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Effect of the $\eta\eta$ channel and interference phenomena in the two-pion transitions of charmonia and bottomonia

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Abstract. The basic shape of di-pion mass spectra in the two-pion transitions of both charmonia and bottomonia states is explained by a unified mechanism based on contributions of the $\pi\pi$, $K\overline{K}$, and $\eta\eta$ coupled channels including their interference. The role of the individual f_0 resonances in shaping the di-pion mass distributions in the charmonia and bottomonia decays is considered.

1 Introduction

In this contribution we report on new results in continuation of our study of scalar meson properties analyzing jointly the data on the isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on the two-pion transitions of heavy mesons, when it is reasonable to consider that the two-pion pair is produced in the S-wave state and the final meson remains a spectator. We analyzed data on the charmonium decay processes — $J/\psi \to \phi\pi\pi$, $\psi(2S) \to J/\psi\pi\pi$ — from the Crystal Ball, DM2, Mark II, Mark III, and BES II Collaborations and practically all available data on two-pion transitions of the Υ mesons from the ARGUS, CLEO, CUSB, Crystal Ball, Belle, and *BaBar* Collaborations — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n, m = 2, 3, 4, 5, n = 1, 2, 3$). Moreover, the contribution of multi-channel $\pi\pi$ scattering (namely, $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$) in the final-state interactions is considered. The multi-channel $\pi\pi$ scattering is described in our model-independent approach based on analyticity and unitarity and using a uniformization procedure. A novel feature in the analysis is accounting for effects from the $\eta\eta$ channel in the indicated two-pion transitions which is assumed not only kinematically, i.e., via including the channel threshold in the uniformizing variable, but also by adding the $\pi\pi \to \eta\eta$ amplitude in the corresponding formulae for the decays.

We showed that the experimentally observed interesting (even mysterious) behavior of the $\pi\pi$ spectra of the Υ-family decays, which starts to be apparent in the second radial excitation and is also seen for the higher states, — a bell-shaped form in the near-$\pi\pi$-threshold region, smooth dips at about

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0.6 GeV in the $\Upsilon(4S,5S) \to \Upsilon(1S)\pi^+\pi^-$, about 0.45 GeV in the $\Upsilon(4S,5S) \to \Upsilon(2S)\pi^+\pi^-$, and at about 0.7 GeV in the $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, and also sharp dips near 1 GeV in the $\Upsilon(4S,5S) \to \Upsilon(1S)\pi^+\pi^-$ — can be explained by the interference between the $\pi\pi$ scattering amplitudes, $K\overline{K} \to \pi\pi$ and $\eta\eta \to \pi\pi$, in the final-state re-scattering (by the constructive interference in the near-$\pi\pi$-threshold region and by the destructive one in the dip regions). Note that in a number of works (see, e.g., [1] and the references therein) various assumptions were made to explain this observed behavior of the di-meson mass distributions.

We have explained the basic shape of di-pion mass spectra in the two-pion transitions of both charmonia and bottomonia on the basis of our previous conclusions on wide resonances without any additional assumptions. In works [2, 3] we have shown: If a wide resonance cannot decay into a channel which opens above its mass, but the resonance is strongly coupled to this channel, then one should consider this resonance as a multi-channel state allowing for this closed channel.

2 The effect of multi-channel $\pi\pi$ scattering in decays of the ψ- and Υ-meson families

We have considered the three-channel case of the multi-channel $\pi\pi$ scattering, i.e. the reactions $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, because it was shown [2] that this is a minimal number of coupled channels needed for obtaining correct values of f_0-resonance parameters. In the combined analysis, the data for the multi-channel $\pi\pi$ scattering were taken from many papers (see in [3]). For the decay $J/\psi \to \phi\pi^+\pi^-$ data were taken from Mark III, DM2 and BES II Collaborations; for $\psi(2S) \to J/\psi(\pi^+\pi^-$ and $\pi^0\pi^0)$ — from Mark II and Crystal Ball(80) (see also in [3]). For $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-$ and $\pi^0\pi^0)$ data were used from ARGUS [4], CLEO [5], CUSB [6], and Crystal Ball [7] Collaborations; for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$ — from CLEO [8, 9]; for $\Upsilon(4S) \to \Upsilon(1S,2S)\pi^+\pi^-$ — from *BaBar* [10] and Belle [11]; for $\Upsilon(5S) \to \Upsilon(1S,2S,3S)\pi^+\pi^-$ — from Belle Collaboration [11].

The di-meson mass distributions in the quarkonia decays are calculated using a formalism analogous to that proposed in [12] for the decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $V' \to V\pi\pi$ $(V = \psi, \Upsilon)$ which is extended with allowing for amplitudes of transitions between the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ channels in decay formulae. It is assumed that the pion pairs in the final state have zero isospin and spin. Only these pairs of pions undergo the final state interactions whereas the final $\Upsilon(nS)$ meson $(n < m)$ remains as a spectator. The decay amplitudes are related with the scattering amplitudes T_{ij} $(i, j = 1-\pi\pi, 2-K\overline{K}, 3-\eta\eta)$ as follows

$$F(J/\psi \to \phi\pi\pi) = c_1(s)T_{11} + \Big(\frac{\alpha_2}{s-\beta_2} + c_2(s)\Big)T_{21} + c_3(s)T_{31}, \tag{1}$$

$$F(\psi(2S) \to \psi(1S)\pi\pi) = d_1(s)T_{11} + d_2(s)T_{21} + d_3(s)T_{31}, \tag{2}$$

$$F(\Upsilon(mS) \to \Upsilon(nS)\pi\pi) = e_1^{(mn)}T_{11} + e_2^{(mn)}T_{21} + e_3^{(mn)}T_{31}, \quad m > n,\ m = 2,3,4,5,\ n = 1,2,3 \tag{3}$$

where $c_i = \gamma_{i0} + \gamma_{i1}s$, $d_i = \delta_{i0} + \delta_{i1}s$ and $e_i^{(mn)} = \rho_{i0}^{(mn)} + \rho_{i1}^{(mn)}s$; indices m and n correspond to $\Upsilon(mS)$ and $\Upsilon(nS)$, respectively. The free parameters α_2, β_2, γ_{i0}, γ_{i1}, δ_{i0}, δ_{i1}, $\rho_{i0}^{(mn)}$ and $\rho_{i1}^{(mn)}$ depend on the couplings of J/ψ, $\psi(2S)$, and $\Upsilon(mS)$ to the channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$. The pole term in (1) in front of T_{21} is an approximation of possible ϕK states, not forbidden by OZI rules.

The amplitudes T_{ij} are expressed through the S-matrix elements

$$S_{ij} = \delta_{ij} + 2i\sqrt{\rho_i\rho_j}T_{ij}, \tag{4}$$

where $\rho_i = \sqrt{1 - s_i/s}$ and s_i is the reaction threshold. The S-matrix elements are taken as the products

$$S = S_B S_{res} \tag{5}$$

where S_{res} represents the contribution of resonances, S_B is the background part. The S_{res}-matrix elements are parameterized on the uniformization plane of the $\pi\pi$-scattering S-matrix element by poles and zeros which represent resonances. The uniformization plane is obtained by a conformal map of the 8-sheeted Riemann surface, on which the three-channel S matrix is determined, onto the plane. In the uniformizing variable used [13]

$$w = \frac{\sqrt{(s - s_2)s_3} + \sqrt{(s - s_3)s_2}}{\sqrt{s(s_3 - s_2)}} \quad (s_2 = 4m_K^2 \text{ and } s_3 = 4m_\eta^2) \tag{6}$$

we have neglected the $\pi\pi$-threshold branch point and allowed for the $K\overline{K}$- and $\eta\eta$-threshold branch points and left-hand branch point at $s = 0$ related to the crossed channels.

Resonance representations on the Riemann surface are obtained using formulae in table 1 [14], expressing analytic continuations of the S-matrix elements to all sheets in terms of those on the physical (I) sheet that have only the resonances zeros (beyond the real axis), at least, around the physical region. In table 1 the Roman numerals denote the Riemann-surface sheets, the superscript I

Table 1. Analytic continuations of the S-matrix elements

Process	I	II	III	IV	V	VI	VII	VIII
$1 \to 1$	S_{11}	$\frac{1}{S_{11}}$	$\frac{S_{22}}{D_{33}}$	$\frac{D_{33}}{S_{22}}$	$\frac{\det S}{D_{11}}$	$\frac{D_{11}}{\det S}$	$\frac{S_{33}}{D_{22}}$	$\frac{D_{22}}{S_{33}}$
$1 \to 2$	S_{12}	$\frac{iS_{12}}{S_{11}}$	$\frac{-S_{12}}{D_{33}}$	$\frac{iS_{12}}{S_{22}}$	$\frac{iD_{12}}{D_{11}}$	$\frac{-D_{12}}{\det S}$	$\frac{iD_{12}}{D_{22}}$	$\frac{D_{12}}{S_{33}}$
$2 \to 2$	S_{22}	$\frac{D_{33}}{S_{11}}$	$\frac{S_{11}}{D_{33}}$	$\frac{1}{S_{22}}$	$\frac{S_{33}}{D_{11}}$	$\frac{D_{22}}{\det S}$	$\frac{\det S}{D_{22}}$	$\frac{D_{11}}{S_{33}}$
$1 \to 3$	S_{13}	$\frac{iS_{13}}{S_{11}}$	$\frac{-iD_{13}}{D_{33}}$	$\frac{-D_{13}}{S_{22}}$	$\frac{-iD_{13}}{D_{11}}$	$\frac{D_{13}}{\det S}$	$\frac{-S_{13}}{D_{22}}$	$\frac{iS_{13}}{S_{33}}$
$2 \to 3$	S_{23}	$\frac{D_{23}}{S_{11}}$	$\frac{iD_{23}}{D_{33}}$	$\frac{iS_{23}}{S_{22}}$	$\frac{-S_{23}}{D_{11}}$	$\frac{-D_{23}}{\det S}$	$\frac{iD_{23}}{D_{22}}$	$\frac{iS_{23}}{S_{33}}$
$3 \to 3$	S_{33}	$\frac{D_{22}}{S_{11}}$	$\frac{\det S}{D_{33}}$	$\frac{D_{11}}{S_{22}}$	$\frac{S_{22}}{D_{11}}$	$\frac{D_{33}}{\det S}$	$\frac{S_{11}}{D_{22}}$	$\frac{1}{S_{33}}$

is omitted to simplify the notation, $\det S$ is the determinant of the 3×3 S-matrix on sheet I, $D_{\alpha\beta}$ is the minor of the element $S_{\alpha\beta}$, that is, $D_{11} = S_{22}S_{33} - S_{23}^2$, $D_{22} = S_{11}S_{33} - S_{13}^2$, $D_{33} = S_{11}S_{22} - S_{12}^2$, $D_{12} = S_{12}S_{33} - S_{13}S_{23}$, $D_{23} = S_{11}S_{23} - S_{12}S_{13}$, etc.

These formulae show how singularities and resonance poles and zeros are transferred from the matrix element S_{11} to matrix elements of coupled processes.

The background is introduced via the S_B-matrix elements in a natural way: on the threshold of each important channel there appears generally speaking a complex phase shift. It is important that we have obtained practically zero background of the $\pi\pi$ scattering in the scalar-isoscalar channel. First, this confirms well our assumption in (5). Second, this shows that the representation of multi-channel resonances by the pole and zeros on the uniformization plane given in table 1 is good and quite sufficient. This result is also a criterion for the correctness of the approach.

In table 2 we show the poles corresponding to f_0 resonances, obtained in the analysis. Generally, *the wide multi-channel states are most adequately represented by poles*, because the poles give the main model-independent effect of resonances and are rather stable characteristics for various models,

Table 2. The poles for resonances on the $\sqrt{s}$-plane. $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$.

Sheet		$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
II	E_r	521.6±12.4	1008.4±3.1			1512.4±4.9	
	$\Gamma_r/2$	467.3±5.9	33.5±1.5			287.2±12.9	
III	E_r	552.5±17.7	976.7±5.8	1387.2±24.4		1506.1±9.0	
	$\Gamma_r/2$	467.3±5.9	53.2±2.6	167.2±41.8		127.8±10.6	
IV	E_r			1387.2±24.4		1512.4±4.9	
	$\Gamma_r/2$			178.2±37.2		215.0±17.6	
V	E_r			1387.2±24.4	1493.9±3.1	1498.8±7.2	1732.8±43.2
	$\Gamma_r/2$			261.0±73.7	72.8±3.9	142.3±6.0	114.8±61.5
VI	E_r	573.4±29.1		1387.2±24.4	1493.9±5.6	1511.5±4.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9		250.0±83.1	58.4±2.8	179.3±4.0	111.2±8.8
VII	E_r	542.5±25.5			1493.9±5.0	1500.4±9.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9			47.8±9.3	99.9±18.0	55.2±38.0
VIII	E_r				1493.9±3.2	1512.4±4.9	1732.8±43.2
	$\Gamma_r/2$				62.2±9.2	298.4±14.5	58.8±16.4

whereas masses and total widths are very model-dependent for wide resonances [16]. The masses, widths, and the coupling constants of resonances should be calculated using the poles on sheets II, IV and VIII, because only on these sheets the analytic continuations have the forms (see table 1):

$$\propto 1/S_{11}^{\mathrm{I}}, \quad \propto 1/S_{22}^{\mathrm{I}} \quad \text{and} \quad \propto 1/S_{33}^{\mathrm{I}},$$

respectively, i.e., the pole positions of resonances are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels.

Further, since studying the decays of charmonia and bottomonia, we investigated the role of the individual f_0 resonances in contributing to the shape of the di-pion mass distributions in these decays, firstly we studied their role in forming the energy dependence of amplitudes of reactions $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. In this case we switched off only those resonances [$f_0(500)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$], removal of which can be somehow compensated by correcting the background (maybe, with elements of the pseudo-background) to have the more-or-less acceptable description of the multi-channel $\pi\pi$ scattering. Below we therefore considered description of the multi-channel $\pi\pi$ scattering for two more cases:

- First, when leaving out a minimal set of the f_0 mesons consisting of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$, which is sufficient to achieve a description of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ with a total $\chi^2/\mathrm{ndf} \approx 1.20$.
- Second, from the above-indicated three mesons only the $f_0(500)$ can be omitted while still obtaining a reasonable description of multi-channel $\pi\pi$ scattering (though with appearance of a pseudo-background) with the total $\chi^2/\mathrm{ndf} \approx 1.43$.

In figure 1 we show the obtained description of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$. One can see that the curves are quite similar in all three cases.

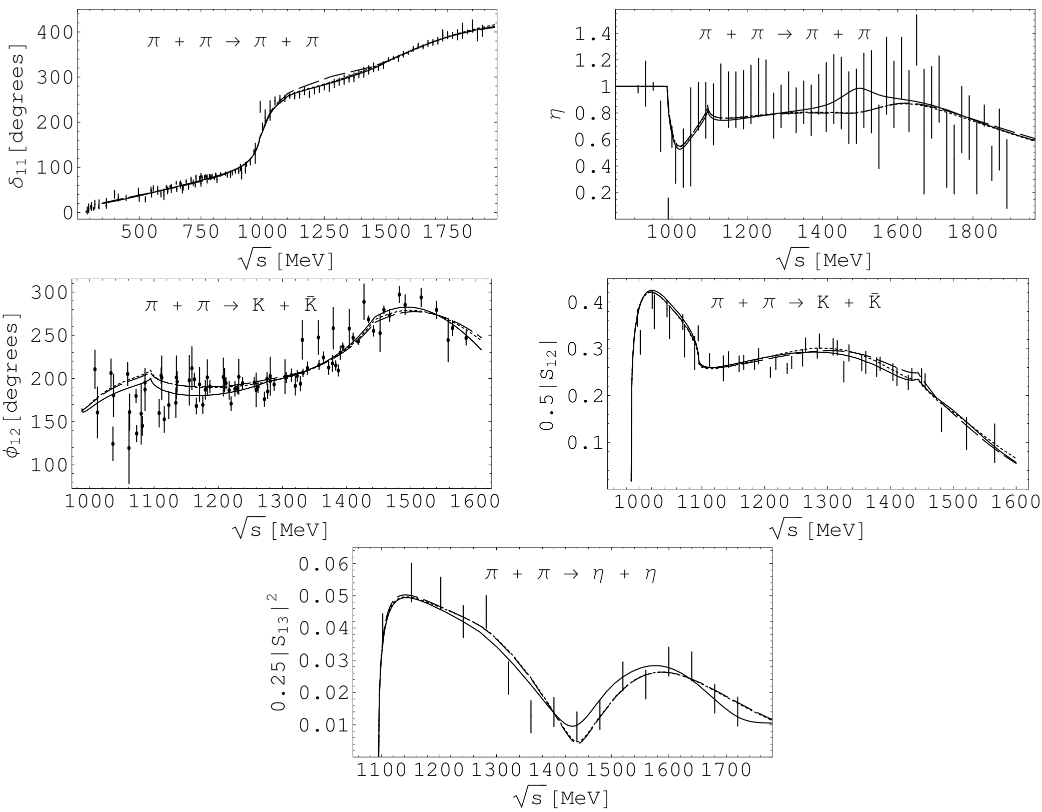


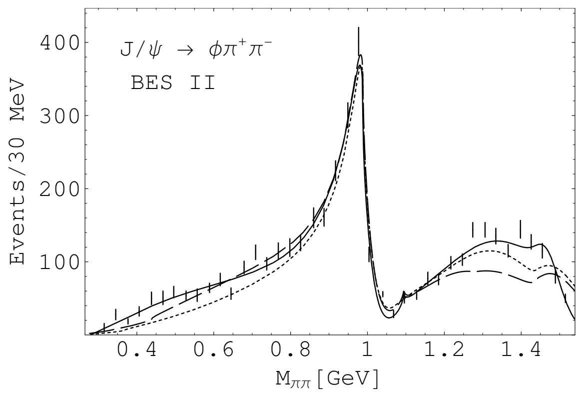
Figure 1. The phase shifts and moduli of the S-matrix element in the S-wave $\pi\pi$-scattering (upper panel), in $\pi\pi \to K\overline{K}$ (middle panel), and the squared modulus of the $\pi\pi \to \eta\eta$ S-matrix element (lower figure).

The di-meson mass distributions in the decay analysis were calculated using the relation

$$N|F|^2 \sqrt{(s-s_1)[m_\psi^2 - (\sqrt{s}-m_\phi)^2][m_\psi^2 - (\sqrt{s}+m_\phi)^2]} \tag{7}$$

for the decay $J/\psi \to \phi\pi\pi$ and with analogous relations for $\psi(2S) \to \psi(1S)\pi\pi$ and $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$. The normalization to the experiment, N is: for $J/\psi \to \phi\pi\pi$ 0.5172 (Mark III), 0.1746 (DM 2) and 3.8 (BES II); for $\psi(2S) \to J/\psi\pi^+\pi^-$ 1.746 (Mark II); for $\psi(2S) \to J/\psi\pi^0\pi^0$ 1.6891 (Crystal Ball(80)); for $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ 4.1758 (ARGUS), 2.0445 (CLEO(94)) and 1.0782 (CUSB); for $\Upsilon(2S) \to \Upsilon(1S)\pi^0\pi^0$ 0.0761 (Crystal Ball(85)); for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-$ and $\pi^0\pi^0)$ 19.8825 and 4.622 (CLEO(07)); for $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-$ and $\pi^0\pi^0)$ 1.6987 and 1.1803 (CLEO(94)); for $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ 4.6827 (*BaBar*(06)) and 0.3636 (Belle(07)); for $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$, 37.9877 (*BaBar*(06)); for $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(3S)\pi^+\pi^-$, respectively 0.2047, 2.8376 and 6.9251 (Belle(12)).

A satisfactory description of all considered processes (including $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$) was obtained with the total $\chi^2/\text{ndf} = 736.457/(710 - 118) \approx 1.24$; for the $\pi\pi$ scattering, $\chi^2/\text{ndf} \approx 1.15$. Results for the distributions are shown in figures 2-4 with the same notation as in figure 1. Here the effects of omitting some resonance are more apparent than in figure 1.



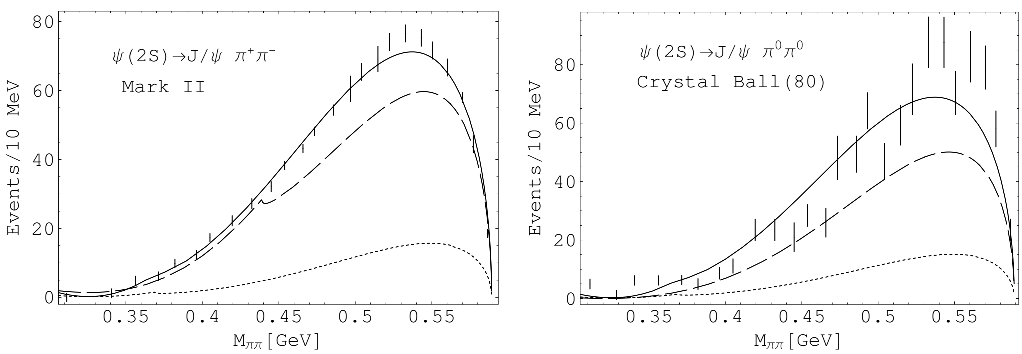


Figure 2. The decays $J/\psi \to \phi\pi\pi$ and $\psi(2S) \to J/\psi\pi\pi$. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

3 Conclusions and Discussion

The combined analysis was performed for the data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on the decays of the charmonia — $J/\psi \to \phi\pi\pi$, $\psi(2S) \to J/\psi\,\pi\pi$ — and of the bottomonia — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2, 3, 4, 5$, $n = 1, 2, 3$) from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, BES II, *BaBar*, and Belle Collaborations.

It is shown that the di-pion mass spectra in the above-indicated decays of charmonia and bottomonia are explained by the unified mechanism which is based on our previous conclusions on wide resonances [2, 3] and is related to contributions of the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels including their interference. It is shown that in the final states of these decays (except $\pi\pi$ scattering) the contribution of coupled processes, e.g., $K\overline{K}, \eta\eta \to \pi\pi$, is important even if these processes are energetically forbidden.

Accounting for the effect of the $\eta\eta$ channel in the considered decays, both kinematically (i.e. via the uniformizing variable) and also by adding the $\pi\pi \to \eta\eta$ amplitude in the formulas for the decays, permits us to eliminate nonphysical (i.e. those related with no channel thresholds) non-regularities in some $\pi\pi$ distributions, which are present without this extension of the description [15]. We obtained a reasonable and satisfactory description of all considered $\pi\pi$ spectra in the two-pion transitions of charmonium and bottomonium.

It was also very useful to consider the role of individual f_0 resonances in contributions to the di-pion mass distributions in the indicated decays. For example, it is seen that the sharp dips near 1 GeV in the $\Upsilon(4S, 5S) \to \Upsilon(1S)\pi^+\pi^-$ decays are related with the $f_0(500)$ contribution to the interfering amplitudes of $\pi\pi$ scattering and $K\overline{K}, \eta\eta \to \pi\pi$ processes. Namely consideration of this role of the $f_0(500)$ allows us to make a conclusion on existence of the sharp dip at about 1 GeV in the di-pion mass spectrum of the $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ decay where, unlike $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$, the scarce data do not permit to draw such conclusions yet.

Also, a manifestation of the $f_0(1370)$ turned out to be interesting and unexpected. First, in the satisfactory description of the $\pi\pi$ spectrum of decay $J/\psi \to \phi\pi\pi$, the second large peak in the 1.4-GeV region can be naively explained as the contribution of the $f_0(1370)$. We have shown that this is not right – the constructive interference between the contributions of the $\eta\eta$ and $\pi\pi$ and $K\overline{K}$ channels plays the main role in formation of the 1.4-GeV peak. This is quite in agreement with our earlier conclusion that the $f_0(1370)$ has a dominant $s\bar{s}$ component [2].

On the other hand, it turned out that the $f_0(1370)$ contributes considerably in the near-$\pi\pi$-threshold region of many di-pion mass distributions, especially making the threshold bell-shaped form of the di-pion spectra in the decays $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n, m = 3, 4, 5, n = 1, 2, 3$). This fact confirms, first, the existence of the $f_0(1370)$ (up to now there is no firm conviction if it exists or not). Second, that the exciting role of this meson in making the threshold bell-shaped form of the di-pion spectra can be explained as follows: the $f_0(1370)$, being predominantly the $s\bar{s}$ state [3] and practically not contributing to the $\pi\pi$-scattering amplitude, influences noticeably the $K\overline{K}$ scattering; e.g., it was shown that the $K\overline{K}$-scattering length is very sensitive to whether this state does exist or not [16]. The interference of contributions of the $\pi\pi$-scattering amplitude and the analytically-continued $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ amplitudes lead to the observed results.

It is important that we have performed a combined analysis of available data on the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, on decays of charmonia $J/\psi \to \phi\pi\pi$, $\psi(2S) \to J/\psi(\pi\pi)$ and of bottomonia $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n, m = 2, 3, 4, 5, n = 1, 2, 3$) from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, BES II, *BABAR*, and Belle Collaborations. The convincing description (including also the $\eta\eta$ channel) of practically all available data on two-pion transitions of the Ψ and the Υ mesons confirmed all our previous conclusions on the unified mechanism of formation of the basic di-pion spectra, which is based on our previous conclusions on wide resonances [2, 3] and is related to contributions of the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels including their interference. This also confirmed all our earlier results on the scalar mesons [3]; the most important results are:

1. Confirmation of the $f_0(500)$ with a mass of about 700 MeV and a width of 930 MeV (the pole position on sheet II is $514.5 \pm 12.4 - 465.6 \pm 5.9$ MeV).

2. An indication that the $f_0(980)$ (the pole on sheet II is $1008.1 \pm 3.1 - i(32.0 \pm 1.5)$ MeV) is neither a $q\bar{q}$ state nor the $K\overline{K}$ molecule.

3. An indication for the $f_0(1370)$ and $f_0(1710)$ to have a dominant $s\bar{s}$ component.

4. An indication for the existence of two states in the 1500-MeV region: the $f_0(1500)$ ($m_{res} \approx$ 1495 MeV, $\Gamma_{tot} \approx 124$ MeV) and the $f_0'(1500)$ ($m_{res} \approx 1539$ MeV, $\Gamma_{tot} \approx 574$ MeV).

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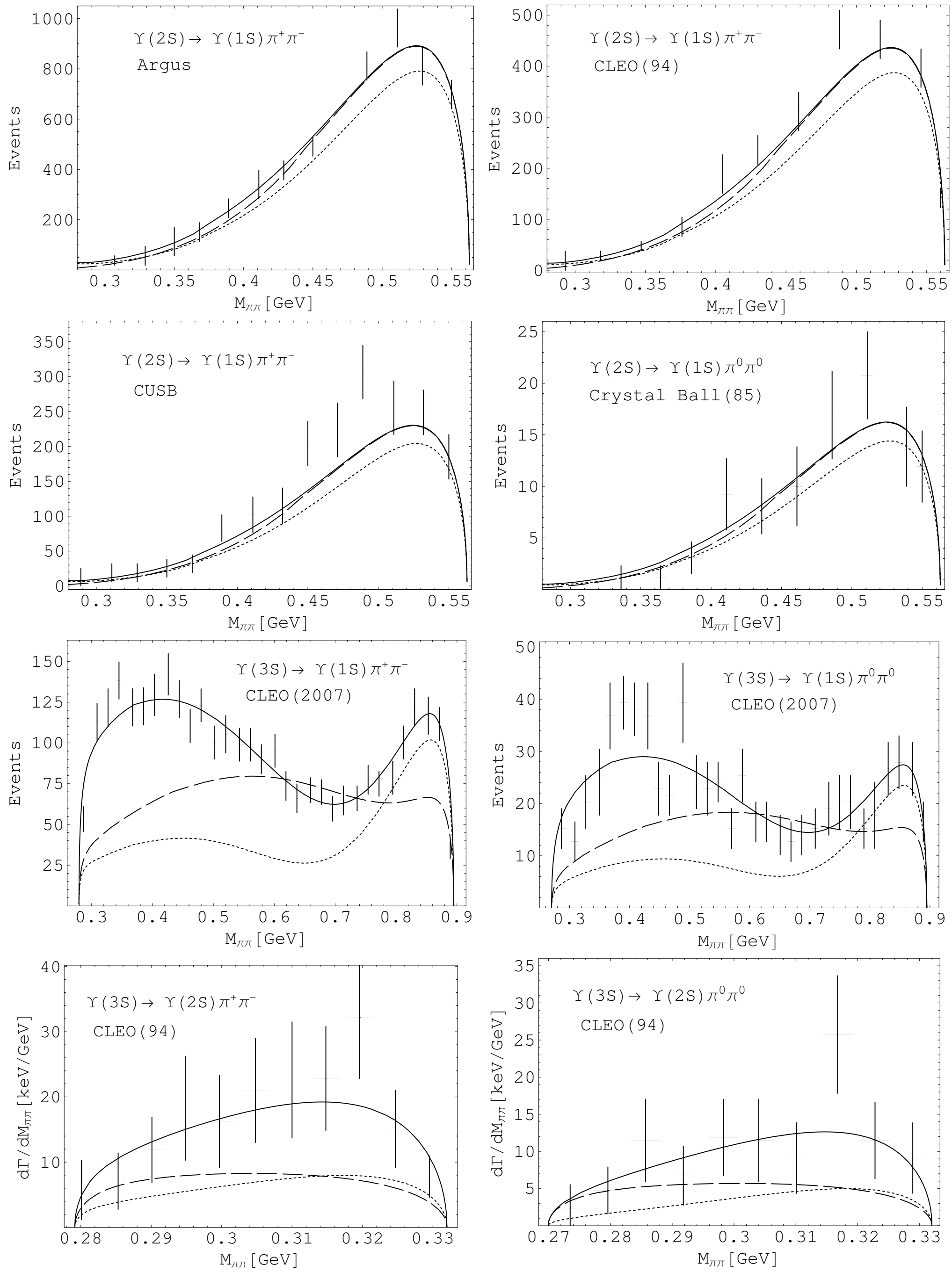

Figure 3. The decays $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ (two upper panels), $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ (middle panel) and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ (lower panel). The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

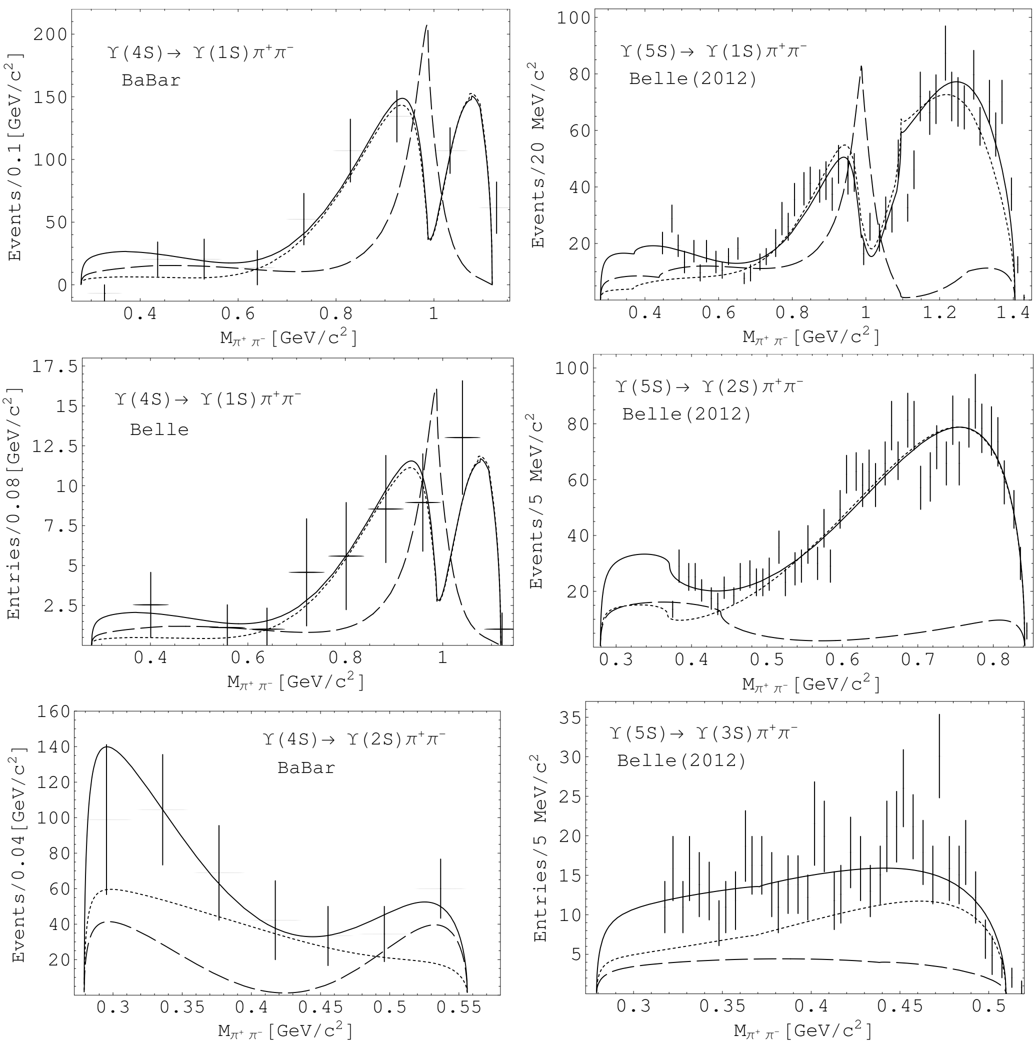

Figure 4. The decays $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ (left-hand) and $\Upsilon(5S) \to \Upsilon(ns)\pi^+\pi^-$ ($n = 1, 2, 3$) (right-hand). The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.