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Asymptotic-Diffusion Analysis of Multiserver Retrial Queue with Two-Way Communication

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Abstract

In this paper, we consider multiserver retrial queue with two-way communication. Incomimg calls arrive according to the Poisson process and reserve the server for an exponentially distributed time. If all of the servers are busy the incoming call joins the orbit and makes a delay for an exponentially distributed time before the next attempt to occupy the server. Idle servers also make outgoing calls following a distinct exponential distribution. Using the asymptotic-diffusion analysis method we derive the approximation for the stationary probability distribution of the number of calls in the orbit.

Keywords: multiserver retrial queue, two-way communication, incoming call, outgoing call, asymptotic-diffusion analysis, diffusion approximation

1. Introduction

Currently, more and more services are partially or fully working in a call-center mode. Banks use call centers to advise customers and to advertise their services; online stores utilize call centers to refine and confirm orders, and call centers also exist independently to conduct social surveys.

A lot of research papers are devoted to modeling call centers. Papers [1, 2, 3, 4] are devoted to the quality of customer service in telephone services. In [5], the authors present a study of incoming processes models in real call-centers. A statistical analysis of the work of call-centers taking into account various aspects of the functioning of the service is presented in the study [6].

Recently, retrial queues with two-way communication have been used as a model of a blended call-center. The most detailed studies on retrial queues can be found

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in [7, 8]. The main feature of model with two-way communication is that during idle time, the server makes outgoing calls and serves them along with incoming calls. As an outgoing call we refer to an operator's call to a client, or any other type of alternative operator's activities rather than serving incoming calls. Thus, retrial queueing models with two-way communication are flexible and allow you to simulate most modern telephone services. In [9] the model of multiserver retrial queue with two-way communication was was proposed and numerical analysis was presented.

In this paper, we consider the same model as in [9] but rather than numerical analysis of the stationary distribution, our focus is to obtain the diffusion limit of the underlying time-dependent Markov process [10]. As a byproduct, the limiting results are then used to approximate the probability distribution of the number of customers in the orbit in the stationary state.

The rest of our paper is organized as follows. Section 2 is devoted to the presentation of the model and preliminary analysis of the underlying Markov chain. Our main results are in Section 3 where we present the asymptotic-diffussion analysis to obtain the diffusion limit of the underlying Markov chain. In Section 4 we describe an algorithm for constructing a probability distribution approximation of the system state. Section 5 shows the diffusion approximation accuracy for several values of system parameters. Finally, Section 6 is devoted to some concluding remarks.

2. Mathematical model

We consider multiserver retrial queue with two-way communication. The input process is a stationary Poisson process with rate λ. Service times of incoming calls are exponentially distributed with rate μ_1. Calls that find servers fully occupied join the orbit and reattempt to access the server after an exponentially distributed delay with rate σ. When the server is idle it makes an outgoing calls with rate α and provides the service for an exponentially distributed time with rate μ_2. We denote N is the number of servers in the system.

Let $n_1(t), n_2(t)$ denote the number of servers busy serving incoming and outgoing calls at the time t, respectively. Also $i(t)$ is a number of calls in the orbit. Thus, we can see that three-dimensional random process $\{n_1(t),\ n_2(t),\ i(t)\}$ is a continuous time Markov chain.

Let $P(n_1, n_2, i, t) = P\{n_1(t) = n_1, n_2(t) = n_2, i(t) = i\}$ denotes the probability distribution of the process $\{n_1(t),\ n_2(t),\ i(t)\}$, which is the solution of Kolmogorov's system of equations

$$\frac{\partial P(0,0,i,t)}{\partial t} = -(\lambda + i\sigma + N\alpha)P(0,0,i,t) + \mu_1 P(1,0,i,t) + \mu_2 P(0,1,i,t),$$

$$\frac{\partial P(n_1,n_2,i,t)}{\partial t} = -(\lambda + i\sigma + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2)P(n_1,n_2,i,t)+$$

$$+\lambda P(n_1-1,n_2,i,t)+(i+1)\sigma P(n_1-1,n_2,i+1,t)+$$
$$+(N-n_1-n_2+1)\alpha P(n_1,n_2-1,i,t)+(n_1+1)\mu_1 P(n_1+1,n_2,i,t)+$$
$$+(n_2+1)\mu_2 P(n_1,n_2+1,i,t),\ 0<n_1+n_2<N,$$
$$\frac{\partial P(n_1,n_2,i,t)}{\partial t}=-(\lambda+n_1\mu_1+n_2\mu_2)P(n_1,n_2,i,t)+\lambda P(n_1-1,n_2,i,t)+$$
$$+\lambda P(n_1,n_2,i-1,t)+(i+1)\sigma P(n_1-1,n_2,i+1,t)+$$
$$+\alpha P(n_1,n_2-1,i,t),\ n_1+n_2=N. \tag{1}$$

Then we transform the system (1) into system for partial characteristic functions $H(n_1,n_2,u,t)=\sum\limits_{i=0}^{\infty}e^{jui}P(n_1,n_2,i,t)$, where $j=\sqrt{-1}$

$$\frac{\partial H(n_1,n_2,u,t)}{\partial t}=-(\lambda+N\alpha)H(n_1,n_2,u,t)+j\sigma\frac{\partial H(n_1,n_2,u,t)}{\partial u}+$$
$$+\mu_1 H(n_1+1,n_2,u,t)+\mu_2 H(n_1,n_2+1,u,t),\ n_1+n_2=0,$$
$$\frac{\partial H(n_1,n_2,u,t)}{\partial t}=-(\lambda+(N-n_1-n_2)\alpha+n_1\mu_1+n_2\mu_2)H(n_1,n_2,u,t)+$$
$$+j\sigma\frac{\partial H(n_1,n_2,u,t)}{\partial u}+\lambda H(n_1-1,n_2,u,t)-j\sigma e^{-ju}\frac{\partial H(n_1-1,n_2,u,t)}{\partial u}+$$
$$+(N-n_1-n_2+1)\alpha H(n_1,n_2-1,u,t)+(n_1+1)\mu_1 H(n_1+1,n_2,u,t)+$$
$$+(n_2+1)\mu_2 H(n_1,n_2+1,u,t),0<n_1+n_2<N,$$
$$\frac{\partial H(n_1,n_2,u,t)}{\partial t}=-(\lambda+n_1\mu_1+n_2\mu_2)H(n_1,n_2,u,t)+\lambda H(n_1-1,n_2,u,t)+$$
$$+\lambda e^{ju}H(n_1,n_2,u,t)-j\sigma e^{-ju}\frac{\partial H(n_1-1,n_2,u,t)}{\partial u}+$$
$$+\alpha H(n_1,n_2-1,u,t),\ n_1+n_2=N. \tag{2}$$

We denote $\mathbf{H}(u,t)$ is a matrix of functions $H(n_1,n_2,u,t)$ and rewrite the system (2) in form of

$$\frac{\partial \mathbf{H}(u,t)}{\partial t}=(\mathbf{A}+\lambda e^{ju}\mathbf{B})\mathbf{H}(u,t)+j\sigma(\mathbf{I}_0-e^{-ju}\mathbf{I}_1)\frac{\partial \mathbf{H}(u,t)}{\partial u}, \tag{3}$$

where $\mathbf{A},\ \mathbf{B},\ \mathbf{I}_0,\ \mathbf{I}_1$ are operators, which set in the following form

$$\mathbf{AH}(u,t)=\begin{cases}-(\lambda+N\alpha)H(n_1,n_2,u,t)+\mu_1H(n_1+1,n_2,u,t)+\\+\mu_2H(n_1,n_2+1,u,t),\ n_1+n_2=0,\\ \\ -(\lambda+(N-n_1-n_2)\alpha+n_1\mu_1+n_2\mu_2)H(n_1,n_2,u,t)+\\+(N-n_1-(n_2-1))\alpha H(n_1,n_2-1,u,t)+\\+\lambda H(n_1-1,n_2,u,t)+(n_1+1)\mu_1H(n_1+1,n_2,u,t)+\\+(n_2+1)\mu_2H(n_1,n_2+1,u,t),\ 0<n_1+n_2<N,\\ \\ -(\lambda+n_1\mu_1+n_2\mu_2)H(n_1,n_2,u,t)+\lambda H(n_1-1,n_2,u,t)+\\+\alpha H(n_1,n_2-1,u,t),\ n_1+n_2=N,\end{cases}\tag{4}$$

$$\mathbf{BH}(u,t)=\begin{cases}0, & n_1+n_2<N,\\ H(n_1,n_2,u,t), & n_1+n_2=N,\end{cases}\tag{5}$$

$$\mathbf{I}_0\mathbf{H}(u,t)=\begin{cases}H(n_1,n_2,u,t), & n_1+n_2<N,\\ 0, & n_1+n_2=N,\end{cases}\tag{6}$$

$$\mathbf{I}_1\mathbf{H}(u,t)=\begin{cases}0, & n_1+n_2=0,\\ H(n_1-1,n_2,u,t), & n_1+n_2>0.\end{cases}\tag{7}$$

We denote $\mathbf{E}$ as an operator that summarizing over all available values of $n_1,\ n_2$ and present the additional equation that we need to provide analysis

$$\mathbf{E}\frac{\partial\mathbf{H}(u,t)}{\partial t}=\mathbf{E}(\mathbf{A}+\lambda e^{ju}\mathbf{B})\mathbf{H}(u,t)+j\sigma\mathbf{E}(\mathbf{I}_0-e^{-ju}\mathbf{I}_1)\frac{\partial\mathbf{H}(u,t)}{\partial u}.\tag{8}$$

We also note that

$$\mathbf{E}(\mathbf{A}+\lambda\mathbf{B})=0,\ \mathbf{E}(\mathbf{I}_0-\mathbf{I}_1)=0.\tag{9}$$

3. Asymptotic-Diffusion Analysis

In operator equations (3) and (8) we introduce the following notations

$$\sigma=\varepsilon,\ u=\varepsilon w,\ \tau=\varepsilon t,\ \mathbf{H}(u,t)=\mathbf{F}(w,\tau,\varepsilon),$$

to obtain the equations

$$\varepsilon\frac{\partial\mathbf{F}(w,\tau,\varepsilon)}{\partial\tau}=(\mathbf{A}+\lambda e^{jw\varepsilon}\mathbf{B})\mathbf{F}(w,\tau,\varepsilon)+j(\mathbf{I}_0-e^{-jw\varepsilon}\mathbf{I}_1)\frac{\partial\mathbf{F}(w,\tau,\varepsilon)}{\partial w},\tag{10}$$

$$\varepsilon\mathbf{E}\frac{\partial\mathbf{F}(w,\tau,\varepsilon)}{\partial\tau}=\mathbf{E}(\mathbf{A}+\lambda e^{jw\varepsilon}\mathbf{B})\mathbf{F}(w,\tau,\varepsilon)+j\mathbf{E}(\mathbf{I}_0-e^{-jw\varepsilon}\mathbf{I}_1)\frac{\partial\mathbf{F}(w,\tau,\varepsilon)}{\partial w}. \quad (11)$$

We solve the system (11) taking the limit as $\varepsilon\to 0$ and present the result in following theorem.

Theorem 1. In considered retrial queue, the stationary probability distribution $R(n_1,n_2)$ of the two-dimensional process $\{n_1(t),\ n_2(t)\}$ is a solution of the system of operator equations

$$(\mathbf{A}+\lambda\mathbf{B}-x(\mathbf{I}_0-\mathbf{I}_1))\mathbf{R}=0,$$
$$\mathbf{E}\mathbf{R}=1, \quad (12)$$

where $\mathbf{R}$ is a matrix of probabilitites $R(n_1,n_2)$, function $x(\tau)$ is a solution of differential equation

$$x'(\tau)=\mathbf{E}\left[\lambda\mathbf{B}-x(\tau)\mathbf{I}_1\right]\mathbf{R}.$$

Denoting

$$a(x)=\mathbf{E}\left[\lambda\mathbf{B}-x\mathbf{I}_1\right]\mathbf{R} \quad (13)$$

and making the following replacements in the operator equations (3) and (8)

$$\mathbf{H}(u,t)=e^{j\frac{u}{\sigma}x(\sigma t)}\mathbf{H}^{(2)}(u,t), \quad (14)$$

we have

$$\frac{\partial\mathbf{H}^{(2)}(u,t)}{\partial t}+jux'(\sigma t)\mathbf{H}^{(2)}(u,t)=$$
$$=(\mathbf{A}+\lambda e^{ju}\mathbf{B}-x(\sigma t)(\mathbf{I}_0-e^{-ju}\mathbf{I}_1))\mathbf{H}^{(2)}(u,t)+j\sigma(\mathbf{I}_0-e^{-ju}\mathbf{I}_1)\frac{\partial\mathbf{H}^{(2)}(u,t)}{\partial u}, \quad (15)$$
$$\mathbf{E}\frac{\partial\mathbf{H}^{(2)}(u,t)}{\partial t}+jux'(\sigma t)\mathbf{E}\mathbf{H}^{(2)}(u,t)=$$
$$=\mathbf{E}(\mathbf{A}+\lambda e^{ju}\mathbf{B}-x(\sigma t)(\mathbf{I}_0-e^{-ju}\mathbf{I}_1))\mathbf{H}^{(2)}(u,t)+j\sigma\mathbf{E}(\mathbf{I}_0-e^{-ju}\mathbf{I}_1)\frac{\partial\mathbf{H}^{(2)}(u,t)}{\partial u}. \quad (16)$$

In operator equations (15) and (16) we introduce the following notations

$$\sigma=\varepsilon^2,\ \tau=\varepsilon^2 t,\ u=\varepsilon w,\ \mathbf{H}^{(2)}(u,t)=\mathbf{F}^{(2)}(w,\tau,\varepsilon), \quad (17)$$

to obtain the equations

$$\varepsilon^2\frac{\partial\mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial\tau}+jw\varepsilon a(x)\mathbf{F}^{(2)}(w,\tau,\varepsilon)=$$
$$=(\mathbf{A}+\lambda e^{jw\varepsilon}\mathbf{B}-x(\mathbf{I}_0-e^{-jw\varepsilon}\mathbf{I}_1))\mathbf{F}^{(2)}(w,\tau,\varepsilon)+$$

$$+j\varepsilon(\mathbf{I}_0 - e^{-jw\varepsilon}\mathbf{I}_1)\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial w}, \tag{18}$$

$$\varepsilon^2\mathbf{E}\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial \tau} + jw\varepsilon a(x)\mathbf{E}\mathbf{F}^{(2)}(w,\tau,\varepsilon) =$$

$$= \mathbf{E}(\mathbf{A} + \lambda e^{jw\varepsilon}\mathbf{B} - x(\mathbf{I}_0 - e^{-jw\varepsilon}\mathbf{I}_1))\mathbf{F}^{(2)}(w,\tau,\varepsilon) +$$

$$+j\varepsilon\mathbf{E}(\mathbf{I}_0 - e^{-jw\varepsilon}\mathbf{I}_1)\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial w}. \tag{19}$$

Solving the equations (18) and (19) taking the limit as $\varepsilon \to 0$ we present the following theorem.

Theorem 2. Probability density of diffusion limit $z(\tau)$ of the number of calls in the orbit given as follows

$$\Pi(z) = \frac{C}{b(z)}\exp\left\{\frac{2}{\sigma}\int_0^z \frac{a(x)}{b(x)}dx\right\}, \tag{20}$$

where C is a normalization factor, function $a(x)$ is defined by (13), function $b(x)$ has the following form

$$b(x) = a(x) + 2[\mathbf{E}(\lambda\mathbf{B} - x\mathbf{I}_1)\mathbf{g} + x\mathbf{E}\mathbf{I}_1\mathbf{R}]. \tag{21}$$

Here $\mathbf{g}$ is the matrix of additional values. It has the same dimension as $\mathbf{R}$ and appears as the solution of the system of operator equations

$$(\mathbf{A} + \lambda\mathbf{B} - x(\mathbf{I}_0 - \mathbf{I}_1))\mathbf{g} = a(x)\mathbf{R} - (\lambda\mathbf{B} - x\mathbf{I}_1)\mathbf{R},$$

$$\mathbf{E}\mathbf{g} = 0. \tag{22}$$

From the obtained probability density $\Pi(z)$ we build the approximation of the probability distribution of the number of calls in the orbit using expression

$$PD(i) = \frac{\Pi(i\sigma)}{\sum\limits_{n=0}^{\infty}\Pi(n\sigma)}. \tag{23}$$

4. Algorithm of Calculating Drift and Diffusion Coefficients

To obtain the function $a(x)$ we need to calculate the elements of the matrix $\mathbf{R}$. We rewrite the system of operator equations (12) in scalar form

$$-(\lambda + N\alpha + x(\tau))R(n_1,n_2) + \mu_1 R(n_1+1,n_2) + \mu_2 R(n_1,n_2+1) = 0,\ n_1+n_2 = 0,$$

$$-(\lambda + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2 + x(\tau))R(n_1, n_2)+$$
$$+(N - n_1 - (n_2 - 1))\alpha R(n_1, n_2 - 1) + (\lambda + x(\tau))R(n_1 - 1, n_2)+$$
$$+(n_1 + 1)\mu_1 R(n_1 + 1, n_2) + (n_2 + 1)\mu_2 R(n_1, n_2 + 1) = 0,\ 0 < n_1 + n_2 < N,$$
$$-(n_1\mu_1+n_2\mu_2)R(n_1, n_2)+(\lambda+x(\tau))R(n_1-1, n_2)+\alpha R(n_1, n_2-1) = 0,\ n_1+n_2 = N.$$
$$\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1} R(n_1, n_2) = 1. \tag{24}$$

We transform the system of equations (24) to the system of linear algebraic equations renumbering the elements of the matrix $\mathbf{R}$ in the following way

$$(n_1, n_2)\ \to 2n_1 + n_2 + \frac{(n_1 + n_2)^2 - (n_1 + n_2)}{2}. \tag{25}$$

Then we obtain the system of equations

$$\widetilde{\mathbf{R}}\mathbf{D}(x) = 0,\ \widetilde{\mathbf{R}}\mathbf{e} = 1, \tag{26}$$

where $\widetilde{\mathbf{R}}$ is a vector of probabilities $R(n_1, n_2)$, $\mathbf{e}$ is a unit vector, the matrix $\mathbf{D}(x)$ is a matrix of the system with renumbered elements.

Solving the system of equations (26) we can express the function $a(x)$ as follows

$$a(x) = (\lambda + x)\widetilde{\mathbf{R}}\mathbf{e}_1 - x, \tag{27}$$

where $\mathbf{e}_1$ is a vector, where the last $N + 1$ elements are ones and the other elements are zeroes.

To derive the function $b(x)$ we need to calculate the elements of the matrix $\mathbf{g}$. We rewrite the system of operator equations (22) in scalar form

$$-(\lambda + N\alpha + x(\tau))g(n_1, n_2) + \mu_1 g(n_1 + 1, n_2) + \mu_2 g(n_1, n_2 + 1) = a(x)R(n_1, n_2),$$
$$n_1 + n_2 = 0,$$
$$-(\lambda+(N-n_1-n_2)\alpha+n_1\mu_1+n_2\mu_2+x(\tau))g(n_1, n_2)+(N-n_1-(n_2-1))\alpha g(n_1, n_2-1)+$$
$$+(\lambda + x(\tau))g(n_1 - 1, n_2) + (n_1 + 1)\mu_1 g(n_1 + 1, n_2) + (n_2 + 1)\mu_2 g(n_1, n_2 + 1) =$$
$$= a(x)R(n_1, n_2) + x(\tau)R(n_1 - 1, n_2),\ 0 < n_1 + n_2 < N,$$
$$-(n_1\mu_1 + n_2\mu_2)g(n_1, n_2) + (\lambda + x(\tau))g(n_1 - 1, n_2) + \alpha g(n_1, n_2 - 1) =$$
$$= (a(x) - \lambda)R(n_1, n_2) + x(\tau)R(n_1 - 1, n_2),\ n_1 + n_2 = N.$$

$$\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1} g(n_1,n_2)=0. \tag{28}$$

We transform the system of the operator equations (22) to the system of linear algebraic equations renumbering the elements of the matrix $\mathbf{g}$ using (25). Thus, we obtain the system of equations

$$\widetilde{\mathbf{g}}\mathbf{D}(x)=\mathbf{d}(x),\ \widetilde{\mathbf{g}}\mathbf{e}=0, \tag{29}$$

where $\widetilde{\mathbf{g}}$ is a vector of elements of the matrix $\mathbf{g}$ with renumbered elements, vector $\mathbf{d}(x)$ is a vector of right parts of the system (28) with renumbered elements.

Solving the system of equations (29) we can express the function $b(x)$ as follows

$$b(x)=a(x)+2[(\lambda+x)\widetilde{\mathbf{g}}\mathbf{e}_1+x(1-\widetilde{\mathbf{R}}\mathbf{e}_1)]. \tag{30}$$

5. Numerical Examples

We fix the number of servers in the system $N=5$. We assume that the rates of service times are $\mu_1=1,\ \mu_2=2$. Outgoing calls rate is $\alpha=1$.

The accuracy of approximation we will determine using Kolmogorov range

$$\Delta=\max_{0\leqslant i\leqslant\infty}\left|\sum_{n=0}^{i}(P(n)-PD(n))\right|, \tag{31}$$

where $P(n)$ is the probability distribution of the number of calls in the orbit, obtained with simulation, $PD(n)$ is a diffusion approximation defined by (23). Calculation of Δ we provide while both $P(n)$ and $PD(n)$ are not equal to zero.

We assume that the approximation is acceptable when its accuracy $\Delta<0.05$. Table 1 depicts the accuracy of the diffusion approximation $PD(n)$ depending on parameters σ and ρ, where ρ characterizes the system load $\rho=\lambda/N\mu_1$.

Δ	$\sigma=5$	$\sigma=2$	$\sigma=1$	$\sigma=0.5$	$\sigma=0.2$	$\sigma=0.1$
$\rho=0.6$	**0,04**	**0,039**	**0,039**	**0,022**	**0,015**	**0,023**
$\rho=0.7$	0,07	0,063	0,051	**0,036**	**0,031**	**0,032**
$\rho=0.8$	0,09	0,078	0,064	**0,049**	**0,039**	**0,041**
$\rho=0.9$	0,081	0,069	0,065	0,053	**0,044**	**0,05**

Table 1. Kolmogorov range

6. Conclusion

We have considered multiserver retrial queue with two-way communication. Using asymptotic-diffusion analysis method we have built the diffusion process the distribution density of which we used to construct the approximation of the number of calls in the orbit.

We have presented numerical experiments where we have estimated the accuracy of the approximation by comparing to simulation.

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