ФГБОУ ВО «МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНОЛОГИЧЕСКИЙ УНИВЕРСИТЕТ «СТАНКИН»

ИНСТИТУТ БИОФИЗИКИ КЛЕТКИ РОССИЙСКОЙ АКАДЕМИИ НАУК – ОБОСОБЛЕННОЕ ПОДРАЗДЕЛЕНИЕ ФЕДЕРАЛЬНОГО ГОСУДАРСТВЕННОГО БЮДЖЕТНОГО УЧРЕЖДЕНИЯ НАУКИ «ФЕДЕРАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ ЦЕНТР «ПУЩИНСКИЙ НАУЧНЫЙ ЦЕНТР БИОЛОГИЧЕСКИХ ИССЛЕДОВАНИЙ – ФИЦ ПНЦБИ РАН (*ПУЩИНО)*

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THE MODELING OF NONLINEAR PROCESSES AND SYSTEMS

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SEMICLASSICAL DYNAMICS OF QUASIPARTICLES IN THE NONLOCAL FISHER-KPP MODEL

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Abstract: We construct asymptotic solutions to the nonlocal Fisher-KPP equation using the formalism of the semiclassical approximation based on the Maslov complex germ method. The attractive feature of the solutions in this work is that their behave as quasiparticles whose dynamics stems from the original FKPP equation.

Keywords: semiclassical asymptotics; Maslov method; nolocal Fisher-KPP equation; weak diffusion; quasiparticles.

Models of a fairly wide class of nonlinear phenomena in physical and biological systems with long-range interaction are formulated mainly in terms of multidimensional integro-differential equations with variable coefficients responsible for the external impact on the system.

The mathematical complexity of such equations extremely limits the ability to study such model equations applying analytical methods. As a rule, the use of analytical approaches is associated with significant simplifications, which often leaves the most interesting aspects of the behavior of the systems under study out of sight.

To cope, perhaps partially, with this immanent obstacle, one can involve effective methods and approaches that have proven themselves in similar problems. In this context, the method of semiclassical asymptotics, based on the WKB-Maslov theory [1,2], proved to be fruitful for a special class of nonlinear evolution equations with nonlocal nonlinearity, which, in particular, include nonlocal generalizations of the nonlinear Schrödinger equation, the Gross-Pitaevskii equation in the theory of Bose-Einstein condensates [3], the generalized nonlocal version of the Fisher-Kolmogorov-Petrovskii-Piskunov (FKPP) population equation [4] and some others.

In this work, we develop this formalism for the nonlocal FKPP equation that reads

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} + a(x,t)u(x,t) - \kappa u(x,t)\int_{-\infty}^{\infty} b(x,y,t)u(y,t)dy.$$

The nonlocal FKPP equation describes one of possible mechanisms of pattern formation that relates to one of the important phenomenon arising in population biology.

Within such model, the coefficient $a(x,t)$ stands for the reproduction rate, $b(x,y,t)$ describes the competition losses, and $u(x,t)$ is the population density. The study of these phenomena is carried out mainly by computer modeling, and known analytical methods provide only indirect information. In comparison, the semiclassical method has an advantage because it has the potential for modification allowing one to describe some aspects of pattern dynamics in terms of the properties of semiclassical asymptotics.

In this work, we extend the method described in detail in [4] to be able to construct quasiparticles-like asymptotic solutions in a special class of semiclassically concentrated functions for the one-dimensional nonlocal FKPP equation in the weak diffusion approximation [5].

In the framework of the approach developed, we deduce a dynamic system that describes the movement of quasiparticles, the interaction of which stems from nonlocal competitive losses included in the generalized FKPP equation under consideration.

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