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В материалах научной конференции DCCN-2021, подготовленных к выпуску к.ф.-м.н. Козыревым Д.В., обсуждены перспективы развития и сотрудничества в этой сфере.

Сборник материалов конференции предназначен для научных работников и специалистов в области управления крупномасштабными системами.

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Asymptotic Diffusion Analysis of an Retrial Queueing System M/M/1 with Impatient Calls

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Abstract

In the paper, the retrial queueing system of M/M/1 type with input Poisson flow of events and impatient calls is considered. The service time, delay time of calls in the orbit and the impatience time of calls in the orbit have exponential distribution. Asymptotic diffusion analysis method is proposed for the solving problem of finding distribution of the number of calls in the orbit under a long delay of calls in orbit and long time patience of calls in the orbit condition.

Keywords: Retrial queueing system, Impatient calls, Asymptotic diffusion analysis

1. Introduction

At the present time retrial queueing systems (RQ-systems) research is in the demand as evidenced by numerous papers in this area and grants support. The systems as mathematical models are very suitable for modern telecommunication systems, networks, mobile networks describing. Along with the construction of mathematical models of RQ-systems, new methods of their study are being developed. A fairly new method is asymptotic diffusion analysis method, as a modification of the asymptotic analysis method. Both of them are suggested by the Tomsk research school, and there are interesting works [1, 2, 3, 4], in which the asymptotic diffusion analysis method is used.

The present paper is devoted to study of the retrial queueing system of M/M/1 with impatient calls by the asymptotic diffusion analysis method.

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2. Mathematical Model

We consider an retrial queueing system with one server and Poisson arrival process with intensity λ . An arriving call (or customer) that has found the service device free takes it for the service for a random time distributed exponentially with parameter μ . If the device is busy, calls that arrive go into the orbit. On the orbit, each call, independently of others, waits for a random time whose duration has an exponential distribution with parameter σ , and then again accesses the device with a second attempt to obtain servicing. If the device is free, the call from orbit occupies it for random servicing time. If the device is busy, call immediately goes into the orbit and wait once more random time. Moreover, a call from the orbit leaves the system after exponential distributed time with parameter α , demonstrating the “impatience” property.

The problem is to find the stationary distribution of the number of calls in the orbit. This problem has been solved in [5] by the asymptotic analysis method under a long time patience of calls in the orbit condition. In the present paper we use asymptotic diffusion analysis under a long delay of calls in orbit and long time patience of calls in the orbit condition to study the stationary distribution $P(i)$ of the number of calls in the orbit.

3. Process of the System States: System of Kolmogorov Differential Equations in terms of Partial Characteristic Functions

Let us consider Markovian process $\{k(t), i(t)\}$ determined states of the considered RQ-system where $i(t)$ is the number of calls in the orbit at the moment t , $i(t) = 0, 1, 2, 3, \dots$, $k(t)$ defines device state at the moment t and takes one of the following values: $k(t) = 0$, if server is free at the moment t , and $k(t) = 1$, if server is busy at the moment t .

Denote as $P_0(i, t) = P\{k(t) = 0, i(t) = i\}$ and $P_1(i, t) = P\{k(t) = 1, i(t) = i\}$ the probability that, at the moment t , there are i calls in the orbit, $i = 0, 1, 2, \dots$, and the service device is free or the server is busy respectively.

Introduce the partial characteristic functions

$$H_k(u) = \sum_{i=0}^{\infty} e^{ju i} P_k(i, t), \quad k = 0, 1, \quad j = \sqrt{-1}. \quad (1)$$

To obtain the probability distribution $P_0(i, t), P_1(i, t)$ for the states of the retrial queue M/M/1 with impatient calls in the orbit, we construct a system of Kolmogorov differential equations [5] and write it in terms of partial characteristic functions (1)

$$\begin{cases} \frac{\partial H_0(u, t)}{\partial t} = -\lambda H_0(u, t) + \mu H_1(u, t) + j(\sigma + \alpha(1 - e^{-ju})) \frac{\partial H_0(u, t)}{\partial u}, \\ \frac{\partial H_1(u, t)}{\partial t} = \lambda H_0(u, t) - \mu H_1(u, t) - \lambda(1 - e^{ju}) H_1(u, t) - j\sigma e^{-ju} \frac{\partial H_0(u, t)}{\partial u} \\ + j\alpha(1 - e^{-ju}) \frac{\partial H_1(u, t)}{\partial u}. \end{cases} \quad (2)$$

In adding the first equation by the second equation of (2) we get (3)

$$\frac{\partial H(u, t)}{\partial t} = (1 - e^{-ju}) \left(\lambda e^{ju} H_1(u, t) + j(\sigma + \alpha) \frac{\partial H_0(u, t)}{\partial u} + j\alpha \frac{\partial H_1(u, t)}{\partial u} \right), \quad (3)$$

where $H(u, t) = H_0(u, t) + H_1(u, t)$.

We use the system (2) and equation (3) for diffusion approximation in three stages: 1) obtaining the drift (transfer) coefficient; 2) centering the process and obtaining the diffusion coefficient; 3) diffusion approximation.

4. Obtaining the Drift (Transfer) Coefficient

In the system (2) and equation (3), we make the substitutions $\sigma = \varepsilon$, $\alpha = q\varepsilon$, $u = \varepsilon w$, $\tau = \varepsilon t$, $H_k(u, t) = F_k(w, \varepsilon, \tau)$, $k = 0, 1$, where ε is infinitesimal value, so

$$\begin{cases} \varepsilon \frac{\partial F_0(w, \varepsilon, \tau)}{\partial \tau} = -\lambda F_0(w, \varepsilon, \tau) + \mu F_1(w, \varepsilon, \tau) + j(1 + q - qe^{-jw\varepsilon}) \frac{\partial F_0(w, \varepsilon, \tau)}{\partial w}, \\ \varepsilon \frac{\partial F_1(w, \varepsilon, \tau)}{\partial \tau} = \lambda F_0(w, \varepsilon, \tau) - \mu F_1(w, \varepsilon, \tau) - \lambda(1 - e^{jw\varepsilon}) F_1(w, \varepsilon, \tau) \\ - j e^{-jw\varepsilon} \frac{\partial F_0(w, \varepsilon, \tau)}{\partial w} + j q (1 - e^{-jw\varepsilon}) \frac{\partial F_1(w, \varepsilon, \tau)}{\partial w}, \\ \varepsilon \frac{\partial F(w, \varepsilon, \tau)}{\partial \tau} = (e^{jw\varepsilon} - 1) \left(\lambda F_1(w, \varepsilon, \tau) + j(1 + q) e^{-jw\varepsilon} \frac{\partial F_0(w, \varepsilon, \tau)}{\partial w} \right. \\ \left. + j q e^{-jw\varepsilon} \frac{\partial F_1(w, \varepsilon, \tau)}{\partial w} \right). \end{cases} \quad (4)$$

Transform the equations of (4) under $\varepsilon \rightarrow 0$ with $F_k(w, \tau) = \lim_{\varepsilon \rightarrow 0} F_k(w, \varepsilon, \tau)$, $k = 0, 1$, and find their solution $F_k(w, \tau)$, $k = 0, 1$, in the form

$$F_k(w, \tau) = R_k \exp \{jwx(\tau)\}, \quad k = 0, 1, \quad (5)$$

where $R_k = H_k(0)$, $k = 0, 1$, $x(\tau)$ - unknown function of time τ .

Substituting (5) in (4) we get the following

$$\begin{cases} R_0 = R_0(x(\tau)) = \frac{\mu}{\lambda + \mu + x(\tau)}, \\ R_1 = R_1(x(\tau)) = \frac{\lambda + x(\tau)}{\lambda + \mu + x(\tau)}, \\ x'(\tau) = a(x(\tau)) = \lambda - qx(\tau) - (\lambda + x(\tau)) R_0(x(\tau)). \end{cases} \quad (6)$$

5. Centering the Process and Obtaining the Diffusion Coefficient

In (2) and (3) we let

$$H_k(u, t) = \exp \left\{ \frac{ju}{\sigma} x(\sigma t) \right\} H_k^{(2)}(u, t), \quad k = 0, 1, \quad (7)$$

and then we make the substitutions $\sigma = \varepsilon^2$, $\alpha = q\varepsilon^2$, $u = \varepsilon w$, $\tau = \varepsilon^2 t$, $H_k^{(2)}(u, t) = F_k^{(2)}(w, \varepsilon, \tau)$, $k = 0, 1$, to obtain the system below after some transformations

$$\left\{ \begin{array}{l} jw\varepsilon a(x(\tau))F_0^{(2)}(w, \varepsilon, \tau) = \mu F_1^{(2)}(w, \varepsilon, \tau) - [\lambda + x(\tau) + qjw\varepsilon x(\tau)]F_0^{(2)}(w, \varepsilon, \tau) \\ + j\varepsilon \frac{\partial F_0^{(2)}(w, \varepsilon, \tau)}{\partial w} + O(\varepsilon^2), \\ jw\varepsilon a(x(\tau))F_1^{(2)}(w, \varepsilon, \tau) = [\lambda + (1 - jw\varepsilon)x(\tau)]F_0^{(2)}(w, \varepsilon, \tau) \\ + [\lambda jw\varepsilon - \mu - qjw\varepsilon x(\tau)]F_1^{(2)}(w, \varepsilon, \tau) - j\varepsilon \frac{\partial F_0^{(2)}(w, \varepsilon, \tau)}{\partial w} + O(\varepsilon^2), \\ \varepsilon^2 \frac{\partial F^{(2)}(w, \varepsilon, \tau)}{\partial \tau} + jw\varepsilon a(x(\tau))F^{(2)}(w, \varepsilon, \tau) = \left(jw\varepsilon + \frac{(jw\varepsilon)^2}{2} \right) \\ \left\{ - (1 + q)(1 - jw\varepsilon)x(\tau)F_0^{(2)}(w, \varepsilon, \tau) + [\lambda - q(1 - jw\varepsilon)x(\tau)]F_1^{(2)}(w, \varepsilon, \tau) \right. \\ \left. + j\varepsilon(1 + q) \frac{\partial F_0^{(2)}(w, \varepsilon, \tau)}{\partial w} + jq\varepsilon \frac{\partial F_1^{(2)}(w, \varepsilon, \tau)}{\partial w} \right\} + O(\varepsilon^3). \end{array} \right. \quad (8)$$

The solution of equations system (8) has the following form

$$\left\{ \begin{array}{l} F_k^{(2)}(w, \varepsilon, \tau) = \Phi(w, \tau) (R_k + jw\varepsilon f_k) + O(\varepsilon^2), \quad k = 0, 1, \\ R_0 + R_1 = 1, \end{array} \right. \quad (9)$$

where $R_k = R_k(x(\tau))$, $k = 0, 1$, are defined above, f_0, f_1 , ($f_0 + f_1 = f$), are constants, and $\Phi(w, \tau)$ is determined function.

Using (6) and (9) in (8) after transformations we can get

$$\left\{ \begin{array}{l} -[\lambda + x(\tau)]f_0 + \mu f_1 = [a(x(\tau)) + qx(\tau)]R_0 - R_0 \frac{\partial \Phi(w, \tau)/\partial w}{w\Phi(w, \tau)}, \\ [\lambda + x(\tau)]f_0 - \mu f_1 = [a(x(\tau)) - \lambda + qx(\tau)]R_1 + x(\tau)R_0 + R_0 \frac{\partial \Phi(w, \tau)/\partial w}{w\Phi(w, \tau)}, \\ \frac{\partial \Phi(w, \tau)}{\partial \tau} = (jw)^2 \Phi(w, \tau) \{ -a(x(\tau))f + qx(\tau)R_1 + [\lambda - qx(\tau)]f_1 \\ - (1 + q)x(\tau)f_0 + (1 + q)x(\tau)R_0 \} - w(1 + q)R_0 \frac{\partial \Phi(w, \tau)}{\partial w} \\ - wqR_1 \frac{\partial \Phi(w, \tau)}{\partial w} + \frac{(jw)^2}{2} a(x(\tau))\Phi(w, \tau). \end{array} \right. \quad (10)$$

The solution of system (10) has the form

$$f_k = CR_k + g_k - \varphi_k \frac{\partial \Phi(w, \tau) / \partial w}{w \Phi(w, \tau)}, \quad k = 0, 1, \quad (11)$$

and after substitution (11) in the first and the second equations of the (10) we obtain the equations systems (12), (13) for the φ_k and g_k , $k = 0, 1$, respectively

$$\begin{cases} -[\lambda + x(\tau)] g_0 + \mu g_1 = [a(x(\tau)) + qx(\tau)] R_0, \\ [\lambda + x(\tau)] g_0 - \mu g_1 = [a(x(\tau)) - \lambda + qx(\tau)] R_1 + x(\tau) R_0, \end{cases} \quad (12)$$

$$\begin{cases} [\lambda + x(\tau)] \varphi_0 - \mu \varphi_1 = -R_0, \\ -[\lambda + x(\tau)] \varphi_0 + \mu \varphi_1 = R_0. \end{cases} \quad (13)$$

Equations (6) and additional condition $g_0 + g_1 = 0$ for the (12) lead us to (14)

$$\begin{cases} \varphi_k = \varphi_k(x(\tau)) = \frac{\partial R_k(x(\tau))}{\partial x(\tau)}, \quad \varphi_0 + \varphi_1 = 0, \quad k = 0, 1, \\ g_0 = g_0(x(\tau)) = -\frac{a(x(\tau)) + qx(\tau)}{\lambda + \mu + x(\tau)} R_0(x(\tau)), \quad g_1 = -g_0. \end{cases} \quad (14)$$

The third equation of the (10) with (6), (11), (14) can be rewritten as

$$\frac{\partial \Phi(w, \tau)}{\partial \tau} = a'(x(\tau)) w \frac{\partial \Phi(w, \tau)}{\partial w} + b(x(\tau)) \frac{(jw)^2}{2} \Phi(w, \tau), \quad (15)$$

where

$$b(x(\tau)) = a(x(\tau)) + 2 \left(qx(\tau) R_1(x(\tau)) + (1 + q)x(\tau) R_0(x(\tau)) + [\lambda + x(\tau)] g_1 \right). \quad (16)$$

6. Diffusion Approximation

Using (15) and (1), (7), (8) we can get the Fokker-Plank equation for the probability density of an diffusion process $y(\tau)$ with drift (transfer) coefficient $a'(x(\tau))y(\tau)$ and diffusion coefficient $b(x(\tau))$

$$\frac{\partial P(y(\tau), \tau)}{\partial \tau} = -a'(x(\tau)) \frac{\partial \{y(\tau) P(y(\tau), \tau)\}}{\partial y(\tau)} + \frac{b(x(\tau))}{2} \frac{\partial^2 P(y(\tau), \tau)}{\partial y^2(\tau)}, \quad (17)$$

and the process $y(\tau)$ is the solution of the stochastic differential equation (18)

$$dy(\tau) = a'(x(\tau)) y(\tau) d\tau + \sqrt{b(x(\tau))} d\omega(\tau), \quad (18)$$

where $\omega(\tau)$ is the Wiener process.

Introduce diffusion process $z(\tau) = x(\tau) + \varepsilon y(\tau)$ and write the stochastic differential equation (19) for $z(\tau)$

$$dz(\tau) = a(z(\tau))d\tau + \varepsilon\sqrt{b(z(\tau))}d\omega(\tau). \quad (19)$$

Denote the probability density of the $z(\tau)$ as $\Pi(z(\tau), \tau) = \frac{\partial P\{z(\tau) < z\}}{\partial z}$ and he Fokker-Plank equation for it can be written as follows

$$\frac{\partial \Pi(z(\tau), \tau)}{\partial \tau} = -\frac{\partial \{a(z(\tau))\Pi(z(\tau), \tau)\}}{\partial z(\tau)} + \frac{\varepsilon^2}{2} \frac{\partial^2 \{b(z(\tau))\Pi(z(\tau), \tau)\}}{\partial z^2(\tau)}. \quad (20)$$

The solution of the equation (20) for stationary probability distribution of the process $z(\tau)$ has the form

$$\Pi(z) = \frac{C}{b(z)} \exp \left\{ \frac{2}{\sigma} \int_0^z \frac{a(x)}{b(x)} dx \right\}, \quad C - \text{constant}. \quad (21)$$

Finally, based on the (21) in (22) we get diffusion approximation $\tilde{P}(i)$ for the stationary distribution $P(i)$ of the number of calls in the orbit

$$\tilde{P}(i) = \frac{\Pi(i\sigma)}{\sum_{k=0}^{\infty} \Pi(k\sigma)}. \quad (22)$$

7. Numerical Results

Preliminary calculations suggest that theoretical results are consistent with simulation ones. To compare the pre-limit probability distribution of the number of calls in the orbit of considered queueing system $P(i)$ calculated via matrix method and its approximation $PD(i)$ constructed by using the asymptotic diffusion analysis method for different values of the system parameters we use Kolmogorov distance Δ between respective distribution functions: $\Delta = \max_{n \geq 0} \left| \sum_{i=0}^n [P(i) - PD(i)] \right|$.

The comparison of the distributions is shown in Figures 1, 2.

8. Conclusion

In the present paper, retrial queueing system of M/M/1 type with impatient customers in the orbit is considered. In the course of the study, the asymptotic diffusion analysis method was used and the diffusion approximation of the stationary probability distribution of the calls number in the orbit was obtained. As a asymptotic

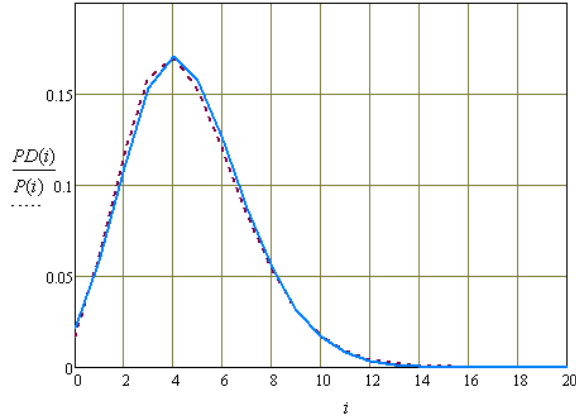


Fig. 1. Comparison of the asymptotic (solid line) and the pre-limit (dashed line) distributions for $\sigma = 0.01$, $\lambda = 0.4$, $\mu = 1$, $q = 2$, $H = 1$, $\Delta = 0.012$.

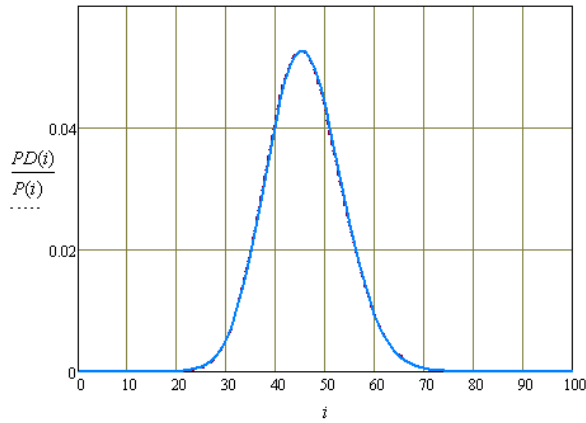


Fig. 2. Comparison of the asymptotic (solid line) and the pre-limit (dashed line) distributions for $\sigma = 0.001$, $\lambda = 0.4$, $\mu = 1$, $q = 2$, $H = 1$, $\Delta = 0.0027$.

condition it was taken condition of a long delay of calls in orbit and a long time patience of calls in the orbit. In further studies it is planned to obtain numerical results about diffusion approximation and compare them with the asymptotic analysis method results from [5]. Based on the works of the other authors from references, for example, it can be assumed that the asymptotic diffusion analysis method is more accurate than asymptotic analysis method under the same asymptotic condition.

REFERENCES

1. Nazarov A. A., Paul S. V., Lizyura O. D. Asymptotic Diffusion Analysis of Retrial Queue M/M/1/1 with Outgoing Calls // In: Vishnevskiy V. M. and Samouylov K. E. Distributed Computer and Communication Networks (DCCN-2019). 2019. P. 148–155. (in Russian)
2. Nazarov A., Phung-Duc T., Paul S., Lizyura O. Asymptotic-Diffusion Analysis of Retrial Queue with Two-Way Communication and Renewal Input // Proceedings of The 5th International Conference on Stochastic Methods (ICSM-5). 2020. P. 339–345.
3. Nazarov A. A., Phung-Duc T., Izmailova Ya. Ye. Asymptotic-Diffusion Analysis of Multiserver Retrial Queueing System with Priority Customers // Proceedings of the XIX International Conference named after A. F. Terpugov. Information Technologies and Mathematical Modelling (ITMM-2020). 2021. P. 88–98.
4. Moiseev A., Nazarov A., Paul S. Asymptotic diffusion analysis of multi-server retrial queue with hyperexponential service // Mathematics. 2020. V. 8, no. 4. P. 531.
5. Nazarov A. A., Fedorova E. A. Asymptotic Analysis of Retrial Queue M/M/1 with Impatient Calls under the Long Patience Time Condition // In: Vishnevskiy V. M. and Samouylov K. E. Distributed Computer and Communication Networks (DCCN-2016). 2016. P. 342–348. (in Russian)

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