

# On Newtonian singularities in higher derivative gravity models

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**ABSTRACT:** We consider the problem of Newtonian singularity in the wide class of higher derivative gravity models, including the ones which are renormalizable and super-renormalizable at the quantum level. The simplest version of the singularity-free theory has four derivatives and is pretty well-known. We argue that in all cases of local higher-derivative theories, when the poles of the propagator are real and simple, the singularities disappear due to the cancelation of contributions from scalar and tensor massive modes.

**KEYWORDS:** Models of Quantum Gravity, Classical Theories of Gravity

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## 1 Introduction

The singularities in the relevant solutions of general relativity indicate the limits of applicability of the theory. For this reason they represent an important motivation to study quantum effects, which are supposed to make the theory free of singularities. For instance, in cosmology we know that this really happens, since in the framework of the complete, quantum theory-based, non-local version of the Starobinsky model [1, 2] there is no initial singularity [3–6]. The exploration of the same issue in the black hole case is much more difficult (see, e.g., [7] and also [8]) and at present there is no comprehensive investigation of the problem in this case. On the other side, the singularity which looks very similar to the one in the black hole solution can be met already in Newton gravity, in the case of a single point-like particle. In the recent works (see also [9–12]), the Newtonian singularity problem has been addressed in the framework of non-local gravity. The same non-local model has been suggested earlier by Tomboulis [16] as a version of super-renormalizable and ghost-free theory. It turns out that at least some special version of the theory, with exponential type of non-locality, the theory is really singularity-free. Let us note that much earlier, the same result concerning the absence of Newtonian singularity has been obtained in [18, 19] for a modified low-energy string effective action, which is essentially equivalent to the model of [16] and [9–12]. In the present work we shall extend this result for a set of local higher-derivative models suggested in [20].

In order to verify the presence of Newtonian singularity, one can consider metric fluctuations around Minkowski space-time,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ . Then the linearized Lagrangian provides the IR Newtonian limit in the amplitude corresponding to the one-graviton exchange between two static masses. Since the Newtonian gravity comes from the linear approximation on the flat background, at the covariant level the result is completely determined by the terms of up to the second order in curvature tensor.

Consider the general version of higher derivative gravitational action of up to the second order in curvature tensor, but without restrictions on the number of derivatives.

The corresponding action has the form

$$S = \frac{1}{4\kappa} \int d^4x \sqrt{-g} \left\{ -2R + R F_1(\square) R + R_{\mu\nu} F_2(\square) R^{\mu\nu} + R_{\mu\nu\alpha\beta} F_3(\square) R^{\mu\nu\alpha\beta} \right\}, \quad (1.1)$$

where we used notation  $\kappa = 8\pi G$ . Let us note that the cosmological constant is not included, because it is known to be very small and also does not affect singularity in the Newtonian potential of the point-like mass.

The expressions such as (1.1) emerge naturally in different physical situations. To start with, the one-loop semiclassical corrections to the gravity action produce the form factors which have exactly the form (1.1), with the non-polynomial functions  $F_{1,2,3} = F_{1,2,3}(\square/m^2)$ , typically with the logarithmic asymptotics in the far UV [21, 22]. The constant values of  $F_{1,2,3}$  correspond to the well-known fourth-derivative models of renormalizable quantum gravity [23], with existing extensive discussion of classical properties in the literature starting from [24]. Furthermore, the polynomial form of the same functions  $F_{1,2,3}$ , if being introduced into the classical action, leads to super-renormalizable models of quantum gravity [20], even if the  $\mathcal{O}(R^3)$  and other higher-order (corresponding to the order of polynomials) terms are included. In both these cases, however, the spectrum of the theory has a set of massive spin-2 excitations, some of them are always unphysical ghosts with negative kinetic energy. The problem of ghosts has a long and interesting history, but since it is not the subject of the present work, let us just readdress the reader to the recent papers [25, 26] of one of us for a brief review and further references. The models of [20] were promoted to the ghost-free non-polynomial form in [16], but the quantum properties of this version, such as (super)renormalizability are not clear yet. One can see the discussion of this question in [25, 26] and also parallel consideration in [27, 28].

The functions  $F_{1,2,3}$  used in [16] to avoid ghosts are exponential, quite different from the asymptotically logarithmic form factors of one-loop semiclassical terms of [21, 22], from the constants in the renormalizable gravity case [23], and from the polynomial form of a super-renormalizable quantum gravity models of [20]. As we have mentioned above, recently the exponential form of these functions has been used in [10–12] to cure the singularity of the Newtonian point-like solution. This result looks quite remarkable. One may think that it indicates that the strong in UV, exponential form-factors are very special and that they are necessary to cure the singularity. If it is really so, this would mean that there is some fundamental physical reason for the absence of singularity in the strongly growing in UV functions  $F_{1,2,3}$ . Then one may expect that the same will happens also in the non-linear regime, when we deal with the full black-hole solution, being it Schwarzschild or Kerr.

In the present work we are going to show that the singularity-free solutions are possible not only for the exponential functions  $F_{1,2,3}$ , but also for the polynomial versions of the functions  $F_{1,2,3}$ , including the constant functions (which is the very well-known result of [24]). Therefore, in this work our purpose is to explore the singularity problem in a different class of higher derivative theories compared to the ones which were considered in [24] and [12]. The paper is organized as follows. In section 2 we present the general theoretical background of the problem. Section 3 contains the main results, including the proof of the non-singular behavior of local higher derivative gravity theories with a real

spectrum. In section 4 we discuss the role of tensor and scalar ghosts in the cancellation of Newtonian singularity and, also, the existing relation between this cancellation and (super)renormalizability of the corresponding quantum theory. Finally, in section 5 we draw our conclusions and also present some discussions.

## 2 Modified Newtonian limit

The Newtonian limit means static weak-field approximation. So, we consider metric fluctuations around Minkowski space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (2.1)$$

To find linearized field equations we need to consider only those terms in the action which are of the second order in the perturbations  $h_{\mu\nu}$ . The following relevant observation is in order. By means of the Bianchi identities and integrations by parts one can prove that for any integer  $N$

$$\int d^4x \sqrt{-g} \left\{ R_{\mu\nu\alpha\beta} \square^N R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} \square^N R^{\mu\nu} + R \square^N R \right\} = \mathcal{O}(R^3) = \mathcal{O}(h^3). \quad (2.2)$$

Assuming that the functions  $F_{1,2,3}$  admit an expansion into power series in  $\square$ , one comes to the conclusion that the Riemann-squared term is not relevant in the linear regime. Then one can simply trade  $F_{1,2,3} \rightarrow \tilde{F}_{1,2,3}$ , where  $\tilde{F}_1 = F_1 - F_3$ ,  $\tilde{F}_2 = F_2 + 4F_3$  and  $\tilde{F}_3 = 0$ . In what follows we effectively use the relations for  $\tilde{F}_{1,2,3}$ , but do not write the tildes for simplicity of notations. So, from now on,  $F_3(\square) \equiv 0$ .

Performing the expansion in  $h_{\mu\nu}$ , the bilinear part of the action (1.1) is given by

$$\begin{aligned} \mathcal{L}_{\text{quadr}} = & -\frac{1}{4\kappa} [h^{\mu\nu} \square h_{\mu\nu} + A_\nu^2 + (A_\nu - \phi_{,\nu})^2] \\ & - \frac{1}{16\kappa} \left[ -\square h_{\mu\nu} F_2(\square) \square h^{\mu\nu} + A_{,\mu}^\mu F_2(\square) A_{,\nu}^\nu + F^{\mu\nu} F_2(\square) F_{\mu\nu} \right. \\ & \left. - (A_{,\alpha}^\alpha - \square \phi) (F_2(\square) + 4F_1(\square)) (A_{,\beta}^\beta - \square \phi) \right], \end{aligned} \quad (2.3)$$

where the vector and antisymmetric tensors are below defined in terms of the gravitational fluctuation,

$$A^\mu = h^{\mu\nu}_{,\nu}, \quad \phi = h^\mu_{,\mu} \quad (\text{trace of } h_{\mu\nu}), \quad F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}. \quad (2.4)$$

Inverting the quadratic operator in (2.3), we find the following two-point function in momentum space in terms of spin-2 projector  $P^{(2)}$  and scalar projector  $P^{(0-s)}$  (see, e.g., [23] or [31] for details),

$$G_2(k) = \frac{P^{(2)}}{k^2 (1 + k^2 F_2(-k^2)/2)} - \frac{P^{(0-s)}}{2k^2 [1 - k^2 (F_2(-k^2) + 3F_1(-k^2))]} \quad (2.5)$$

Since we are interested to find a solution for the point-like static mass source, let us take

$$T_{\mu\nu} = \rho \delta_\mu^0 \delta_\nu^0 = M \delta^3(\mathbf{r}) \delta_\mu^0 \delta_\nu^0. \quad (2.6)$$

One can then easily calculate the solution of the linear equations of motion coming from (2.3) by means of the Fourier transform of the potential. The Lagrangian for the graviton fluctuation and matter source reads

$$\mathcal{L}_h = h_{\mu\nu} (G_2^{-1})^{\mu\nu,\rho\sigma} h_{\rho\sigma} - 4\kappa h_{\mu\nu} T^{\mu\nu}. \quad (2.7)$$

Therefore in short notation,

$$\hat{h} = 2\kappa \hat{G}_2 \hat{T} \implies \varphi(r) = -\frac{h_{00}}{2}. \quad (2.8)$$

In this way, after some algebra it is possible to express the potential as

$$\varphi(r) = -\frac{2GM}{\pi r} \int_0^\infty \frac{dp}{p} \sin(pr) \left\{ \frac{4}{3(1+p^2 F_2/2)} - \frac{1}{3[1-p^2(F_2+3F_1)]} \right\}, \quad (2.9)$$

where  $F_1 = F_1(-p^2)$  and  $F_2 = F_2(-p^2)$ . We can introduce the short notation

$$a(\square) \equiv 1 - \square F_2(\square)/2 \text{ and } c(\square) \equiv 1 + \square (F_2(\square) + 3F_1(\square)), \quad (2.10)$$

for future reference. For the special case where  $a = c$  the potential is given by

$$\varphi(r) = -\frac{2Gm}{\pi r} \int_0^\infty \frac{dp}{p} \frac{\sin(pr)}{a(-p^2)}. \quad (2.11)$$

Some extra notes about the  $a = c$  case are in order. This condition can be achieved if we choose the functions  $F_i(\square)$  according to

$$F_1(\square) = \frac{a(\square) - 1}{\square}, \quad F_2(\square) = -2F_1(\square), \quad F_3(\square) = 0. \quad (2.12)$$

This means, for the non-linear case, the special form of the higher derivative part of the gravitational action

$$S = -\frac{1}{2\kappa} \int d^4x \sqrt{-g} \left\{ R + G_{\mu\nu} \frac{a(\square) - 1}{\square} R^{\mu\nu} \right\}, \quad (2.13)$$

where  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$  is the Einstein tensor. It is important to stress that, despite the relation (2.2) holds in the linear approximation, in the non-linear regime,  $F_3(\square) = 0$  can be achieved only in an *ad hoc* manner, exactly as the second relation in (2.12). In what follows we will not apply the constraint  $a = c$  and will mainly deal with the general case, except some places where it is specially indicated.

The propagator of the gravitational field in the theory (2.13) simplifies to the following form,

$$G_2(k) = \frac{1}{k^2 a(-k^2)} \left[ P^{(2)} - \frac{1}{2} P^{(0-s)} \right], \quad (2.14)$$

which has an algebraic structure that does not depend too much on the form of the function  $a(-k^2)$ , since the last enters this expression as an overall factor. The general relativity propagator can be recovered if we set  $a = 1$ . Then, in order to have the correct general relativity limit, one has to assume that in the infrared, when  $k^2 \rightarrow 0$ , this function must

satisfy the condition  $a(-k^2) \rightarrow 1$ . This requirement means that  $a(\square)$  should be a non-singular analytic function at  $k^2 = 0$  and cannot contain non-local operators such as  $1/\square$ . Furthermore, if the residue of the  $P^{(2)}$ -term coefficient at  $k^2 = 0$  is negative, the theory contains a higher derivative ghost. On the other hand, by choosing  $a(\square)$  to be an entire function, one can construct a theory being free from higher derivative ghosts [16].

Essentially the same example of entire function has been considered in refs. [9, 12, 15] (see also [13, 14], including for the non-commutative geometry case), namely

$$a(\square) = e^{-\square/m^2}. \quad (2.15)$$

For the function (2.15) the solution for the modified Newtonian potential (2.11) is

$$\varphi(r) = -\frac{GM}{r} \operatorname{erf}\left(\frac{mr}{2}\right). \quad (2.16)$$

Since  $\operatorname{erf}(r) \rightarrow r$  when  $r \rightarrow 0$ , the modified Newtonian potential has a non-singular behavior.

Is it true that the singularity disappears due to the non-locality and that the effect depends on the presence of the exponential form factor? In order to answer this question, in the next section we are going to construct other examples of the functions  $F_{1,2,3}(\square)$  leading to a non-singular Newtonian limit in  $r = 0$  and also have a correct infrared behavior at  $r \rightarrow \infty$ .

### 3 Polynomial functions

As we have already mentioned in the Introduction, the main advantage of the polynomial form factors is that the corresponding theory is (super)renormalizable, that is not certain yet for the nonlocal ghost-free models. Consider the most general local action [20]

$$\begin{aligned} S = \frac{1}{4\kappa} \int d^4x \sqrt{-g} \Big\{ & -2R + \alpha_0 R_{\mu\nu}^2 + \beta_0 R^2 + \gamma_0 R_{\mu\nu\alpha\beta}^2 + \dots \\ & + \alpha_1 R_{\mu\nu} \square R^{\mu\nu} + \beta_1 R \square R + \gamma_1 R_{\mu\nu\alpha\beta} \square R^{\mu\nu\alpha\beta} + \mathcal{O}(R_{\dots}^3) + \dots \\ & + \alpha_N R_{\mu\nu} \square^N R^{\mu\nu} + \beta_N R \square^N R + \gamma_N R_{\mu\nu\alpha\beta} \square^N R^{\mu\nu\alpha\beta} + \dots + \mathcal{O}(R_{\dots}^{N+2}) \Big\}. \end{aligned} \quad (3.1)$$

As we have already explained above, only the terms quadratic in curvature tensor may be relevant for deriving the modified Newtonian potential. Moreover, due to the relation (2.2) one can safely omit the terms with the squares of the Riemann tensor, so the result will depend only on the coefficients  $\alpha_i$  and  $\beta_i$ , with  $i = 0, 1, \dots, N$ .

The  $N = 0$  model is the 4th-order gravity [23]. For this case the solution for the modified Newtonian potential is pretty well-known [23, 24],

$$\varphi(r) = -GM \left( \frac{1}{r} - \frac{4}{3} \frac{e^{-m_{(2)}r}}{r} + \frac{1}{3} \frac{e^{-m_{(0)}r}}{r} \right). \quad (3.2)$$

The mass parameters are defined by  $m_{(2)} = (\frac{1}{2}\alpha_0)^{-1/2}$  for the tensor mode and by  $m_{(0)} = [-(3\beta_0 + \alpha_0)]^{-1/2}$  for the scalar mode. The scalar sector has a gauge-fixing ambiguity [31], but this has no importance for the scattering amplitude behind the result (3.2). For the sake

of simplicity we can say that the two parameters  $m_{(2)}$  and  $m_{(0)}$  correspond, respectively, to the masses of tensor and scalar massive degrees of freedom in the propagator of the theory.

At large distances the effects of the Yukawa corrections in (3.2) disappear and one meets a standard Newton limit in the gravitational potential. On the other hand, in the short-distance regime the situations depends on the coefficients  $\alpha_0$  and  $\beta_0$ . At the origin  $r = 0$ , expanding the exponential into power series one can easily check that the contributions of higher derivative terms to the Newtonian potential make it regular. The modified potential tends to the constant value

$$\varphi(r) = -\frac{1}{3} GM [4m_{(2)} - m_{(0)}] + \mathcal{O}(r). \quad (3.3)$$

This well-known example shows that the theory without any kind of non-locality can be free from the Newton singularity. The singularity cancellation occurs because the zero-order terms of the two different Yukawa potentials combine exactly into the coefficient  $-4/3 + 1/3 = -1$ , to cancel the original Newtonian term.

The solution for the Newtonian potential in the theory which has only Einstein-Hilbert and the square of scalar curvature terms in the action (i.e., when  $\alpha_i = \gamma_i = 0$  for  $i = 0, 1, \dots, N$  in (3.1)) was previously considered in ref. [32]. It was shown that in this case the modified Newtonian potential gains a higher derivative contribution given by a sum of Yukawa potentials, namely

$$\varphi(r) = -GM \left( \frac{1}{r} + \sum_{i=0}^N \frac{c_i}{r} e^{-m_{(0)i} r} \right), \quad (3.4)$$

where the coefficients  $c_i$  satisfy the condition

$$\sum_{i=0}^N c_i = \frac{1}{3}. \quad (3.5)$$

For the full theory with at least  $\alpha_N \neq 0$  we expect a number of new terms with Yukawa potentials coming for the Ricci tensor-squared terms. And if the coefficients of these Yukawa potentials satisfy some kind of relation like (3.5) but with the sum of coefficients equal to  $-4/3$ , then the Newtonian potential is singularity free for the general local higher derivative gravitational action of the form (1.1). The proof of this statement is the main purpose of this section.

In the linear regime the action (1.1) is equivalent to the action (3.1) with

$$F_2(\square) = \alpha_0 + \alpha_1 \square + \dots + \alpha_N \square^N, \quad (3.6)$$

$$F_1(\square) = \beta_0 + \beta_1 \square + \dots + \beta_N \square^N. \quad (3.7)$$

Consider the integral (2.9) for the Newtonian potential  $\varphi(r)$ . It is easy to see that for the case of our interest there is the following relation (2.9), (2.10):

$$\left( \frac{4}{3a} - \frac{1}{3c} \right) = \left[ \frac{4}{3} \frac{1}{P_{2N+2}} - \frac{1}{3} \frac{1}{Q_{2N+2}} \right], \quad (3.8)$$

where  $P_{2N+2}$  and  $Q_{2N+2}$  are polynomials of  $\square$  of the corresponding order. Due to eqs. (3.6)–(3.7), the functions  $P_{2N+2}$  and  $Q_{2N+2}$  can be written in terms of the coefficients  $\alpha_N$  and  $\beta_N$  as

$$P_{2N+2} = 1 + \frac{1}{2} [\alpha_0 p^2 - \alpha_1 p^4 + \cdots + (-1)^N \alpha_N p^{2N+2}], \quad (3.9)$$

$$Q_{2N+2} = 1 - (3\beta_0 + \alpha_0)p^2 + (3\beta_1 + \alpha_1)p^4 + \cdots + (-1)^{N+1} (3\beta_N + \alpha_N)p^{2N+2}. \quad (3.10)$$

Let us assume that the coefficients of the polynomials (3.9), (3.10) do not vanish, i.e.  $\alpha_i \neq 0$  and  $\alpha_i + 3\beta_i \neq 0$  for  $i = 0, 1, \dots, N$ . According to the fundamental theorem of algebra, the polynomials  $P_{2N+2}$  and  $Q_{2N+2}$  can be factorized as

$$P_{2N+2} = \frac{1}{m_{(2)0}^2 m_{(2)1}^2 \cdots m_{(2)N}^2} \times (p^2 + m_{(2)0}^2) \times (p^2 + m_{(2)1}^2) \times \cdots \times (p^2 + m_{(2)N}^2), \quad (3.11)$$

$$Q_{2N+2} = \frac{1}{m_{(0)0}^2 m_{(0)1}^2 \cdots m_{(0)N}^2} \times (p^2 + m_{(0)0}^2) \times (p^2 + m_{(0)1}^2) \times \cdots \times (p^2 + m_{(0)N}^2). \quad (3.12)$$

Here the square of the roots of eqs.  $P_{2N+2} = 0$  and  $Q_{2N+2} = 0$  are  $-m_{(2)N}^2$  and  $-m_{(0)N}^2$ , correspondingly.

In what follows we assume that by adjusting the coefficients  $\alpha_N$  and  $\alpha_N + 3\beta_N$  of subleading terms of the polynomials  $P_{2N+2}$  and  $Q_{2N+2}$  it is possible to provide that all  $m_{(k)j}$  are real quantities and

$$0 < m_{(k)0}^2 < m_{(k)1}^2 < \cdots < m_{(k)N}^2, \quad (3.13)$$

$$m_{(k)i} \neq m_{(k)j}, \quad i \neq j \quad (3.14)$$

for  $k = 0, 2$ . The last condition means that all the poles of the propagator are simple. From the physical side  $m_{(2)N}$  and  $m_{(0)N}$  corresponds to the masses of spin-2 and spin-0 massive extra degrees of freedom in the propagator of the theory (3.1).

Using the identity (3.8) and formulas (3.11), (3.12) the Newtonian potential (2.9) can be cast into the form

$$\varphi(r) = -\frac{2GM}{\pi r} \left[ \frac{4}{3} I_{(2)} - \frac{1}{3} I_{(0)} \right], \quad (3.15)$$

where

$$I_{(2)} = \int_0^\infty dp \frac{(m_{(2)0}^2 m_{(2)1}^2 \cdots m_{(2)N}^2) \sin(pr)}{p (p^2 + m_{(2)0}^2) (p^2 + m_{(2)1}^2) \cdots (p^2 + m_{(2)N}^2)} \quad (3.16)$$

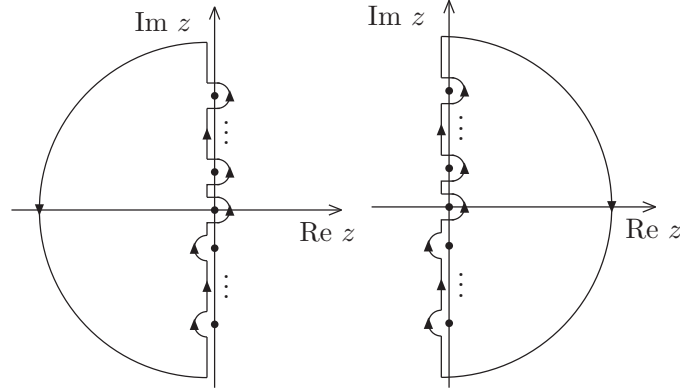
and

$$I_{(0)} = \int_0^\infty dp \frac{(m_{(0)0}^2 m_{(0)1}^2 \cdots m_{(0)N}^2) \sin(pr)}{p (p^2 + m_{(0)0}^2) (p^2 + m_{(0)1}^2) \cdots (p^2 + m_{(0)N}^2)}. \quad (3.17)$$

To evaluate the integrals  $I_{(2)}$ ,  $I_{(0)}$  we perform an analytic continuation  $p \rightarrow z$  to the complex plane  $\mathbb{C}$ . Then the integral  $I_{(2)}$  can be written as

$$I_{(2)} = \frac{W_1 - W_2}{4i}, \quad (3.18)$$





**Figure 1.** The first and second curves of the integration on the complex plane. On the left the poles at  $z = 0$  and  $z = +im_{(2)N}$  are inside the contour. On the right the poles at  $z = -im_{(2)N}$  are inside the contour.

where

$$W_1 = \oint_{\Gamma} dz \frac{(m_{(2)0}^2 m_{(2)1}^2 \cdots m_{(2)N}^2) e^{izr}}{z (z^2 + m_{(2)0}^2) (z^2 + m_{(2)1}^2) \cdots (z^2 + m_{(2)N}^2)}, \quad (3.19)$$

$$W_2 = \oint_{\Gamma} dz \frac{(m_{(2)0}^2 m_{(2)1}^2 \cdots m_{(2)N}^2) e^{-izr}}{z (z^2 + m_{(2)0}^2) (z^2 + m_{(2)1}^2) \cdots (z^2 + m_{(2)N}^2)}. \quad (3.20)$$

Since the masses  $m_{(2)j}$  are different, the integrals  $W_1, W_2$  have simple poles at the points  $z = 0$  and  $z^2 = -m_j^2$ , where  $j = 0, 1, \dots, N$ . Let  $\Gamma$  be a positively oriented simple closed path in  $\mathbb{C}$  which passes on the left of the poles on the lower half plane  $z = -im_{(2)j}$  and on the right of the poles at the points  $z = 0$  on the upper half plane  $z = +im_{(2)j}$ .

For  $W_1$  which contains  $e^{irp}$ , the contour  $\Gamma$  should be chosen in such a way that it encircles the poles at  $z = 0$  and  $z = +im_{(2)j}$ . One can see the left plot of figure 1 for the illustration. Then, using the Cauchy's residue theorem we find

$$\begin{aligned} W_1 &= +2\pi i \left\{ \text{Res} \left[ \frac{(m_{(2)0}^2 \cdots m_{(2)N}^2) e^{izr}}{(z^2 + m_{(2)0}^2) \cdots (z^2 + m_{(2)N}^2)}, z = 0 \right] \right. \\ &\quad + \text{Res} \left[ \frac{(m_{(2)0}^2 \cdots m_{(2)N}^2) e^{izr}}{z (z + im_{(2)0}) \cdots (z^2 + m_{(2)N}^2)}, z = +im_{(2)0} \right] \\ &\quad \left. + \cdots + \text{Res} \left[ \frac{(m_{(2)0}^2 \cdots m_{(2)N}^2) e^{izr}}{z (z^2 + m_{(2)0}^2) \cdots (z + im_{(2)N})}, z = +im_{(2)N} \right] \right\} \\ &= 1 - \frac{(m_{(2)1}^2 \cdots m_{(2)N}^2) e^{-m_{(2)0}r}}{2 (m_{(2)1}^2 - m_{(2)0}^2) \cdots (m_{(2)N}^2 - m_{(2)0}^2)} + \cdots - \frac{(m_{(2)0}^2 \cdots m_{(2)N-1}^2) e^{-m_{(2)N}r}}{2 (m_{(2)0}^2 - m_{(2)N}^2) \cdots (m_{(2)N}^2 - m_{(2)N-1}^2)}. \end{aligned} \quad (3.21)$$

The integral  $W_1$  is calculated in counterclockwise direction which we define to be positive.

For  $W_2$  which has  $e^{-irp}$ , the path  $\Gamma$  is chosen in such way that encircles the poles at  $z = -im_{(2)j}$ . One can look at the plot on the right in figure 1 for the illustration. The integral is evaluated in clockwise direction, then we find

$$\begin{aligned}
 W_2 &= -2\pi i \left\{ +\text{Res} \left[ \frac{\left(m_{(2)0}^2 \cdots m_{(2)N}^2\right) e^{-izr}}{z(z - im_{(2)0}) \cdots (z^2 + m_{(2)N}^2)}, z = -im_{(2)0} \right] \right. \\
 &\quad \left. + \cdots + \text{Res} \left[ \frac{\left(m_{(2)0}^2 \cdots m_{(2)N}^2\right) e^{-izr}}{z(z^2 + m_{(2)0}^2) \cdots (z - im_{(2)N})}, z = -im_{(2)N} \right] \right\} \\
 &= \frac{\left(m_{(2)1}^2 \cdots m_{(2)N}^2\right) e^{-m_{(2)0}r}}{2\left(m_{(2)1}^2 - m_{(2)0}^2\right) \cdots \left(m_{(2)N}^2 - m_{(2)0}^2\right)} + \cdots + \frac{\left(m_{(2)0}^2 \cdots m_{(2)N-1}^2\right) e^{-m_{(2)N}r}}{2\left(m_{(2)0}^2 - m_{(2)N}^2\right) \cdots \left(m_{(2)N}^2 - m_{(2)N-1}^2\right)}.
 \end{aligned} \tag{3.22}$$

Now, by using eqs. (3.21), (3.22) and (3.18) we obtain

$$I_{(2)} = \frac{\pi}{2} \left[ 1 - \sum_{i=0}^N \left( \prod_{j \neq i} \frac{m_{(2)j}^2}{m_{(2)j}^2 - m_{(2)i}^2} e^{-m_{(2)i}r} \right) \right]. \tag{3.23}$$

By an analogous consideration it is possible to evaluate  $I_{(0)}$ . We are going to leave this calculation to be an exercise for an interested reader, the answer is

$$I_{(0)} = \frac{\pi}{2} \left[ 1 - \sum_{i=0}^N \left( \prod_{j \neq i} \frac{m_{(0)j}^2}{m_{(0)j}^2 - m_{(0)i}^2} e^{-m_{(0)i}r} \right) \right]. \tag{3.24}$$

Finally, from (3.15), (3.23) and (3.24) we arrive to the final answer for the modified Newtonian potential

$$\begin{aligned}
 \varphi(r) &= -GM \left\{ \frac{1}{r} - \frac{4}{3} \sum_{i=0}^N \prod_{j \neq i} \frac{m_{(2)j}^2}{m_{(2)j}^2 - m_{(2)i}^2} \frac{e^{-m_{(2)i}r}}{r} \right. \\
 &\quad \left. + \frac{1}{3} \sum_{i=0}^N \prod_{j \neq i} \frac{m_{(0)j}^2}{m_{(0)j}^2 - m_{(0)i}^2} \frac{e^{-m_{(0)i}r}}{r} \right\}.
 \end{aligned} \tag{3.25}$$

Now let us study the behavior of potential (3.25) near the origin. When  $r \rightarrow 0$

$$\varphi(r) \rightarrow \frac{1}{r} - \frac{4}{3r} \sum_{i=0}^N \prod_{j \neq i} \frac{m_{(2)j}^2}{m_{(2)j}^2 - m_{(2)i}^2} + \frac{1}{3r} \sum_{i=0}^N \prod_{j \neq i} \frac{m_{(0)j}^2}{m_{(0)j}^2 - m_{(0)i}^2} + \text{const.} \tag{3.26}$$

For any set of numbers  $a_j$  the following relation is valid:

$$\sum_{i=0}^N \prod_{j \neq i} \frac{a_j}{a_j - a_i} = 1. \tag{3.27}$$

With this relation, one can see that the limit (3.26) goes to a constant and the modified Newtonian potential is regular in  $r = 0$ .

To illustrate the consideration of this section, let us present an exact solution for the sixth-order gravity, corresponding to  $N = 1$ . In this case the masses of the spin-2 particles are given by

$$m_{(2)0}^2 = \frac{-\alpha_0 - \sqrt{\alpha_0^2 + 4\alpha_1}}{2\alpha_1}, \quad m_{(2)1}^2 = \frac{-\alpha_0 + \sqrt{\alpha_0^2 + 4\alpha_1}}{2\alpha_1}. \quad (3.28)$$

In order for these solutions to define two different non-zero real masses, the parameters should satisfy the conditions

$$\alpha_0 > 0, \quad \alpha_1 < 0, \quad \alpha_0^2 + 4\alpha_1 > 0. \quad (3.29)$$

For the massive scalar particle, defining  $\omega_N \equiv 3\beta_N + \alpha_N$ , we have

$$m_{(0)0}^2 = \frac{\omega_0 - \sqrt{\omega_0^2 - 4\omega_1}}{2\omega_1}, \quad m_{(0)1}^2 = \frac{\omega_0 + \sqrt{\omega_0^2 - 4\omega_1}}{2\omega_1}. \quad (3.30)$$

For real different masses we need to impose

$$\omega_0 < 0, \quad \omega_1 > 0, \quad \omega_0^2 + 4\omega_1 > 0. \quad (3.31)$$

Since  $\alpha_0$  must be positive and  $\alpha_1$  must be negative, these relations are true only if

$$\beta_0 < 0, \quad \beta_1 > 0 \quad (3.32)$$

and if their absolute values satisfy

$$|\beta_0| > \frac{1}{3}|\alpha_0|, \quad |\beta_1| < \frac{1}{3}|\alpha_1|. \quad (3.33)$$

## 4 Ghosts and repulsion forces

As we know from [23], the cancellation of Newtonian singularity in the four-derivative case is due to the opposite signs of the contribution of graviton and scalar degree of freedom from one side, and the massive tensor ghost from another side. It would be interesting and useful to understand whether a similar relation takes place for the higher derivative models of [20], especially in view of the absence of singularities that takes place for the ghost-free theory of [18, 19] and [9, 15, 16].

In the case of fourth-order gravity, the eq. (3.2) shows that the massive spin-2 ghost particle contributes with an opposite sign, different from the contribution of graviton and scalar massive particle. When a test particle is approaching to the origin  $r = 0$  the gravitational force applied to it tends to zero because the repulsive force due to the ghosts cancels the attractive force of graviton plus an extra massive scalar degree of freedom. Let us show that the same situation holds for the more complicated case of superrenormalizable gravity theory. In this case, again, one can say that all ghost particles contribute with repulsive force, while the non-ghost degrees of freedom always contribute to the attractive force.

To prove this statement, let us begin considering the scalar sector of the theory. Consider the propagator of the scalar part,

$$G_2^{(0)}(k) = \left[ \frac{A_0}{k^2 + m_{(0)0}^2} + \frac{A_1}{k^2 + m_{(0)1}^2} + \cdots + \frac{A_N}{k^2 + m_{(0)N}^2} \right] P^{(0-s)}, \quad (4.1)$$

according to [20], the residues of the propagator satisfy  $A_j \cdot A_{j+1} < 0$ . For the scalar degree of freedom we have  $A_0 > 0$ . As a consequence, the residue  $A_k$  with an odd  $k$  always has a negative sign and represents a ghost particle, while the even components are always a non-ghost degrees of freedom. It proves useful to rewrite the contribution to the gravitational potential coming from the  $i$ -th massive scalar particle, that is the last term in eq. (3.26), in the form with an explicit sign dependence,

$$\varphi_{(0)i}(r) = -\frac{GM}{3}(-1)^i \prod_{j \neq i} \left| \frac{m_{(0)j}^2}{m_{(0)i}^2} - 1 \right|^{-1} \frac{e^{-m_{(0)i}r}}{r}. \quad (4.2)$$

In the last equation we used the relation (3.13) and the sign of each product is shown explicitly. For an odd  $i$  we have a ghost, that is the sign in (4.2) is positive and there is a repulsive potential. At the same time, the massive healthy particles contribute to an attractive force.

For the tensorial part the situation is similar. The total propagator of the spin-2 massive particles can be written as

$$G_2^{(2)}(k) = \left[ \frac{B_0}{k^2 + m_{(2)0}^2} + \frac{B_1}{k^2 + m_{(2)1}^2} + \cdots + \frac{B_N}{k^2 + m_{(2)N}^2} \right] P^{(2)}, \quad (4.3)$$

where the residues satisfy  $B_j \cdot B_{j+1} < 0$  [20]. Since for the spin-2 massive particles we have  $B_0 < 0$ , each  $B_k$  with an even index has a negative sign and represents a ghost. The gravitational potential for the  $i$ -th spin-2 massive particle can be written as

$$\varphi_{(2)i}(r) = +\frac{4GM}{3}(-1)^i \prod_{j \neq i} \left| \frac{m_{(2)j}^2}{m_{(2)i}^2} - 1 \right|^{-1} \frac{e^{-m_{(2)i}r}}{r}. \quad (4.4)$$

For the ghost potentials, when  $i$  is even, we have a positive sign in (4.4) and, consequently, a repulsive force.

With the simple consideration presented above, we have shown that for a point-like source the ghosts always induce a repulsive Newtonian potential. As in the fourth-order gravity, in the superrenormalizable models of [20] the singularity of the potential disappears because the repulsive force acting on a test particle due to the ghosts cancels with the attractive force of graviton and non-ghosts massive particles near  $r = 0$ .

The main point of the above consideration is that the singularity cancellation only occurs because for each massive spin-2 ghost particle we have a non-ghost massive scalar, and vice-versa. This structure of cancellation has an important consequence. If we recast the relevant part of the action (1.1) in the form

$$S = \frac{1}{4\kappa} \int d^4x \sqrt{-g} \left\{ -2R + R \Phi_1(\square) R + \frac{1}{2} C_{\mu\nu\alpha\beta} \Phi_2(\square) C^{\mu\nu\alpha\beta} \right\}, \quad (4.5)$$

where  $\Phi_2 = F_2$ ,  $\Phi_1 = F_1 + F_2/3$  and  $C^\mu_{\nu\alpha\beta}$  is the Weyl tensor, then where the form factor  $\Phi_2$  alone will define the tensor sector and the form factor  $\Phi_1$  alone, the scalar sector. Imagine that the two functions  $\Phi_1$  and  $\Phi_2$  are polynomials of the *different* orders. Then the pairs scalar particle — tensor ghost and tensor particle — scalar ghost will be broken and there will be no singularities cancellation. The effect takes place *only* when the two polynomials are of the same order. It is interesting that this corresponds, in principle, to the condition of superrenormalizability as it follows from the consideration of [20]. In case of the different orders of the two polynomials one meets non-homogeneous propagators and vertices and it is certainly possible to have some diagrams with the growing power counting index. This means that there is a direct relation between the cancellation of Newtonian singularity and quantum renormalizability properties. Of course, this relation is a kind of a *post factum* feature, which may not have deep physical meaning, but it gives, anyway, a certain hint to the quantum properties of the ghost-free theory with an exponential form-factor, as suggested in [12, 15]. As we have noted in [25, 26], the power counting in this theory is indefinite, of the  $\infty - \infty$  type, but we can not exclude at the moment a consistent way to define a quantum field theory of gravity with asymptotically exponential growth. On the other hand, the theory is perfectly well defined for exponential form factors asymptotically polynomial [9, 16]. In this case the theory is unitary and superrenormalizable or finite at quantum level [9, 16, 17]. At the same time, the absence of Newtonian singularity for the class of theories in [12, 15] tells us that the UV behavior of this theory is the right one, corresponding to the (super)renormalizable models of quantum gravity. In our opinion, this gives a strong hope and motivation to study the quantum UV divergences of this model in more details than it was done until now.

Another possibility to interpret the role of ghosts in the cancellation of singularities concerns the proposal of [33, 34] that the consistent quantum theory must describe ghosts not as individual particles, but as part of a pair of ghost and graviton. As we have seen, this idea is not working for the cancellation of Newtonian singularity and one can easily show that it is nor working also for the super-renormalizable models suggested in [20]. The consideration presented above shows that the role of the ghosts and normal particles in the singularity cancellation requires that these particles should actually enter by the pairs of scalar particle plus tensor ghost and tensor particle plus scalar ghost. This may mean that the proposal of [33, 34] should be modified accordingly.

## 5 Conclusions and discussions

We have considered the problem of point-mass singularity in the wide class of higher derivative models, including fourth derivative ones, and the higher than four-derivative theories, of the polynomial (superrenormalizable at quantum level) type. The singularity in the modified Newtonian potential disappears in the theories of (3.1), due to the cancellation of the contributions from scalar and tensor modes to the Yukawa-type potential with the initial Newtonian singularity.

The cancellation can be easily provided in the ghost-free model of [12, 16], eq. (2.15), in the fourth derivative model of [23] and in the superrenormalizable gravity. For the last two

cases the tensors and scalars contribute respectively with coefficients  $-4/3$  and  $1/3$  near the origin, and this leads to the cancellation of Newtonian singularity. Compared to the case of exponential form factors considered in [12], we see that the presence of non-polynomial and therefore non-local terms is not really necessary for the cancellation of Newtonian singularity. At the same time, it is remarkable that the local and non-local theories manifest the same property. In particular, one may expect that the non-singular feature of local theories will hold under semiclassical [21, 22] and quantum gravity corrections to the terms quadratic in curvature. At the same time, the definite answer to this question can be obtained only after more detailed analysis, which we postpone for the future work.

The effect of singularity cancellation is essentially a linear effect involving the independent contributions of scalars and tensors. Therefore, it is not certain that the cancellation may hold in this theories at the non-linear level, e.g., for the black hole solutions. Of course, the singularity avoidance in black holes by means of higher derivatives is natural and is expected to be possible, but in order to verify this phenomenon one has to go beyond the linear approximation, which works so well in the modified Newtonian case. Let us note that some aspects of the full non-linear solutions for the static spherically symmetric case in the fourth-order gravity ( $F_i = \text{constant}$ , in our notations) were considered in [24], then in [8] and, most recently, in [35]. Through the study of the asymptotic behavior of the solutions it was shown that there are different families of solutions, some equivalent to the Schwarzschild solution in GR. According to [24], one of these solutions is regular at the origin. However, in order to determine whether the solutions that match to the modified Newton solution at infinity still have singularity at  $r = 0$  or not, a more detailed analytical or numerical investigation is needed. In the more complicated theories with higher than four derivatives, which we consider here, one can expect a similar general situation in the non-linear regime and hence a complicated analysis of the general field equations is necessary to achieve concrete results about the black hole singularities. The first step in this direction was done in [36], where the theory under investigation is apparently the one of [20]. Indeed, the comprehensive investigation of the problem of  $r = 0$  singularity is very difficult and still not completed task in both cases. An important aspect is that the divergences of quantum theory are apparently related to the Newtonian singularities and not to the ones at the non-linear level of modified.

One may think about some relation between the absence of Newtonian singularity in classical theory and asymptotic freedom at quantum level. Such a relation would be somehow natural, because Newtonian singularity is indeed the simplest UV divergence due to the interaction. So, when the singularity disappears, it looks like a kind of an UV screening of the interaction [37]. However, our results show that the relation with asymptotic freedom is not so relevant. Indeed, the cancellation of singularity occurs in all superrenormalizable models of [20] if the massive spin-2 and spin-0 excitations correspond to real simple poles. At the same time, most of the coupling constants in the theories described in [20] are not renormalized. For example, in case of  $N \geq 3$  the one-loop  $\beta$ -functions are exact and they are non-zero only for the zero-, two- and four-derivative terms. Furthermore, these  $\beta$ -functions depend on the coupling in the highest derivative sectors (but not on the gauge fixing!), and their sign can be deliberately changed by tuning

these highest derivative couplings, while the couplings in the higher than four-derivative sector are not renormalized. As we saw, this does not affect the cancellation of classical Newtonian singularity.

Another possibility is to look for some relation with the (super)renormalizability of the theory. Indeed, that the singularities are canceled in the renormalizable theory fourth-derivative is known already from the first works of Stelle [23, 24]. We can say that this is also true for at least some of the superrenormalizable models of [20], and for the non-local model of [9, 16]. So, in reality there is a strong correspondence between quantum and classical properties in this case. However, the complete answer to this question is possible only after further analysis of the problem, taking into account the theories with complex and multiple poles in the propagator. One observation concerning this issue has been done recently by one of us in [38]. The cancelation of Newtonian singularity in the exponential gravity of [18, 19] and [16] can be actually seen as an effect of an infinite amount of hidden ghost-like complex-poles states in this theory, presumably acting in a way similar to eqs. (4.4) and (4.2). This situation represents an additional strong motivation to explore all aspects of the theories with complex poles with a special attention.

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## References

- [1] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett. B* **91** (1980) 99 [[INSPIRE](#)].
- [2] M.V. Fischetti, J.B. Hartle and B.L. Hu, *Quantum Effects in the Early Universe. I. Influence of Trace Anomalies on Homogeneous, Isotropic, Classical Geometries*, *Phys. Rev. D* **20** (1979) 1757 [[INSPIRE](#)].
- [3] P.R. Anderson, *Effects of quantum fields on singularities and particle horizons in the early universe*, *Phys. Rev. D* **28** (1983) 271 [[INSPIRE](#)].
- [4] P.R. Anderson, *Effects of quantum fields on singularities and particle horizons in the early universe. II*, *Phys. Rev. D* **29** (1984) 615 [[INSPIRE](#)].



- [5] P.R. Anderson, *Effects of Quantum Fields on Singularities and Particle Horizons in the Early Universe. III. The Conformally Coupled Massive Scalar Field*, *Phys. Rev. D* **32** (1985) 1302 [[INSPIRE](#)].
- [6] P.R. Anderson, *Effects of Quantum Fields on Singularities and Particle Horizons in the Early Universe. IV. Initially Empty Universes*, *Phys. Rev. D* **33** (1986) 1567 [[INSPIRE](#)].
- [7] V.P. Frolov and G.A. Vilkovisky, *Spherically Symmetric Collapse in Quantum Gravity*, *Phys. Lett. B* **106** (1981) 307 [[INSPIRE](#)].
- [8] V.P. Frolov and I.L. Shapiro, *Black Holes in Higher Dimensional Gravity Theory with Quadratic in Curvature Corrections*, *Phys. Rev. D* **80** (2009) 044034 [[arXiv:0907.1411](#)] [[INSPIRE](#)].
- [9] L. Modesto, *Super-renormalizable Quantum Gravity*, *Phys. Rev. D* **86** (2012) 044005 [[arXiv:1107.2403](#)] [[INSPIRE](#)].
- [10] T. Biswas, A. Mazumdar and W. Siegel, *Bouncing universes in string-inspired gravity*, *JCAP* **03** (2006) 009 [[hep-th/0508194](#)] [[INSPIRE](#)].
- [11] T. Biswas, T. Koivisto and A. Mazumdar, *Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity*, *JCAP* **11** (2010) 008 [[arXiv:1005.0590](#)] [[INSPIRE](#)].
- [12] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, *Towards singularity- and ghost-free theories of gravity*, *Phys. Rev. Lett.* **108** (2012) 031101 [[arXiv:1110.5249](#)] [[INSPIRE](#)].
- [13] P. Nicolini, A. Smailagic and E. Spallucci, *Noncommutative geometry inspired Schwarzschild black hole*, *Phys. Lett. B* **632** (2006) 547 [[gr-qc/0510112](#)] [[INSPIRE](#)].
- [14] L. Modesto, J.W. Moffat and P. Nicolini, *Black holes in an ultraviolet complete quantum gravity*, *Phys. Lett. B* **695** (2011) 397 [[arXiv:1010.0680](#)] [[INSPIRE](#)].
- [15] L. Modesto, *Super-renormalizable Higher-Derivative Quantum Gravity*, [arXiv:1202.0008](#) [[INSPIRE](#)].
- [16] E.T. Tomboulis, *Superrenormalizable gauge and gravitational theories*, [hep-th/9702146](#) [[INSPIRE](#)].
- [17] L. Modesto and L. Rachwal, *Super-renormalizable and finite gravitational theories*, *Nucl. Phys. B* **889** (2014) 228 [[arXiv:1407.8036](#)] [[INSPIRE](#)].
- [18] A.A. Tseytlin, *On singularities of spherically symmetric backgrounds in string theory*, *Phys. Lett. B* **363** (1995) 223 [[hep-th/9509050](#)] [[INSPIRE](#)].
- [19] W. Siegel, *Stringy gravity at short distances*, [hep-th/0309093](#) [[INSPIRE](#)].
- [20] M. Asorey, J.L. Lopez and I.L. Shapiro, *Some remarks on high derivative quantum gravity*, *Int. J. Mod. Phys. A* **12** (1997) 5711 [[hep-th/9610006](#)] [[INSPIRE](#)].
- [21] E.V. Gorbar and I.L. Shapiro, *Renormalization group and decoupling in curved space*, *JHEP* **02** (2003) 021 [[hep-ph/0210388](#)] [[INSPIRE](#)].
- [22] E.V. Gorbar and I.L. Shapiro, *Renormalization group and decoupling in curved space. 2. The standard model and beyond*, *JHEP* **06** (2003) 004 [[hep-ph/0303124](#)] [[INSPIRE](#)].
- [23] K.S. Stelle, *Renormalization of Higher Derivative Quantum Gravity*, *Phys. Rev. D* **16** (1977) 953 [[INSPIRE](#)].



- [24] K.S. Stelle, *Classical Gravity with Higher Derivatives*, *Gen. Rel. Grav.* **9** (1978) 353 [[INSPIRE](#)].
- [25] F. de O. Salles and I.L. Shapiro, *Do we have unitary and (super)renormalizable quantum gravity below the Planck scale?*, *Phys. Rev. D* **89** (2014) 084054 [[arXiv:1401.4583](#)] [[INSPIRE](#)].
- [26] I.L. Shapiro, A.M. Pelinson and F. de O. Salles, *Gravitational Waves and Perspectives for Quantum Gravity*, *Mod. Phys. Lett. A* **29** (2014) 1430034 [[arXiv:1410.2581](#)] [[INSPIRE](#)].
- [27] L. Modesto, *Super-renormalizable Multidimensional Quantum Gravity*, [arXiv:1202.3151](#) [[INSPIRE](#)].
- [28] L. Modesto, *Finite Quantum Gravity*, [arXiv:1305.6741](#) [[INSPIRE](#)].
- [29] T. Biswas, T. Koivisto and A. Mazumdar, *Nonlocal theories of gravity: the flat space propagator*, [arXiv:1302.0532](#) [[INSPIRE](#)].
- [30] T. Biswas, A. Conroy, A.S. Koshelev and A. Mazumdar, *Generalized ghost-free quadratic curvature gravity*, *Class. Quant. Grav.* **31** (2014) 015022 [Erratum *ibid.* **31** (2014) 159501] [[arXiv:1308.2319](#)] [[INSPIRE](#)].
- [31] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, *Effective Action in Quantum Gravity*, IOP Publishing, Bristol, U.K. (1992).
- [32] I. Quandt and H.-J. Schmidt, *The Newtonian limit of fourth and higher order gravity*, *Astron. Nachr.* **312** (1991) 97 [[gr-qc/0109005](#)] [[INSPIRE](#)].
- [33] S.W. Hawking, *Who's Afraid Of (higher Derivative) Ghosts?*, in *Quantum Field Theory and Quantum Statistics* **2** (1986) 129–139 [[INSPIRE](#)].
- [34] S.W. Hawking and T. Hertog, *Living with ghosts*, *Phys. Rev. D* **65** (2002) 103515 [[hep-th/0107088](#)] [[INSPIRE](#)].
- [35] H. Lü, A. Perkins, C.N. Pope and K.S. Stelle, *Black Holes in Higher-Derivative Gravity*, [arXiv:1502.01028](#) [[INSPIRE](#)].
- [36] B. Holdom, *On the fate of singularities and horizons in higher derivative gravity*, *Phys. Rev. D* **66** (2002) 084010 [[hep-th/0206219](#)] [[INSPIRE](#)].
- [37] C. Bambi, D. Malafarina and L. Modesto, *Terminating black holes in asymptotically free quantum gravity*, *Eur. Phys. J. C* **74** (2014) 2767 [[arXiv:1306.1668](#)] [[INSPIRE](#)].
- [38] I.L. Shapiro, *Counting ghosts in the “ghost-free” non-local gravity*, *Phys. Lett. B* **744** (2015) 67 [[arXiv:1502.00106](#)] [[INSPIRE](#)].