [14] collaborations extracted in Ref. [8]. Also we show the curves for asymptotical cross sections [see Eq. (7)]. One can see that since values $\sqrt{s} \simeq 2.4 \text{ GeV}$ our exact results for integral cross sections in case of the $f_0(980)$ and $a_0(980)$ coincide with asymptotical ones.

In Figs. 5 and 6 we present our results for other isoscalar [f_0 -family: $f_0(1370)$, $f_0(1500)$, $f_0(1710)$] and isovector [a_0 -family: $a_0(980)$, $a_0(1450)$, $a_0(1950)$] states. In case of f_0 family we present the results the ratio of the cross section and two-photon decay width $\Gamma(f_0 \rightarrow \gamma\gamma)$ because there are no data for the $\Gamma(f_0 \rightarrow \gamma\gamma)$. Note that the form factors for the f_0 and a_0 families are chosen as for the case of

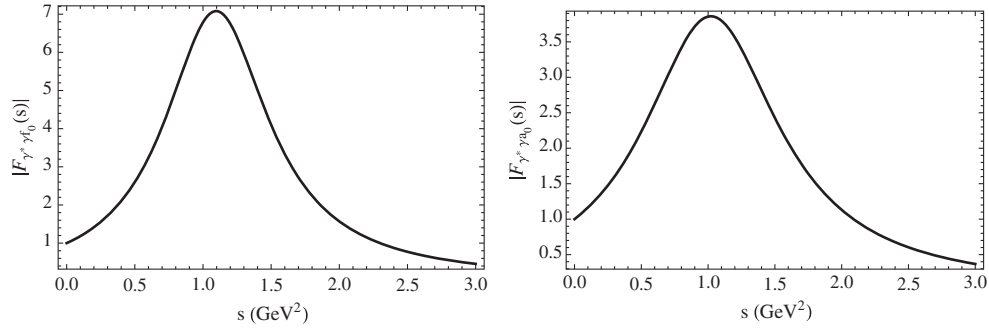


FIG. 2. Form factors $F_{\gamma^*\gamma f_0(980)}(s)$ and $F_{\gamma^*\gamma a_0(980)}(s)$ at $\Lambda_{f_0} = 350 \text{ MeV}$ and $\Lambda_{a_0} = 250 \text{ MeV}$.

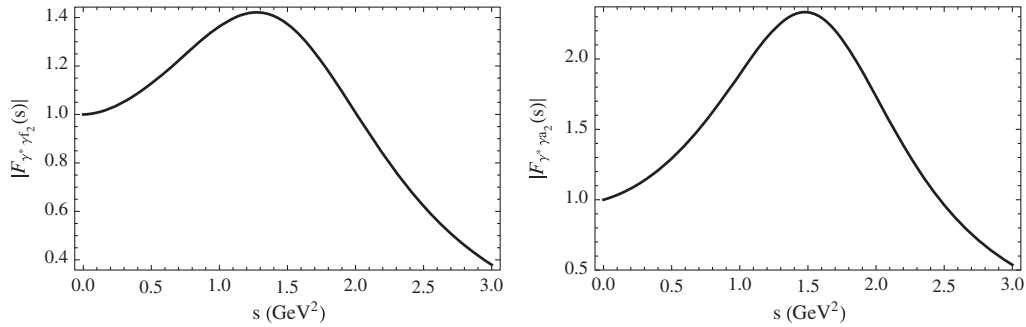


FIG. 3. Form factors $F_{\gamma^*\gamma f_2(1270)}(s)$ and $F_{\gamma^*\gamma a_2(1320)}(s)$ at $\Lambda_{f_2} = 950 \text{ MeV}$ and $\Lambda_{a_2} = 1000 \text{ MeV}$.

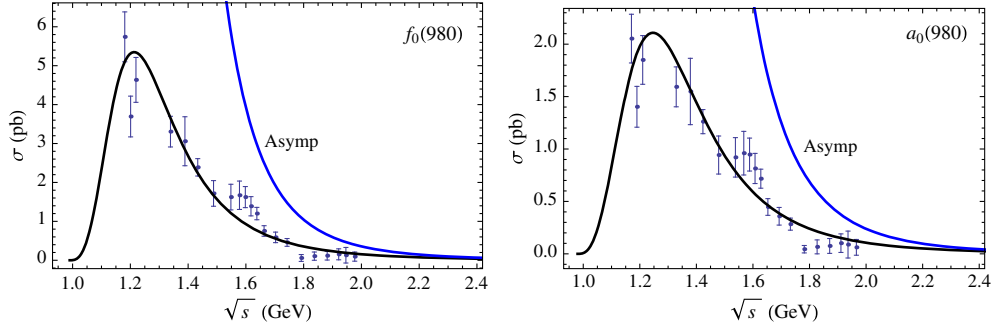


FIG. 4. $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma f_0(980))$ and $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma a_0(980))$ at $\Lambda_{f_0} = 350$ MeV and $\Lambda_{a_0} = 250$ MeV. Data are taken from Ref. [8].

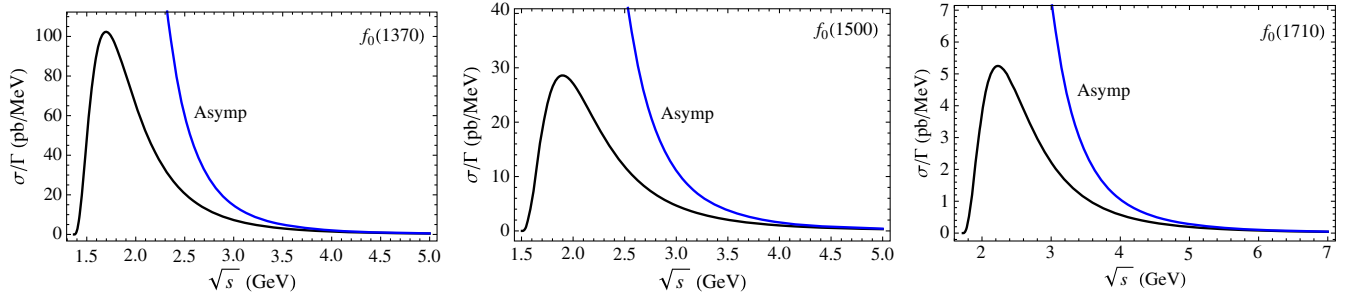


FIG. 5. The integral cross section divided by two-photon decay width $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma f_0)/\Gamma(f_0 \rightarrow \gamma\gamma)$ for $f_0 = f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ with $\Lambda_{f_0} = 350$ MeV.

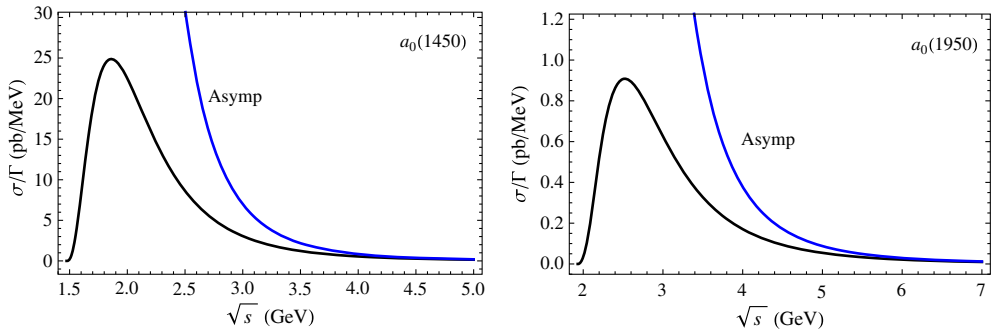


FIG. 6. The integral cross section divided by two-photon decay width $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma a_0)/\Gamma(a_0 \rightarrow \gamma\gamma)$ for $a_0 = a_0(1450)$ and $a_0(1950)$ with $\Lambda_{a_0} = 250$ MeV.

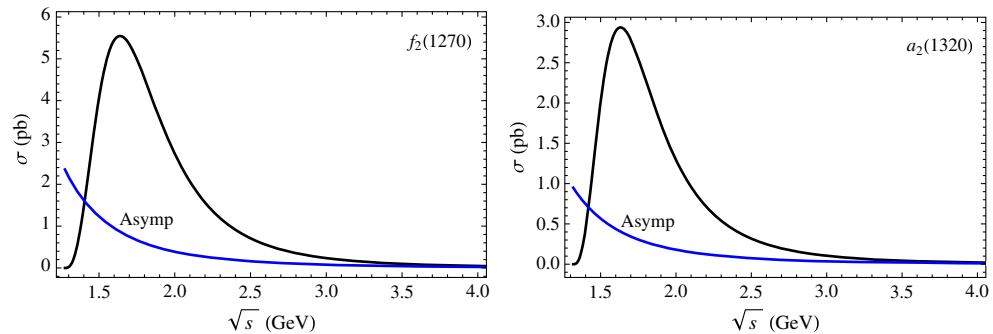


FIG. 7. $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma f_2(1270))$ and $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma a_2(1320))$ at $\Lambda_{f_2} = 950$ MeV and $\Lambda_{a_2} = 1000$ MeV.

the $f_0(980)$ and $a_0(980)$, respectively, otherwise assuming the same QCD compositeness for all the scalar state. One can see that perturbative regime there asymptotical cross sections coincide with exact calculation starts approximately from $\sqrt{s} \simeq 4$ GeV in case of the $f_0(1370)$, $f_0(1500)$, and $a_0(1450)$ states and from $\sqrt{s} \simeq 5$ GeV and $\sqrt{s} \simeq 5.5$ GeV in case of the $f_0(1710)$ and $a_0(1950)$ states, respectively.

Finally, in Fig. 7 we present our predictions for the integral cross sections of tensor mesons. For a comparison we present results for asymptotic cross sections given by Eq. (7). One can see that for tensor meson the perturbative regime starts from $\sqrt{s} \simeq 3.5$ GeV.

In conclusion, we summarize the main results of the paper. Using quark counting rules for the differential cross section of the scalar and tensor mesons in single-photon annihilation processes we constrained the scaling of the $\gamma^* \rightarrow \gamma S$ and $\gamma^* \rightarrow \gamma T$ transition form factors. We showed that in case of quark-antiquark picture or tetraquark system of two tightly bound color diquarks they should scale as $1/s$ and $1/s^2$ for large s , respectively. In case of other tetraquark configurations and two-hadron molecule configurations according to counting of the constituents (this result is known from Refs. [4–6]) they get extra $1/s$ falloff. Restricting to the first choice (quark-antiquark picture or tetraquark system of two tightly bound color diquarks) we extracted from data the behavior of these form factors with respect to the s variable, using a double-pole formula with

3 free parameters. Using the obtained form factors for $f_0(980)$ and $a_0(980)$ we predict the integral cross sections for the members of their families—isoscalar [$f_0(1370)$, $f_0(1500)$, $f_0(1710)$] and isovector [$a_0(1450)$, $a_0(1950)$] states. Here we assumed that the transition form factors do not depend on the mass of the corresponding scalar meson. Similar analysis done for the tensor states $f_2(1270)$ and $a_2(1320)$. We think that our predictions for the transition form factors and integral cross sections of scalar and tensor mesons will be useful for future experiments at electron-positron colliders.

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