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# Numerical modelling of pollutant propagation in Lake Baikal during the spring thermal bar

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**Abstract**

In this paper, the phenomenon of the thermal bar in Lake Baikal and the propagation of pollutants from the Selenga River are studied with a nonhydrostatic mathematical model. An unsteady flow is simulated by solving numerically a system of thermal convection equations in the Boussinesq approximation using second-order implicit difference schemes in both space and time. To calculate the velocity and pressure fields in the model, an original procedure for buoyant flows, SIMPLED, which is a modification of the well-known Patankar and Spalding's SIMPLE algorithm, has been developed. The simulation results have shown that the thermal bar plays a key role in propagation of pollution in the area of Selenga River inflow into Lake Baikal.

*Keywords:* pollutant transport, thermal bar, numerical modelling, Lake Baikal, Selenga River

## 1 Introduction

The mechanisms of natural convection in freshwater ecosystems are of great scientific and practical interest, in particular in relation to human impacts on the environment. In particular, the thermal bar phenomenon can greatly affect the processes of pollutant propagation in freshwater bodies. A thermal bar is a narrow zone in a lake in temperate latitudes where maximum-density waters sink from the surface to the bottom. Our understanding of thermal bars in deep lakes is still scant: there are sparse observational data and very few high-accuracy mathematical models to describe hydrodynamic processes in deep lakes in given hydrological and meteorological conditions. The propagation of Selenga river water in the shallow coastal band of Lake Baikal during the spring-summer warming was numerically studied by T.E. Ovchinnikova and O.B. Bocharov [3] and P.R. Holland et al. [1], but these authors did not consider the relationship between river inflow and pollution spread. A major source of pollution in Lake Baikal is its

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largest tributary, the Selenga, providing over 50% of the total river runoff. About 60% of the total pollution in Baikal is brought by the Selenga. Therefore, it is important to assess some qualitative characteristics of pollutant transport from the Selenga River to Lake Baikal. Also in this part of the lake the thermal bar is most pronounced during the spring-summer warming, and its effects are most suitable for numerical simulation.

The aim of the present paper is to numerically simulate the "river - lake" interaction and study the impact of the spring riverine thermal bar on pollutant propagation in Lake Baikal.

# 2 Numerical model

## 2.1 Governing equations

We consider hydrodynamic processes in a cross-section perpendicular to the shore of a deep lake. We take coordinates $x$, $y$, $z$ in the offshore, alongshore and vertical directions respectively, with $u$, $v$, $w$ as the respective velocity components. Our nonhydrostatic mathematical model, which includes the Coriolis force due to Earth's rotation, is written in the Boussinesq approximation with the following equations:

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uw}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(K_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial u}{\partial z}\right)+2\Omega_z v-2\Omega_y w;$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial wv}{\partial z}=\frac{\partial}{\partial x}\left(K_x\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial v}{\partial z}\right)+2\Omega_x w-2\Omega_z u;$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial w^2}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left(K_x\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial w}{\partial z}\right)-\frac{g\rho}{\rho_0}+2\Omega_y u-2\Omega_x v;$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0;$$

$$\frac{\partial T}{\partial t}+\frac{\partial uT}{\partial x}+\frac{\partial wT}{\partial z}=\frac{\partial}{\partial x}\left(D_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial T}{\partial z}\right)+\frac{1}{\rho_0 c_p}\frac{\partial H_{\mathrm{sol}}}{\partial z};$$

equations of salinity balance and pollutant concentration in the lake ($\Phi = S, C$)

$$\frac{\partial \Phi}{\partial t}+\frac{\partial u\Phi}{\partial x}+\frac{\partial w\Phi}{\partial z}=\frac{\partial}{\partial x}\left(D_x\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial z}\left(D_z\frac{\partial \Phi}{\partial z}\right).$$

Here $\Omega_x$, $\Omega_y$ , and $\Omega_z$ are the vector components of Earth's rotation angular velocity; $g$ is the acceleration of gravity; $c_p$ is the specific heat capacity; $T$ is the temperature; $S$ is the salinity; $C$ is the pollutant concentration; $p$ is the pressure; $\rho_0$ is the water density at standard atmospheric pressure, temperature $T_L$, and salinity $S_L$ ($T_L$ and $S_L$ are a reference temperature and salinity of the lake, respectively). To close the system of equations, a two-parameter $k-\omega$ model of turbulence developed by D.C. Wilcox [7] is used. Absorption of shortwave radiation $H_{\mathrm{sol}}$ is calculated according to the Bouguer-Lambert-Beer law. The Chen-Millero equation, adopted by UNESCO, was taken as the equation of state $\rho = \rho\,(T, S, p)$.

### 2.2 Initial and boundary conditions

Initial conditions for equations are

$$u = 0;\;\; v = 0;\;\; w = 0;\;\; T = T_L;\;\; S = S_L;\;\; C = 0;\;\; k = 0;\;\; \omega = \omega_L \;\; at \;\; t = 0,$$

where $T_L$, $S_L$ are the initial temperature and salinity in the lake, respectively, and $t$ is the time.

On all solid boundaries, conditions of no-slip and zero flux of scalar variables are set. At the free surface, a wind stress condition is used in conjunction with the rigid lid approximation. Heating of the lake is simulated with shortwave, longwave, sensible, latent heat fluxes, predicted on real local weather conditions. On the open boundary special numerical conditions of the radiation type are specified.

## 3 Numerical method

The above-formulated problem is solved by a finite volume method. The scalar quantities (temperature, salinity, etc.) are calculated in the center of a grid cell, and the velocity vector components at the mid-points of the cell boundaries. To approximate the lake coastal profile, a method of blocking of fictitious domains [4] is used: the velocity components in a dead zone are set to zero by using large values of the viscosity coefficients in this zone.

The numerical algorithm for finding the flow and temperature fields is based on a Crank-Nicholson difference scheme. The convective terms in the equations are approximated with a second-order upstream scheme, QUICK.

To calculate the velocity and pressure fields, a procedure for buoyant flows, SIMPLED (Semi-Implicit Method for Pressure Linked Equations with Density correction), which is a modification of the well-known Patankar's method SIMPLE [4], has been developed. The algorithm of SIMPLED is based on a cyclic "prediction-correction" sequence and takes into account density correction in the buoyancy term in comparison with the algorithm SIMPLE.

The systems of grid equations at each time step are solved by the relaxation method with red-black ordering and using of OpenMP programming tools for multicore processors.

The numerical algorithm has been tested for the case of a square cavity with isothermal lateral boundaries. Also, the results of mathematical simulation of the thermal bar in Lake Kamloops by this method [6] are in good agreement with the results of calculations performed by P.R. Holland et al.

## 4 Results and Discussion

The Srednyaya arm (Selenga mouth) - Buguldeika cross-section (see Figure 1), near the boundary between the southern and central basins of Lake Baikal, was taken for the study. Bottom topography data for this cross-section were taken from a bathymetrical electronic map of Lake Baikal [5]. The Selenga shallow water basin is located between 51.9° - 52.5° N. and 106.1° - 106.9° E.: from the southern part of Istoksky Sor to Cape Oblom.

A uniform value $T_L = 3°\mathrm{C}$ is taken as the initial temperature distribution in Lake Baikal in the calculations. The initial water temperature in the Selenga River is 5°C. The temperature of the river water increases by 0.4°C every day, which corresponds to the real temperature regime in May. The river flows into the lake at a velocity of 0.015 m/s. The water mineralization in the lake is $S_L = 0.096$ g/kg, whereas it linearly increases from 0.140 g/kg to 0.150 g/kg in the river. Short- and longwave radiation, fluxes of latent and sensible heat, as well as the wind action at the

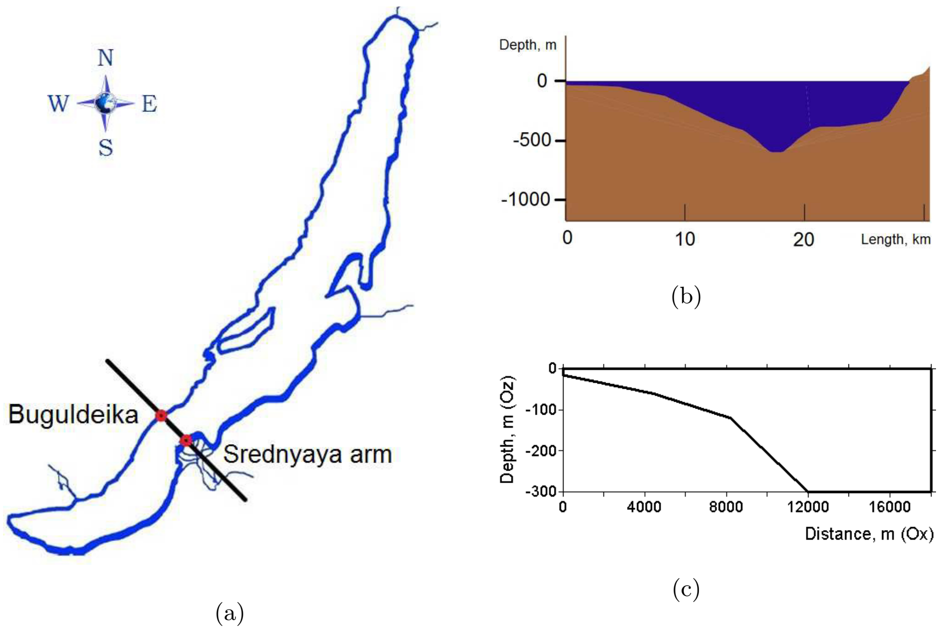


Figure 1: Srednyaya arm - Buguldeika cross-section: (a) Lake Baikal cross-section; (b) bottom topography; (c) calculation domain

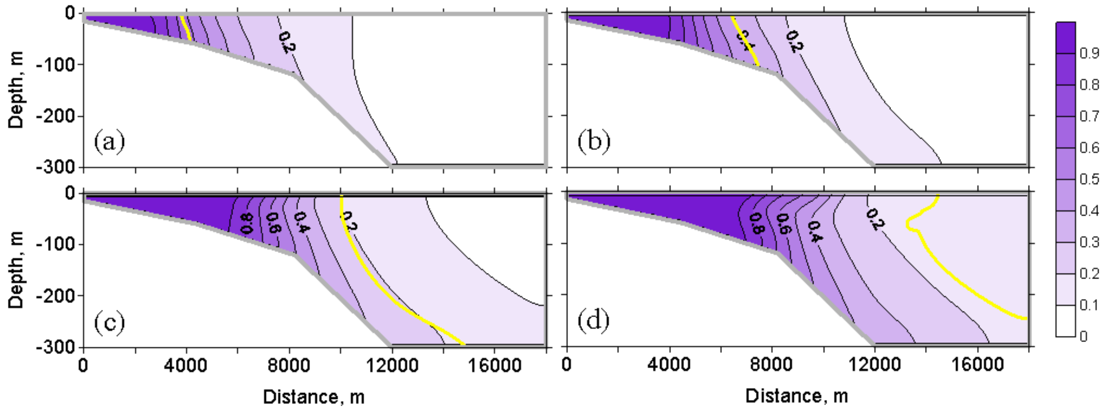


Figure 2: Concentration of pollutants from the Selenga River and maximum density temperature (yellow line) after (a) 5, (b) 10, (c) 20, (d) 30 days

water-air interface are calculated from the available observation data from the Baikalsk meteorological station archive from 01.05.2002 to 30.05.2002 (http://meteo.infospace.ru). Geothermal heat, $H_{\text{geo}} = 0.1$ W/m$^2$, is specified on the bottom. The Srednyaya arm - Buguldeika cross-section corresponds to the geographical latitude $\phi = 52.33°$, and the cross-section angle with respect to East is 142°.

The calculation domain is 18 km long and 300 m deep (Figure 1c). The open boundary at the river outflow (at the left of the domain) is 15 m deep. The calculation domain (see Figure 1c) is covered by a uniform orthogonal grid with dimensions $h_x = 50$ m and $h_z = 5$ m. The time step $\Delta t = 60$ s. The calculations are made with a Tomsk State University supercomputer “SKIF Cyberia”.

The overall dynamics of pollutant propagation in the zone of Selenga inflow into Lake Baikal is presented in Figure 2. We assume that the pollutants dissolved in the water are neutrally buoyant, so they neither rise to the surface nor are they deposited on the bed. Figure 2 shows that on the 5th day water with a pollutant concentration of 90% of the level in the river reaches a distance of 2.6 km from the Selenga mouth; on the 10th day, it reaches 4 km; on the 20th day, 6 km, and on the 30th day, 7.2 km. There is a clear tendency of the pollutant to move down the slope in accordance with the thermal bar evolution. Thus, it can be concluded that the thermal bar plays a key role in pollution propagation in the zone of Selenga inflow into Lake Baikal. Hubbard and Spain [2] have similarly observed that the propagation of dissolved materials from river runoff in Lake Superior is controlled by the thermal bar in spring.

# 5 Conclusions

With the above nonhydrostatic numerical model, the major thermohydrodynamic processes related to the thermal bar dynamics in the vicinity of the Selenga inflow into Lake Baikal have been simulated. The numerical experiments have shown that the incoming river water (which differs from the lake water in mineralization and temperature) contributes to the formation of the thermal bar and its further movement to the center of the lake. A simulation of the propagation of pollutants coming with the Selenga waters has shown that this is closely correlated with the thermal bar development. The dynamics of pollutant propagation in the area of Selenga inflow into Lake Baikal shows that on the 30th day water with a pollutant concentration of 90% of the level in the river reaches a distance of 7.2 km from the Selenga mouth.

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