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Распределенные компьютерные и телекоммуникационные сети: управление, вычисление, связь (DCCN-2021) = Distributed computer and communication networks: control, computation, communications (DCCN-2021) **:** материалы XXIV Междунар. научн. конфер, 20–24 сент. 2021 г., Москва / под общ. ред. В.М. Вишневского, К.Е. Самуйлова; Ин-т проблем упр. им. В.А. Трапезникова Рос. акад. наук Минобрнауки РФ – Электрон. текстовые дан. (1 файл: 24,9 Мб). – М.: ИПУ РАН, 2021. – 1 электрон. опт. диск (CD-R). – Систем. требования: Pentium 4; 1,3 ГГц и выше; Acrobat Reader 4.0 или выше. – Загл. с экрана. – ISBN 978-5-91450-258-1. – № госрегистрации 0322103543. – Текст : электронный.

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Waiting Time Asymptotic Analysis of a M/GI/1 Retrial Queue System

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Abstract

In this paper we deal with M/GI/1 retrial queueing system and conduct asymptotic analysis of the waiting time. The main result of this analysis is the asymptotic characteristic function of the waiting time distribution under heavy load condition . Also during the analysis the asymptotic distribution of the number of returns of the tagged request to the orbit and the asymptotic distributions of the number of requests in the orbit were obtained.

Keywords: Retrial queue, Asymptotic analysis, Waiting time, Number of returns, Number of Retrials

1. Introduction

In the retrial queue system theory it is hard to deal with the waiting time distribution because of the random order service of customers from the orbit. Different approaches to the investigation of the waiting time in $M/GI/1$ RQ - systems can be found in Artalejo, Gomez-Corral [1], Falin, Fricker [2], Nobel [3], Lee, Kim, Kim [4].

In this work we find the asymptotic characteristic function of the waiting time distribution under heavy load condition. As known the number of returns distribution is a counterpart of waiting time distribution and so we investigate both of them. The asymptotic distributions of the number of requests in the orbit under heavy load condition was also obtained during the analysis.

2. Mathematical model

In this paper we consider a $M/GI/1$ retrial queuing (RQ) system. Requests arrive in a Poisson process with intensity $\tilde{\lambda} = \rho\lambda$. If the server is idle at the moment of request arrival this request occupies the server and the service starts immediately. The service time of request follows a common probability law with arbitrary distribution function $B(x)$.

Served request leaves the system. If the server is busy at the moment of request arrival, the request joins to the orbit. Each request from the orbit after a random delay, that has exponential distribution with rate σ, retries to get accesses to the server. At the retrial moment server again can be idle or busy. In the first case this request occupies the server for a random service time; otherwise, it instantly returns to the orbit for a next random delay.

We define W - the waiting time of the tagged request in the orbit as the length of the interval from the moment the request arrives in the system till the start of the service. Also in our research, the following notations are used: $\tilde{\nu}$ - the number of transitions of the tagged request to the orbit; r - the probability that the server is busy at the moment the request arrives at the system. Obviously, $\tilde{\nu} = 0$ with the probability $(1-r)$, that the request finds the server idle at the moment of the arrival to the system; $\nu(t)$ - the number of returns of the tagged request to the orbit from the moment t until the start of the service. Following this logic, we can write for $\tilde{\nu}$:

$$\tilde{\nu} = \begin{cases} 0, \quad \text{with probability } (1-r), \\ 1+\nu(t), \quad \text{with probability } r. \end{cases}$$

Using above notations, the characteristic function for W can be written in the following form:

$$\begin{aligned} G(u) = E\left\{e^{juW}\right\} = (1-r) + r\sum_{n=0}^{\infty} E\left\{e^{juW}/\tilde{\nu} = 1+n\right\} P\left\{\nu(t) = n\right\} = \\ = (1-r) + r\sum_{n=0}^{\infty}\left(\frac{\sigma}{\sigma - ju}\right)^{1+n} P\left\{\nu(t) = n\right\}. \end{aligned} \quad (1)$$

The aim of our study is to find asymptotic characteristic function of W under heavy load condition. Obviously, for this purpose it is enough to find the probability r and the probability distribution $P\{\nu(t) = n\}$ under limiting condition.

3. Kolmogorov's equations

Let's denote by $i(t)$ the number of requests in the orbit at time t and by $k(t)$ - the state of the server at time t:

$$k(t) = \begin{cases} 0, \quad \text{if the server is idle,} \\ 1, \quad \text{if the server is busy.} \end{cases}$$

We introduce a process $y(t)$ - the elapsed service time at the moment t for a request standing on the server, and the conditional rate $\mu(x) = \frac{B'(x)}{1-B(x)}$ of service of a request standing on the server in case that the elapsed service time is equal to x.

We do not define process $y(t)$ when the server is free. Thus, we investigate a random process with a variable number of components $\{k(t), i(t), y(t)\}$, which forms a continuous time Markov process. Let's assume that the stationary probability distribution of the states of this process is exist.

Let's denote:

$$P_0(i,t) = P\{k(t) = 0, i(t) = i\},$$
$$P_1(i,y,t) = \frac{\partial P\{k(t) = 1, i(t) = i, y(t) < y\}}{\partial y}.$$

In stationary regime for $P_0(i,t)$ and $P_1(i,y,t)$ we get the following system of equations:

$$\begin{gathered} -(\tilde{\lambda} + i\sigma)P_0(i) + \int_0^\infty P_1(i,y)\mu(y)dy = 0, \\ \frac{\partial P_1(i,y)}{\partial y} = -(\tilde{\lambda} + \mu(y))P_1(i,y) + \tilde{\lambda}P_1(i-1,y), \\ P_1(i,0) = \tilde{\lambda}P_0(i) + (i+1)\sigma P_0(i+1). \end{gathered} \tag{2}$$

Let's introduce steady-state partial characteristic functions:

$$H_0(u) = \sum_{i=0}^{\infty} e^{jui}P_0(i), H_1(u,y) = \sum_{i=0}^{\infty} e^{jui}P_1(i,y), \tag{3}$$

After some actions on (2) using (3) the following system of equations has been composed for $H_0(u)$ and $H_1(u,y)$:

$$\begin{gathered} -\tilde{\lambda}H_0(u) + j\sigma\frac{\partial H_0(u)}{\partial u} + \int_0^\infty H_1(u,y)\mu(y)dy = 0, \\ \frac{\partial H_1(u,y)}{\partial y} = ((e^{ju} - 1)\tilde{\lambda} - \mu(y))H_1(u,y), H_1(u,0) = \tilde{\lambda}H_0(u) - j\sigma e^{-ju}\frac{\partial H_0(u)}{\partial u}, \\ \tilde{\lambda}H_1(u) + e^{-ju}j\sigma H'_0(u) = 0. \end{gathered} \tag{4}$$

Characteristic function for $\nu(t)$ in stationary mode can be represented as follows:

$$G(u) = E\left\{e^{ju\nu(t)}\right\} = \sum_{i=0}^{\infty}\left[G_0(i,u)P_0(i) + \int_0^\infty G_1(i,u,y)P_1(i,y)dy\right],$$

where $G_0(i,u)$ and $G_1(i,u,y)$ are conditional characteristic functions:

$$G_0(i,u,t) = E\left\{e^{juv(t)}/k(t) = 0, i(t) = i\right\},$$
$$G_1(i,u,y,t) = E\left\{e^{juv(t)}/k(t) = 1, i(t) = i, y(t) = y\right\}.$$

We compose a system of inverse Kolmogorov equations for the conditional characteristic functions $G_0(i,u,t)$and $G_1(i,u,y,t)$ and taking into account that the system is functioning in a stationary mode, for $G_0(i,u)$ and $G_k(i,u,y)$ we obtain the following system of equations:

$$-(\tilde{\lambda} + i\sigma)G_0(i,u) + \tilde{\lambda}G_1(i,u,0) + (i-1)\sigma G_1(i-1,u,0) + \sigma = 0, \tag{5}$$

$$\frac{dG_1(i,u,y)}{dy} - (\tilde{\lambda} + \sigma(1-e^{ju}) + \mu(y))G_1(i,u,y) + \\ + \tilde{\lambda}G_1(i+1,u,y) + \mu(y)G_0(i,u) = 0. \tag{6}$$

4. Asymptotic analysis

Theorem 1. Conditional asymptotic characteristic function under the condition that $y(t) = y$ for limiting value of the number of requests in the orbit in RQ system $M/GI/1$ in heavy load case ($\rho \to 1$) has the following form:

$$\lim_{\rho\to 1} M\left\{e^{jw(1-\rho)i(t)}\,|y(t) = y\right\} = \frac{1}{b_1}\left(1 - \frac{b_2}{2{b_1}^2}jw\right)^{-\left(\frac{2b_1}{\sigma b_2}+1\right)}(1 - B(y)) \tag{7}$$

where b_1 and b_2 are the first and the second order moments of service time respectively, $\rho = \lambda b_1$.

Theorem 1 will be used in proof of the next theorem about the asymptotic characteristic function of the number of returns of the tagged request to the orbit.

Integrating (7) over y, we find asymptotic characteristic function under heavy load condition:

$$F(w) = \left(1 - \frac{b_2}{2{b_1}^2}jw\right)^{-\left(\frac{2b_1}{\sigma b_2}+1\right)}$$

$F(w)$ has the form of gamma distribution with density:

$$f_{\alpha,\beta}(x) = \frac{\alpha^\beta}{\Gamma(\beta)}x^{\beta-1}e^{-\alpha x}, x \geq 0, \tag{8}$$

where $\alpha = \frac{2{b_1}^2}{b_2}$ - scale parameter, $\beta = \frac{2b_1}{\sigma b_2} + 1$ - shape parameter.

The expression for $F(w)$ match with the result obtained in [5], in which in order to find the asymptotic characteristic function under heavy load condition was used the method of residual service time. Accordingly, [5] does not contain the result (7) for the conditional asymptotic characteristic function $F_1(w, y)$, under the condition that the elapsed service time $y(t) = y$.

Theorem 2. The characteristic function $\tilde{G}(u)$ of the limit value of the number $\nu(t)$ of returns of the request to the orbit in RQ system $M/GI/1$ in heavy load case has the following form:

$$\tilde{G}(u) = \int\limits_0^\infty \frac{(1-\rho)/\sigma x b_1}{(1-\rho)/\sigma x b_1 - ju} f_{\alpha,\beta}(x)dx.$$

The density of such distribution will have the following form:

$$\tilde{P}(z) = \int\limits_0^\infty \frac{(1-\rho)}{\sigma x b_1} e^{-\frac{(1-\rho)}{\sigma x b_1}z} f_{\alpha,\beta}(x)dx. \quad (9)$$

5. Asymptotic probability distribution of the waiting time of the customer in the orbit

Using the found distribution density (9), we compose a discrete approximation:

$$P(n) = \tilde{P}(n) \cdot \left(\sum_{m=0}^{\infty} \tilde{P}(m)\right)^{-1},$$

where $P(n)$ - discrete approximation of asymptotic probability distribution of $\nu(t)$ the number of returns of the tagged request to the orbit: Let's substitute the resulting distribution into (1):

$$G(u) = E\left\{e^{juW}\right\} = (1-r) + r\sum_{n=0}^{\infty} E\left\{e^{juW}/\nu = 1+n\right\} P\left\{\nu(t) = n\right\} =$$
$$= (1-r) + r\sum_{n=0}^{\infty}\left(\frac{\sigma}{\sigma - ju}\right)^{1+n} P(n).$$

Thus, we have found the asymptotic characteristic function of the waiting time of the request in the RQ system $M/GI/1$ under heavy load condition. By performing the inverse Fourier transform, one can obtain the asymptotic distribution of the waiting time of the request in the orbit.

6. Conclusion

In this paper was presented an asymptotic analysis of the waiting time and the number of returns of a M/GI/1 retrial queueing system under heavy load condition. As a result the asymptotic characteristic function of the waiting time was found.

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