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Предназначен для студентов, аспирантов, молодых ученых, преподавателей в области естественных наук и высшей математики.

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СВЯЗЬ МЕЖДУ ПРОИЗВОДЯЩЕЙ ФУНКЦИЕЙ И ГЕНЕРАТОРОМ КАНОНИЧЕСКИХ (АНТИКАНОНИЧЕСКИХ) ПРЕОБРАЗОВАНИЙ

О.В.Колесников
Научный руководитель: профессор, д.ф.-м.н. П.М. Лавров
Национальный исследовательский Томский государственный университет,
Россия, г. Томск, пр. Ленина, 36, 634050
E-mail: kolesnikov-ov@bk.ru

RELATION BETWEEN GENERATING FUNCTION AND GENERATOR OF CANONICAL (ANTICANONICAL) TRANSFORMATIONS

O.V.Kolesnikov
Scientific Supervisor: Prof., Dr. P.M. Lavrov
National Research Tomsk State University, Russia, Tomsk, Lenin str., 36, 634050
E-mail: kolesnikov-ov@bk.ru

Annotation*. В работе изучаются канонические (антиканонические) преобразования при наличии антикоммутирующих переменных. Устанавливается связь между описанием этих преобразований в терминах производящих функций и в терминах генераторов.*

Canonical transformations play an important role in classical mechanics (see, for example, Arnold, 1978) and field theories (see Rothe, Rothe, 2010 and recent paper by Batalin, Lavrov, Tyutin, 2015). Canonical transformations are defined as change of canonical variables preserving the Poisson bracket. We consider the general case, when canonical variables include on equal footing commuting and anticommuting ones. Quantum theory of general gauge fields gives us another type of transformations based on using the so-called antibracket (Batalin, Vilkovysky, 1981, 1983). These transformations are called anticanonical ones and leave the antibracket invariant. On the other side the antibracket can be considered as counterpart to the Poisson (super)bracket. It is known that canonical (anticanonical) transformations can be described in terms of generating function. Also there is another way to describe these transformations, which involves special function called generator. In the present paper we study connection between these two functions.

We work with commuting and anticommuting variables, which can be described in terms of Berezin algebra. For any element θ of the algebra we can define Grassmann parity $\varepsilon(\theta)$ by the rule

$$\varepsilon(\theta) = \begin{cases} 1, & \textit{for odd element} \\ 0, & \textit{for even element} \end{cases}.$$

Grassmann parity of

$$\varepsilon(\theta_3) = (\varepsilon(\theta_1) + \varepsilon(\theta_2))(\text{mod } 2).$$

Using Grassmann parity we can write rule

$$\theta_1\theta_2 = (-1)^{\varepsilon(\theta_1)\varepsilon(\theta_2)}\theta_2\theta_1.$$

It can be shown, that any function of anticommuting elements of Berezin algebra is a polynomial with defined degree. Considering this, derivative can be defined as usual, inspite the fact, that there are two kinds of derivatives (left and right), which have a relation between them:

$$\theta\frac{\overleftarrow{\partial}}{\partial x} = (-1)^{\varepsilon(x)(\varepsilon(\theta)+1)}\frac{\overrightarrow{\partial}}{\partial x}\theta.$$

Here $\frac{\overleftarrow{\partial}}{\partial x}$ and $\frac{\overrightarrow{\partial}}{\partial x}$ are right and left derivatives respectively, x is some variable with defined Grassmann parity.

First of all we introduce variables used in the paper: (q^i, p_i) is a set of canonical variables. Coordinates and momentums are supposed to be variables of the same Grassmann parity:

$$\varepsilon(q_i) = \varepsilon(p_i).$$

We can include the whole set of variables in X^A:

$$X^A = (q^i, p_i).$$

Similarly we introduce anticanonical variables (φ^i, φ^*_i):

$$\varepsilon(\varphi^*_i) = \varepsilon(\varphi_i) + 1,$$

which also can be denoted as Z^A:

$$Z^A = (\varphi^i, \varphi^*_i).$$

It is important to note that anticanonical variables have different parity in contrast with canonical ones.

Let us define symplectic structures called Poisson bracket and antibracket

$$\{F, G\} = F\left(\frac{\overleftarrow{\partial}}{\partial q^i}\frac{\overrightarrow{\partial}}{\partial p_i} - \frac{\overleftarrow{\partial}}{\partial p_i}\frac{\overrightarrow{\partial}}{\partial q^i}\right)G,$$

$$(F, G) = F\left(\frac{\overleftarrow{\partial}}{\partial \varphi^i}\frac{\overrightarrow{\partial}}{\partial \varphi^*_i} - \frac{\overleftarrow{\partial}}{\partial \varphi^*_i}\frac{\overrightarrow{\partial}}{\partial \varphi^i}\right)G$$

respectively.

Grassmann parity of the structures:

$$\varepsilon(\{F, G\}) = \varepsilon(F) + \varepsilon(G),$$

$$\varepsilon((F, G)) = \varepsilon(F) + \varepsilon(G) + 1.$$

Using the notations

$$X^A = (q^i, p_i)$$
$$Z^A = (\varphi^i, \varphi^*_i),$$

we can write down the useful presentation of Poisson bracket and antibracket:

$$\{F, G\} = F\frac{\overleftarrow{\partial}}{\partial X^A}E^{AB}\frac{\overrightarrow{\partial}}{\partial X^B}G,$$

$$(F, G) = F\frac{\overleftarrow{\partial}}{\partial Z^A}D^{AB}\frac{\overrightarrow{\partial}}{\partial Z^B}G,$$

where

$$E^{AB} = \{X^A, X^B\}, E^{AB} = -(-1)^{\varepsilon(X^A)\varepsilon(X^B)}E^{BA},$$

$D^{AB} = (Z^A, Z^B)$, $D^{AB} = -(-1)^{(\varepsilon(x^A)+1)(\varepsilon(x^B)+1)} D^{BA}$.

Let us introduce canonical transformations:

$$\begin{cases} q^i \to Q^i \\ p_i \to P_i \end{cases},$$

and anticanonical transformations:

$$\begin{cases} \varphi^i \to \Phi^i \\ \varphi^*{}_i \to \Phi^*{}_i \end{cases}.$$

Using the notations described before:

$$X^A \to \overline{X}^A,$$

where

$$\overline{X}^A = (Q^i, P_i)$$
$$\{\overline{X}^A, \overline{X}^B\} = E^{AB}.$$

Similarly for anticanonical transformations:

$$Z^A \to \overline{Z}^A$$
$$\overline{Z}^A = (\Phi^i, \Phi^*{}_i)$$
$$(\overline{Z}^A, \overline{Z}^B) = D^{AB}.$$

In terms of generating function $Y = Y(q, P)$:

$$Q^i = \frac{\vec{\partial} Y}{\partial P_i}, p_i = Y \frac{\overleftarrow{\partial}}{\partial q^i}.$$

Similarly for anticanonical transformations with generating function $K = K(\varphi, \Phi^*)$:

$$\Phi^i = \frac{\vec{\partial} K}{\partial \Phi^*{}_i}, \varphi^*{}_i = K \frac{\overleftarrow{\partial}}{\partial \varphi^i}.$$

There exist another way to describe these transformations. Let us introduce function $F = F(q, p)$ called generator for canonical transformations and generator $G = G(\varphi, \varphi^*)$ for anticanonical ones.

Then we have

$$\overline{X}^A = \overline{X}^A(X, a) = e^{a\hat{F}} X^A,$$

where $\hat{F}$ is a first order differential operator:

$$\hat{F}(M) = \{F, M\}.$$

By the same way we define anticanonical transformations using generator G :

$$\overline{Z}^A = \overline{Z}^A(Z, b) = e^{b\hat{G}} Z^A,$$

where $\hat{G}$ is a first order differential operator as well:

$$\hat{G}(M) = (G, M).$$

In present paper we study canonical and anticanonical transformations of variables paying attention to Grassmann parity of them. Those transformations are considered in terms of generating functional and in terms of generators. As a result we obtain relations between generating functions and generators of canonical (and anticanonical, as well) transformations.

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