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Electroproduction of lightest nucleon resonances up to Q*2=12 GeV*2 in quark models at light front

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Abstract. The lightest nucleon resonances are described at light front as mixed states of the $3q$ cluster ("quark core") possessing a definite value of the inner orbital momentum $L = 0{,}1$ and a hadron molecular state, $N+\sigma$ or $\Lambda+K$. Helicity amplitudes of the resonance electroproduction off the proton are calculated at large Q^2 up to 12 GeV^2 and compared to the last CLAS data. At this basis we have estimated the probability of quark core in lightest nucleon resonances and predicted the high Q^2 behaviour of the resonance electrocoupling.

1 Introduction

The last decade has been marked by a significant progress in the experimental study of low-lying baryon resonances. Specifically new insights have been obtained in π and 2π electroproduction off the proton with the polarized electron beam at JLab (CLAS/CLAS12 Collaborations, see, e.g., recent overviews [1, 2]). In the context of projected extensive studies of baryons with $J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$, etc., there is an interest in calculation of electrocouplings of baryons at large Q^2.

There are many theoretical approaches to the problem, which start from the first principles, Lattice QCD, Dyson-Schwinger (DS)E or Bethe-Salpeter (BS) equations, Light-front QCD, AdS/QCD etc., but some rough estimates could be made on the basis of a light-front quark model. It implies the construction of a good basis of light-front quark configurations possessing a definite value of the orbital angular momentum (L) and satisfying the Pauli exclusion principle. Our approach is to fit parameters of light-front quark configurations to the elastic nucleon form factors and use them to construct a correct basis for excited ($L = 0{,}1$) nucleon states to calculate the transition form factors at large Q^2 up to 12 GeV^2.

Light-front quark wave functions were successfully used by many authors for description of nucleon form factors and transition amplitudes as before appearance of the polarized electron data [3–5] as well as after these (taking into account new high-quality data) [6–9]. However these works used the data up to $Q^2 \lesssim$ 3-4 GeV^2 which were only available. Now CLAS12 plans measurements up to 12

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GeV^2 that initiates the theoretical study of the resonance electroproduction at higher Q^2 in terms of relativistic approaches.

In our recent work [10] we have generalized our earlier non-relativistic approach to the Roper resonance electroproduction at $Q^2 < 4\,\mathrm{GeV}^2$ [11] by going to higher Q^2 in terms of light-front quark configurations. Solving this problem we have run into another difficulty: the contribution of excited relativistic quark configurations (at least for $L = 0{,}1$) overestimates the transition amplitudes $N+\gamma^* \rightarrow N^*$, while the elastic $N + \gamma^* \rightarrow N$ form factors calculated at the same basis are in a good agreement with the data. It follows from these results that, possibly, other (softer) degrees of freedom should be taken into consideration for excited states along with the quark core. Then at high Q^2 the contribution of such soft components should be quickly dying out, and only the quark core contribution will survive with a relatively small weight. Here we consider this possibility.

2 Nucleon and Roper resonance wave functions at light front

In the case of the Roper resonance the solution of the problem is almost evident: its inner structure cannot be adequately described in terms of constituent quark degrees of freedom only. As a result the quark model fails to explain the observed mass and decay width of the Roper resonance. Starting from this we have considered the Roper resonance $R = N_{1/2^+}(1440)$ as a mixed state of the radially excited quark configuration $3q^* = sp^2[3]_X$ and the “hadron molecule” (a loosely bound state of the nucleon and σ meson $(N\sigma)_{mol} = |N + \sigma\rangle$):

$$R = \cos\theta\,|3q^*\rangle + \sin\theta\,|N + \sigma\rangle \tag{1}$$

The parameter θ was adjusted to optimize the description of the helicity amplitude $A_{1/2}$ only. We found that at the value of $\cos\theta$ =0.7 - 0.8 this model correlates well with the recent CLAS data on the both $A_{1/2}$ and $S_{1/2}$ helicity amplitudes [11].

However, in [11] we have used non-relativistic h.o. quark configurations (i.e. the Gaussians) for the quark core of baryons, and thus such calculations would be senseless at $Q^2 \gtrsim$ 3-4 GeV^2. At high Q^2 the contribution of soft components of the baryon (the meson cloud, “molecular” admixtures, etc.) to transition form factors falls off in comparison to the quark core contribution. Hence only the quark contribution should be considered at high Q^2.

Unfortunately, the form factors defined with a Gaussian as a quark core wave function (with the scale parameter normalized on the nucleon radius) are also quickly dying out at $Q^2 \gtrsim$ 3-4 GeV^2. Possible alternatives to the Gaussian wave function have been considered in several works: a superposition of many Gaussians [6], a pole-like w.f. [3] and a model with the running quark mass [7]. We have chosen a pole-like form of the w.f. The pole-like form of the nucleon ground state wave function Φ_{0S}

$$\Phi_{0S}(\xi, \eta, k_\perp, K_\perp) = \frac{\mathcal{N}_{0S}}{(1 + \mathcal{M}_0^2/\beta^2)^\gamma}, \tag{2}$$

where $\mathcal{M}_0^2 = \frac{M^2+k_\perp^2}{\eta\xi(1-\xi)} + \frac{\eta M^2+K_\perp^2}{\eta(1-\eta)}$, was firstly fitted to the elastic nucleon form factors by Schlumpf[3] with γ =3.5 and $\beta \approx$2M. Here $\boldsymbol{k}$, $\boldsymbol{K}$ are (relativistic) relative moments in quark pair "1-2” and in $2q$ cluster – third quark pair "(12)-3” respectively; the $\xi, \eta, \boldsymbol{k}_\perp, \boldsymbol{K}_\perp$ are the light-front variables with $x_1 = \xi\eta$, $x_2 = (1-\xi)\eta$, $x_3 = 1-\eta$, and $\mathcal{M}_0$ is the mass of free $3q$ system (M is the constituent quark mass).

Such form of Φ_{0S} is as yet unjustified, but it should be noticed that at least in the meson sector the pole-like form of the pion $\bar{q}q$ w.f. $f_\pi\varphi_\pi(x, k_\perp^2) = \frac{9}{4\pi^2}\left(1 + \frac{k_\perp^2}{4M^2x(1-x)}\right)^{-\kappa}$ with κ =1 was recently reconstructed [12] starting from the Bethe-Salpeter wave function projected onto the light front (there are also approximations with κ =1 - 2). The nucleon pole-like w.f. $\Phi_{0S}(\xi, \eta, k_\perp, K_\perp)$ looks like a generalization of the pion $\bar{q}q$ w.f. for the case of $3q$ system.

Starting from the "+" component of the quark current on light front

$$I^+ = \sum_{j=1}^{3} I_j^+, \quad I_j^+ = e_{qj}\left(f_{1j} + i\,\hat{n}_z \cdot [\boldsymbol{\sigma}_j \times \boldsymbol{q}_\perp] f_{2j}\right) \tag{3}$$

(without quark form factors f_i, but with a small anomalous quark magnetic moment $\varkappa_q$, i.e. $f_1 = 1$, $f_2 = \varkappa_q$), we have fitted free parameters of the model to the modern data on nucleon form factors within the full measured range $0 \le Q^2 \le 32$ GeV^2 including the electron polarization data on the ratio G_E/G_M at $Q^2 \lesssim$ 6-8 GeV^2. With the values γ =3.51, M =0.251 GeV, $\varkappa_u$ =-0.0028, $\varkappa_d$ =0.0224, β_u =0.579 GeV, β_d =0.5 GeV we have obtained quiet good description [10] of all the nucleon data including the static electromagnetic properties of nucleons.

For the Roper resonance we used a light-front analogue of the radially excited quark configuration

$$\Phi_{2S} = \mathcal{N}_{2S}\left(1 - c_R \frac{\mathcal{M}_0^2}{\beta^2}\right)\Phi_{0S}, \qquad \langle \Phi_{2S}|\Phi_{0S}\rangle = 0 \tag{4}$$

and for the negative parity baryons, e.g. $N_{1/2^-}(1535)$, we used

$$\Phi_{1P} = \mathcal{N}_{1P}|\boldsymbol{K}|\Phi_{0S} \quad \text{and} \quad \mathcal{N}_{1P}|\boldsymbol{k}|\Phi_{0S} \tag{5}$$

After comparison with the data it turns out that the quark-core weight (cosθ) in the mixed model (1) should be reduced from cos$\theta \simeq$ 0.7-0.8 of the nonrelativistic (Gaussian) model to the value of cos$\theta \lesssim$ 0.5 for the relativistic (pole-like) model. Moreover the transition form factors for negative parity nucleon resonances (at least for $L = 1$, next Section) behave at large Q^2 similarly to the Roper one (see figure 1). At the same time the nucleon elastic form factors were successfully described without reducing the quark core weight.

3 High Q^2. Quark configurations and Melosh transformations

A good basis of relativistic quark configurations possessing the definite value of orbital momentum L and satisfying the Pauli exclusion principle is needed for baryon resonances with J^P = $1/2^\pm$, $3/2^\pm$, $5/2^\pm$. We start from non-relativistic shell-model configurations and change the h.o. wave functions by the light-front ones (Gaussian or pole-like) dependent on the relativistic relative moments $\boldsymbol{k}$**,** $\boldsymbol{K}$ and expressed in light-front invariants ξ, η, $\boldsymbol{\lambda}_\perp$, $\boldsymbol{\Lambda}_\perp$. At this stage, as usual, there are problems with boosts (in the instant form of dynamics) or rotations (at the light front). In both cases generators of transformations depend on the dynamics, and thus such shell-model basis is only useful for transformations of kinematic subgroups of the full Lorentz group. Nevertheless some difficulties can be resolved going to the rest frame for definition of the total angular momentum $J = L + S$ or partial one $j = l + s$ for each two-body subsystem of the $3q$ bound state and going to the Breit frame for description of the ep collision. However such technique could only be considered as a reasonable approximation to "true" dynamical calculations.

Recall that the spin $\vec{s}_i$ of the particle is uniquely determined in its own rest frame, where $p_i^\nu = \overset{\circ}{p}{}^\nu{}_i \equiv \{M, 0, 0, 0\}$. In the moving frame the canonical spin state is determined by a rotationless Lorentz boost $\lambda(\boldsymbol{p}_i \leftarrow \overset{\circ}{p}_i)$ (we use notations of [13, 14]): $|\boldsymbol{p}_i; s_i\mu_i\rangle_c = U(\lambda(\boldsymbol{p}_i \leftarrow \overset{\circ}{p}_i))|\,\overset{\circ}{p}_i; s_i\mu_i\rangle$, while the light-front spin state is determined by another type of Lorentz transformation, which leads to the same momentum p_i^ν, but would be represented as a two-step process [13]: $l(\boldsymbol{p}_i \leftarrow \overset{\circ}{p}_i) = \lambda(\boldsymbol{p}_i \leftarrow \boldsymbol{p}_\infty)\lambda(\boldsymbol{p}_\infty \leftarrow \overset{\circ}{p}_i)$ where $\boldsymbol{p}_\infty = p_\infty \hat{\boldsymbol{n}}_z$ is the quark momentum at the infinite momentum frame, $|\boldsymbol{p}_i; s_i\mu_i\rangle_f = U(l(\boldsymbol{p}_i \leftarrow \overset{\circ}{p}_i))|\ \overset{\circ}{p}_i; s_i\mu_i\rangle$ [the full manifold of l transformations forms a subgroup of the Lorentz group, which is a kinematic subgroup for the light-front dynamics, — an analog of the rotational subgroup used in the instant form]. As a result, canonical and front states are related by a specific rotation

$$|\boldsymbol{p}_i, s_i\mu_i\rangle_c = \sum_{\mu_i'} |\boldsymbol{p}_i, s_i\mu_i'\rangle_f\, D^{s_i}_{\mu_i'\mu_i}[R(\boldsymbol{p}_i, M_i)], \quad R(\boldsymbol{p}_i, M_i) = \lambda(\overset{\circ}{p}_i \leftarrow p_i)\lambda(\boldsymbol{p}_i \leftarrow \boldsymbol{p}_\infty)\lambda(\boldsymbol{p}_\infty \leftarrow \overset{\circ}{p}_i), \qquad (6)$$

which is known as the Melosh transformation. For the quark ($s_i = 1/2$) one can write: $D^{\frac{1}{2}}_{\mu_i'\mu_i}[R^{-1}(\boldsymbol{p}_i, M_i)] = \frac{M_i + p_i^+ + i\hat{\boldsymbol{n}}_z\cdot[\sigma\times \boldsymbol{p}_{i\perp}]}{\sqrt{(M+p_i^+)^2 + p_{i\perp}^2}}$. The Lorentz transformation $l(\boldsymbol{p}_i' \leftarrow \boldsymbol{p}_i)$ does not rotate the front spin that is very convenient for description of the states in moving reference frames. But the front basis is not convenient for construction of rotationally covariant states. For the quark pair "12" a rotationally covariant state can be constructed in the proper rest frame of the pair (cm), where $(p_1+p_2)^\nu_{cm} \equiv \mathcal{P}^\nu_{12cm} = \overset{\circ}{\mathcal{P}}{}^\nu{}_{12} = \{\mathcal{M}_{0(12)}, 0, 0, 0\}$:

$$|\overset{\circ}{\mathcal{P}}_{12}, j_{12}j_{12}^z(ls_{12})\rangle = \sum_{\mu_1\mu_2\mu_{12}m} (s_1\mu_1 s_2\mu_2|s_{12}\mu_{12})(lms_{12}\mu_{12}|j_{12}j_{12}^z) \int d^2\hat{k} Y_{l\mu_l}(\hat{\boldsymbol{k}})|\boldsymbol{p}_{1cm}\mu_1\rangle_c|\boldsymbol{p}_{2cm}\mu_2\rangle_c \qquad (7)$$

It is well known [13–15] that in the case of free particles the Lorentz boost does not destroy this state. The right side of (7) is transforming as the state of an elementary particle with total spin j_{12} and mass $\mathcal{M}_{0(12)} = \sqrt{\frac{M^2+k_\perp^2}{\xi(1-\xi)}}$. Hence, to construct a $3q$ state with the total angular momentum J, we should go to the center-of-mass frame of $3q$ system (CM) and use the same formula as (7) for another "two-body state": cluster 12+quark 3. This can be realized by substitution of responding CM moments, $\mathcal{P}_{12CM}$ and $\boldsymbol{p}_{3CM}$, to (7): $\boldsymbol{p}_{1cm} \to \boldsymbol{p}_{3CM}$, $\boldsymbol{p}_{2cm} \to \mathcal{P}_{12CM}$, $Y_{l\mu_l}(\hat{\boldsymbol{k}}) \to Y_{L\mu_L}(\hat{\boldsymbol{K}})$, where $\boldsymbol{p}_{3CM} = \boldsymbol{K}$, $\mathcal{P}_{12CM} = -\boldsymbol{K}$, $\boldsymbol{p}_{1cm} = \boldsymbol{k}$, $\boldsymbol{p}_{2cm} = -\boldsymbol{k}$.

Using the light-front boosts for the transition from frame $cm(12)$ to frame CM(12-3) (and further from frame CM(12-3) to the Breit frame) implies several Melosh rotations which change the wave functions of initial and final states. Fortunately the Melosh rotation of all spins does not violate the Pauli exclusion principle for the full $3q$ wave function (see, e.g., a discussion in [10]), though the calculation technique (fraction parentage coefficients, etc.) becomes more cumbersome, especially in the case of nonzero orbital moments L and l.

4 Wave functions and matrix elements

In the long run we have obtained the expressions for spin-orbital parts of searched wave functions $\Psi^{(0)}_{J(LS(s_{12}))\mu_J}$ (for $l=L=0$), $\Psi^{(1)}_{J(LS(s_{12}))\mu_J}$ (for $L=1$, $l=0$) and $\Psi^{(2)}_{J(lS(s_{12}))\mu_J}$ (for $L=0$, $l=1$) used in the matrix element of current (3) for the third quark in the transition $N \to N^*$,

$$\langle \Psi^{(1)}_{J'(L'S'(s_{12}'))\mu_J'}|\hat{I}_3^+|\Psi^{(0)}_{J(LS(s_{12}))\mu_J}\rangle = \delta(\mathcal{P}_B' - \mathcal{P}_B - \boldsymbol{q}_\perp)\sum_{\{\mu_i\}} e_{q_3}\left(f_1\delta_{\mu_3'\mu_3} + f_2 q_\perp(-1)^{1/2-\mu_3}\delta_{-\mu_3'\mu_3}\right)$$

$$\times \int \frac{d^2\lambda_\perp d^2\Lambda_\perp d\xi d\eta}{\xi(1-\xi)\eta(1-\eta)} \Psi^{(1)}_{J'(L'S'(s_{12}'))\mu_J'}(\mathcal{P}_B', \boldsymbol{K}', \boldsymbol{k}'; \mu_1, \mu_2, \mu_3')\Psi^{(0)}_{J(LS(s_{12}))\mu_J}(\mathcal{P}_B, \boldsymbol{K}, \boldsymbol{k}; \mu_1, \mu_2, \mu_3) \qquad (8)$$

Here $\Psi^{(0)}$ is the nucleon w.f. , $\Psi^{(1)}$ and $\Psi^{(2)}$ are wave functions of a $N^*_{J'^-}$ resonance with orbital momentum $L' = 1$ and $J' = 1/2$ or $3/2$. Superscripts 1 and 2 denote the type of permutational symmetry in the coordinate space for the first two quarks (1 for the symmetric and 2 for the antisymmetric state, i.e. Yamanouchi symbols 112 and 121 respectively). $\boldsymbol{\mathcal{P}}_B$ and $\boldsymbol{\mathcal{P}}'_B$ are moments of initial nucleon and outgoing resonance in the Breit frame respectively, $\boldsymbol{\mathcal{P}}_B = -\alpha \boldsymbol{q}_\perp/2$, $\boldsymbol{\mathcal{P}}'_B = (1-\alpha)\boldsymbol{q}_\perp/2$, $\alpha = \frac{q_\perp^2 + M_{N^*}^2 - M_N^2}{2q_\perp}$, $q_B^\nu = \{0, \boldsymbol{q}_\perp, 0\}$, $Q^2 = \boldsymbol{q}_\perp^2$.

$$\begin{aligned}
\Psi^{(1)}_{J(LS(s_{12}))\mu_J}(\boldsymbol{\mathcal{P}}, \boldsymbol{K}, \boldsymbol{k}; \mu_1, \mu_2, \mu_3) &= {}_f\langle \boldsymbol{p}_1\mu_1, \boldsymbol{p}_2\mu_2, \boldsymbol{p}_3\mu_3 | \boldsymbol{\mathcal{P}}, J(LS(s_{12}))\mu_J\rangle_f \\
&= \delta(\boldsymbol{\mathcal{P}} - \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3) K \Phi_0(\mathcal{M}_0) \sum_{\mu_S \mu_L} Y_{L\mu_L}(\hat{K})(L\mu_L S\mu_S | J\mu_J) \sum_{\{\bar{\mu}\}} (s_1\bar{\mu}_1 s_2\bar{\mu}_2 | s_{12}\bar{\mu}_{12})(s_{12}\bar{\mu}_{12} s_3\bar{\mu}_3) \\
&\times D^{s_{12}}_{\mu_{12}\bar{\mu}_{12}}[R^{-1}(-\boldsymbol{K}, \mathcal{M}_{0(12)})] D^{s_1}_{\mu_1\bar{\mu}_1}[\tilde{R}^{-1}(\boldsymbol{k}, M_1)] D^{s_2}_{\mu_2\bar{\mu}_2}[\tilde{R}^{-1}(-\boldsymbol{k}, M_2)] D^{s_3}_{\mu_3\bar{\mu}_1}[R^{-1}(\boldsymbol{K}, M_3)],
\end{aligned} \tag{9}$$

$$\begin{aligned}
\Psi^{(2)}_{J(lS(s_{12}))\mu_J}(\boldsymbol{\mathcal{P}}, \boldsymbol{K}, \boldsymbol{k}; \mu_1, \mu_2, \mu_3) &= \sum_{j_{12}} (-1)^{s_{12}+s_3+S+j_{12}} \sqrt{(2S+1)(2j_{12}+1)} \begin{Bmatrix} l & J & S \\ s_3 & s_{12} & j_{12} \end{Bmatrix} \\
&\times \Psi^{(2)}_{J(j_{12}(ls_{12})s_3)\mu_J}(\boldsymbol{\mathcal{P}}, \boldsymbol{K}, \boldsymbol{k}; \mu_1, \mu_2, \mu_3),
\end{aligned} \tag{10}$$

$$\begin{aligned}
\Psi^{(2)}_{J(j_{12}(ls_{12})s_3)\mu_J}(\boldsymbol{\mathcal{P}}, \boldsymbol{K}, \boldsymbol{k}; \mu_1, \mu_2, \mu_3) &= {}_f\langle \boldsymbol{p}_1\mu_1, \boldsymbol{p}_2\mu_2, \boldsymbol{p}_3\mu_3 | \boldsymbol{\mathcal{P}}, J(j_{12}(ls_{12})s_3)\mu_J\rangle_f \\
&= k\Phi_0(\mathcal{M}_0) \sum_{\mu_{12}\mu_l} Y_{l\mu_l}(\hat{k}) \sum_{\{\bar{\mu}\}} (s_1\bar{\mu}_1 s_2\bar{\mu}_2 | s_{12}\mu_{12})(l\mu_l s_{12}\mu_{12} | j_{12}\bar{\mu}_{j_{12}})(|j_{12}\bar{\mu}_{j_{12}} s_3\bar{\mu}_3 | J\mu_J) \\
&\times D^{j_{12}}_{\mu_{j_{12}}\bar{\mu}_{j_{12}}}[R^{-1}(-\boldsymbol{K}, \mathcal{M}_{0(12)})] D^{s_1}_{\mu_1\bar{\mu}_1}[\tilde{R}^{-1}(\boldsymbol{k}, M_1)] D^{s_2}_{\mu_2\bar{\mu}_2}[\tilde{R}^{-1}(-\boldsymbol{k}, M_2)] D^{s_3}_{\mu_3\bar{\mu}_1}[R^{-1}(\boldsymbol{K}, M_3)].
\end{aligned} \tag{11}$$

Jacobians, as evident factors, are omitted in (9) – (11) to simplify expressions. It should be noticed that the angular part of w.f.'s (9) – (11), $kY_{l\mu_l}(\hat{\boldsymbol{k}})$ and $KY_{L\mu_L}(\hat{\boldsymbol{K}})$, depends on the canonical 3-momenta $\boldsymbol{k}$ and $\boldsymbol{K}$

$$\boldsymbol{k} \equiv \boldsymbol{p}_{1cm} = \left\{ \boldsymbol{\lambda}_\perp, -\frac{1-2\xi}{2}\mathcal{M}_{0(12)} \right\}, \quad \boldsymbol{K} \equiv \boldsymbol{p}_{3CM} = \left\{ \boldsymbol{\Lambda}_\perp, \left(\frac{1-2\eta}{2}\mathcal{M}_0 + \frac{\mathcal{M}^2_{0(12)} - M^2}{2\mathcal{M}_0} \right) \right\}, \tag{12}$$

which are expressed through light-front invariants $\xi, \eta, \boldsymbol{\lambda}_\perp, \boldsymbol{\Lambda}_\perp, \mathcal{M}_0, \mathcal{M}_{0(12)}$, where

$$\boldsymbol{\lambda}_\perp = \frac{x_2\boldsymbol{p}_{1\perp} - x_1\boldsymbol{p}_{2\perp}}{x_1 + x_2}, \quad \boldsymbol{\Lambda}_\perp = \frac{(x_1+x_2)\boldsymbol{p}_{3\perp} - x_3(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp})}{x_1 + x_2 + x_3}, \tag{13}$$

Spin-orbital basis (9) – (11) given in the coordinate-spin (XS) space was used to construct quark configurations satisfying the Pauli exclusion principle. In the product of XS and isospin (T) spaces (XST) the fully symmetric states with Young scheme $[3]_{XST}$ are represented by expressions

$$\Psi_N(56^+) = \sqrt{\frac{1}{2}} \left[\Psi^{(0)}_{s_{12}=1} |[21]_T\, y_T^{(1)}\rangle + \Psi^{(0)}_{s_{12}=0} |[21]_T\, y_T^{(2)}\rangle \right] \tag{14}$$

for the nucleon and

$$\Psi_{N^*}(70^-) = \frac{1}{2}\left[\Psi^{(1)}_{s_{12}=1} - \Psi^{(2)}_{s_{12}=0} \right] |[21]_T\, y_T^{(1)}\rangle + \frac{1}{2}\left[\Psi^{(1)}_{s_{12}=0} + \Psi^{(2)}_{s_{12}=1} \right] |[21]_T\, y_T^{(2)}\rangle. \tag{15}$$

for negative parity resonances $N^*_{J^-}$ ($J = 1/2, 3/2$) from the 70^- multiplet of $SU(6)$ group classification. Here the isospin w.f. for $T = 1/2$ are marked by the Young scheme (partition) $[21]_T$ and

Yamanouchi symbols $y_T^{(1)} = (112)$ and $y_T^{(2)} = (121)$. For the positive charge ($T_z = +1/2$) these w.f. are given by the well known expressions:

$$|[21]_T\, y_T^{(1)} T_z = +1/2\rangle = \sqrt{\frac{2}{3}} uud - \sqrt{\frac{1}{3}} \frac{ud + du}{\sqrt{2}} u, \quad |[21]_T\, y_T^{(2)} T_z = +1/2\rangle = \frac{ud - du}{\sqrt{2}} u \qquad (16)$$

Spin-orbital states $\Psi^{(n)}_{s_{12}}$ for $n = 1{,}2$ are given in (9)-(11) and for $n = 0$ one can use $\Psi^{(2)}_{s_{12}}$ with $L = 0$ and Φ_{0S} instead of Φ_{1P}. In (15) we omitted all subscripts at $\Psi^{(n)}$, except s_{12}, since the values of $s_{12} = 0$ or 1 define the Yamanouchi symbol in the spin space, $y_S^{(1)} = (112)$ or $y_S^{(2)} = (121)$ respectively (in the case of the total spin $S = 1/2$, i.e. for the Young scheme $[21]_S$). Therefore the relations

$$\Psi^{(1)}_{s_{12}=1} = |[21]_X\, y_X^{(1)}\rangle|[21]_S\, y_S^{(1)}\rangle, \quad \Psi^{(1)}_{s_{12}=0} = |[21]_X\, y_X^{(1)}\rangle|[21]_S\, y_S^{(2)}\rangle,$$
$$\Psi^{(2)}_{s_{12}=1} = |[21]_X\, y_X^{(2)}\rangle|[21]_S\, y_S^{(1)}\rangle, \quad \Psi^{(2)}_{s_{12}=0} = |[21]_X\, y_X^{(2)}\rangle|[21]_S\, y_S^{(2)}\rangle, \qquad (17)$$

are implied in (14) - (15).

Matrix element (8) should be rewritten for physical initial and final states defined by (14) -(15) with covariant w.f.s (9)-(11) to calculate the amplitudes of resonance electroproduction (one can follow the technique developed in [10]).

5 Results and outlook

Matrix elements of the I^+ component of quark current (8) allowed us to calculate the transverse ($A_{1/2}$) and longitudinal ($S_{1/2}$) helicity amplitudes for electroproduction of the lightest nucleon resonances $N^*_{1/2^+}(1440)$ and $N^*_{1/2^-}(1535)$ in a large Q^2 interval up to 12 GeV^2. We used the same technique (fractional parentage coefficients etc.) as in our recent work [10] and the same parameters of the light-front quark model. For the $N^*(1535)$ we used a mixed model analogous to that of the Roper resonance:

$$N^*(1535) = \cos\theta^* |3q^*\rangle + \sin\theta^* |\Lambda + K^+\rangle \qquad (18)$$

with the $|3q^*\rangle$ defined by (15). Parameter of mixing θ^* was adjusted to optimize the description of the helicity amplitude $A_{1/2}$, and we have obtained $\cos\theta^* = 0.6$. We also updated the fit of parameter θ for the Roper resonance reducing $\cos\theta = 0.57$ of [16] to the new value $\cos\theta = 0.5$ which is closer to the CLAS data at $Q^2 \gtrsim 3$ - 4 GeV^2. The results are shown in figure 1.

In summary it can be concluded that

1. The lightest nucleon resonances are described at light front as mixed states of the $3q$ cluster possessing a definite value of the inner orbital momentum $L = 0{,}1$ and a hadron molecular state, $N + \sigma$ or $\Lambda + K$.
2. Nucleon elastic form factors are successfully described in a large interval of Q^2 by the lowest light-front quark configuration without any hadron-molecular admixtures.
3. But in the case of nucleon resonances the quark core overestimates the transition amplitudes at large Q^2, and thus the weight of the quark core in the resonance should be relatively small because of excitation of higher Fock states.

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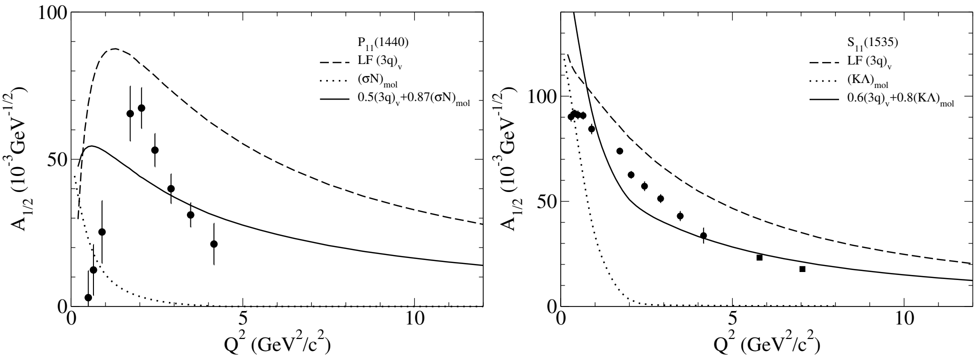

Figure 1. Transverse helicity amplitude $A_{1/2}$ for electroproduction of the Roper resonance (left panel) and the $N^*_{1/2^-}(1535)$ (right panel). Dashed lines are the 100% quark core contribution, dotted lines are the 100% hadron molecule contribution, solid lines are results for mixed models (1) and (18) with $\cos\theta = 0.5$ and $\cos\theta^* = 0.6$ respectively. The data are taken from [17, 18].

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