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Научно-производственное объединение
«Информационные и сетевые технологии» («ИНСЕТ»)**

РАСПРЕДЕЛЕННЫЕ КОМПЬЮТЕРНЫЕ И ТЕЛЕКОММУНИКАЦИОННЫЕ СЕТИ: УПРАВЛЕНИЕ, ВЫЧИСЛЕНИЕ, СВЯЗЬ (DCCN-2020)



**МАТЕРИАЛЫ XXIII МЕЖДУНАРОДНОЙ НАУЧНОЙ
КОНФЕРЕНЦИИ
(14–18 СЕНТЯБРЯ 2020 г., МОСКВА, РОССИЯ)**

Под общей редакцией д.т.н. В.М. Вишневого, д.т.н. К.Е. Самуйлова

НАУЧНОЕ ЭЛЕКТРОННОЕ ИЗДАНИЕ

**Москва
ИПУ РАН
2020**

Russian Academy of Sciences (RAS)
V.A. Trapeznikov Institute of Control Sciences of RAS (ICS RAS)
Peoples' Friendship University of Russia (RUDN University)
Institute of Information and Communication Technologies of Bulgarian
Academy of Sciences (Sofia, Bulgaria)
National Research Tomsk State University (NR TSU)
Research and development company
“Information and networking technologies”

DISTRIBUTED COMPUTER AND COMMUNICATION NETWORKS: CONTROL, COMPUTATION, COMMUNICATIONS (DCCN-2020)



**PROCEEDINGS OF THE XXIII INTERNATIONAL SCIENTIFIC
CONFERENCE**
(September 14–18, 2020, Moscow, Russia)

*Under the general editorship of D.Sc. V.M. Vishnevskiy,
D.Sc. K.E. Samouylov*

**MOSCOW
ISC RAS
2020**

УДК 004.7:004.4].001:621.391:007

ББК 32.973.202:32.968

Р 24

Распределенные компьютерные и телекоммуникационные сети: управление, вычисление, связь (DCCN-2020) = Distributed computer and communication networks: control, computation, communications (DCCN-2020) [Электронный ресурс] : материалы XXIII Междунар. научн. конфер, 14–18 сент. 2020 г., Москва / под общ. ред. В.М. Вишневого, К.Е. Самуйлова; Ин-т проблем упр. им. В.А. Трапезникова Рос. акад. наук. – Электрон. текстовые дан. (1 файл: 44,9 Мб). – М.: ИПУ РАН, 2020. – 1 электрон. опт. диск (CD-R). – Систем. требования: Pentium 4; 1,3 ГГц и выше; Internet Explorer; Acrobat Reader 4.0 или выше. – Загл. с экрана. – ISBN 978-5-91450-248-2. – № госрегистрации 0322002892.

В научном электронном издании представлены материалы XXIII Международной научной конференции «Распределенные компьютерные и телекоммуникационные сети: управление, вычисление, связь» по следующим направлениям:

- Алгоритмы и протоколы телекоммуникационных сетей
- Управление в компьютерных и инфокоммуникационных системах
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В материалах научной конференции DCCN-2020, подготовленных к выпуску к.ф.-м.н. Козыревым Д.В., обсуждены перспективы развития и сотрудничества в этой сфере.

Сборник материалов конференции предназначен для научных работников и специалистов в области управления крупномасштабными системами.

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Утверждено к изданию Программным комитетом конференции

Содержание / Contents

1. Markovich N.M., Krieger U.R. STATISTICAL ANALYSIS OF THE END-TO-END DELAY OF PACKET TRANSFERS IN A PEER-TO-PEER NETWORK.....	1
2. Melikov A., Aliyeva S., Shahmaliyev M. SPACE MERGING APPROACH TO ANALYSIS OF QUEUING SYSTEM WITH HETEROGENEOUS SERVERS AND N-POLICY.....	9
3. Chernyshova E., Lisovskaya E., Moiseeva S., Pagano M. ON A TOTAL AMOUNT OF OCCUPIED RESOURCE IN THE SYSTEM WITH PARALLEL SERVICE AND RENEWAL ARRIVAL PROCESS.....	17
4. Galileyskaya A., Lisovskaya E., Pagano M., Moiseeva S. RESOURCE QS WITH THE REQUESTS DUPLICATION AT THE SECOND PHASE AND RENEWAL ARRIVAL PROCESS.....	25
5. Shchetinin E.Yu., Sevastianov L.A., Ayrjan E.A., Demidova A.V. MELANOMA DETECTION WITH DEEP NEURAL NETWORKS.....	33
6. Shchetinin E.Yu., Sevastianov L.A., Kulyabov D.S., Ayrjan E.A., Demidova A.V. PARALINGUISTIC MODEL FOR EMOTIONS RECOGNITION WITH DEEP NEURAL NETWORKS.....	42
7. Houankpo H.G.K., Kozyrev D.V., Nibasumba E., Mouale M.N.B., Sergeeva I.A. A SIMULATION APPROACH TO RELIABILITY ASSESSMENT OF A REDUNDANT SYSTEM WITH ARBITRARY DISTRIBUTIONS OF UPTIME AND REPAIR TIME OF ITS ELEMENTS.....	51
8. Bakanova N.B., Volchkov D.V., Bakanov A.S. CREATION AND VISUALIZATION OF THE SUBJECT AREA MODEL.....	60
9. Noskov I.I., Bogatyrev V.A. FAULTLESS AND TIMELY MULTIPATH PACKETS DELIVERY PROBABILITY IN COMPUTER NETWORKS USING UDP-BASED PROTOCOL.....	65
10. Andronov A., Dalinger Ia., Santalova D. OVERBOOKING'S PROBLEM FOR A CASE OF A RANDOM ENVIRONMENT EXISTENCE	75
11. Milovanova T.A., Razumchik R.V., Kozyrev D.V. MODELING D2D-ENHANCED IOT CONNECTIVITY.....	83
12. Dudin A.N., Dudin S.A., Dudina O.S. OPTIMIZATION OF A SIGNAL PROCESSING STRATEGY IN SENSOR NODES WITH ENERGY HARVESTING AND CONSUMPTION FOR ADMISSION AND TRANSMISSION.....	90
13. Antonova V.M., Kuznetsova A.M. ИЗУЧЕНИЕ СБОЕВ ПРИ РАБОТЕ ТЕХНОЛОГИИ MIMO.....	98
14. Brokarev I.A., Vaskovskii S.V. INFORMATION-PROCESSING SYSTEM FOR NATURAL GAS QUALITY ANALYSIS.....	106

15. Mandel A.S., Laptin V.A. CHANNEL SWITCHING STRATEGIES FOR MULTISTEP MARKOVIAN CONTROLLABLE QUEUING SYSTEMS (QS) PROBLEMS.....	114
16. Мандель А.С., Лаптин В.А., СТРАТЕГИИ ПЕРЕКЛЮЧЕНИЯ КАНАЛОВ В МНОГОШАГОВЫХ МАРКОВСКИХ ЗАДАЧАХ УПРАВЛЕНИЯ СМО.....	122
17. Shchetinin E.Yu., Sevastianov L.A., Kulyabov D.S., Ayrjan E.A. ON IMPROVING THE ACCURACY OF THE CLASSIFICATION ON IMBALANCED CLASSES WITH MACHINE LEARNING.....	130
18. Grusho A.A., Grusho N.A., Zabezhaiko M.I., Timonina E.E. GENERATION OF METADATA FOR INFORMATION TECHNOLOGY CONTROL.....	139
19. Vishnevsky V.M., Mukhtarov A.A., Pershin O.Yu. ЗАДАЧА ОПТИМАЛЬНОГО РАЗМЕЩЕНИЯ БАЗОВЫХ СТАНЦИЙ ШИРОКОПОЛОСНОЙ СЕТИ ДЛЯ КОНТРОЛЯ ЛИНЕЙНОЙ ТЕРРИТОРИИ ПРИ ОГРАНИЧЕНИИ НА ВЕЛИЧИНУ МЕЖКОНЦЕВОЙ ЗАДЕРЖКИ.....	148
20. Ivanova N.M. MODELING AND SIMULATION OF RELIABILITY FUNCTION OF A K-OUT- OF-N:F SYSTEM WITH PARTIAL REPAIR.....	156
21. Stepanov S.N., Stepanov M.S., Andrabi U., Ndayikunda Ju. THE MODELING OF RESOURCE SHARING FOR HETEROGENEOUS DATA STREAMS OVER 3GPP LTE WITH NB-IOT FUNCTIONALITY.....	164
22. Stepanov S.N., Stepanov M.S., Shishkin M.O. ESTIMATION OF PERFORMANCE MEASURES OF EMERGENCY SERVICES FOR OVERLOAD OF CALLS.....	173
23. Ivanov A., Ziazina N., Antonova V. PERFORMANCE OF MATLAB CLUSTERING ALGORITHMS	181
24. Nazarov A.A., Phung-Duc T., Izmailova Y.E. GAUSSIAN ASYMPTOTICS FOR A MULTICLASS M/M/1/1 RETRIAL QUEUEING SYSTEM.....	189
25. Namiot D., Sneps-Snepp M. HOW TO BUILD A HYPER-LOCAL INTERNET	197
26. Goldstein B.S., Kislyakov S.V. FORECASTING THE INCOMING LOAD OF A CONTACT CENTER USING CHAOS THEORY METHODS.....	207
27. Kalimulina E.Yu. ON ERGODICITY OF SOME STOCHASTIC NETWORKS AND ITS APPLICATIONS.....	215
28. Sopin E.S., Darmolad A.V., Bixalina D.N. QUANTIFYING THE ROUND-TRIP DELAY IN CLOUD-RAN.....	222
29. Sopin E., Begishev V., Moltchanov D., Samuylov A. RESOURCE QUEUING SYSTEM WITH PREEMPTIVE PRIORITY FOR URLLC AND EMBB COEXISTENCE IN 5G NR.....	229

30. Markovich N.M., Ryzhov M.S.	
LEADER ELECTION IN COMMUNITIES FOR INFORMATION SPREADING.....	236
31. Klimenok V.I., Dudin A.N., Vishnevsky V.M.	
СИСТЕМА МАССОВОГО ОБСЛУЖИВАНИЯ ММАР/PH _{1,2} /N/0 С НЕОДНОРОДНЫМИ ЗАПРОСАМИ И ПРИОРИТЕТАМИ.....	244
32. Nosova M.G.	
RESEARCH OF DEMOGRAPHIC PROCESSES BY METHODS OF QUEUING THEORY	252
33. Sztrik J., Tóth Á., Pintér Á., Bács Z.	
RELIABILITY ANALYSIS OF FINITE-SOURCE RETRIAL QUEUEING SYSTEMS WITH TWO-WAY COMMUNICATIONS TO THE ORBIT AND BLOCKING USING SIMULATION.....	260
34. Rykov V.V., Ivanova N.M., Kozyrev D.V.	
SENSITIVITY ANALYSIS OF CHARACTERISTICS OF A K-OUT-OF-N:F SYSTEM TO SHAPES OF LIFE AND REPAIR TIMES DISTRIBUTIONS OF ITS COMPONENTS.....	268
35. Bogatyrev V.A., Bogatyrev A.V., Bogatyrev S.V.	
TIMELINESS OF REDUNDANT SERVICE OF A HETEROGENEOUS REQUEST FLOW BY A SEQUENCE OF NODES OF THE INFO-COMMUNICATION SYSTEM.....	276
36. Mishkoy G.K., Mitev L.M.	
COMPUTATIONAL ASPECTS OF MODELLING PERFORMANCE CHARACTERISTICS FOR POLLING MODELS WITH SEMI-MARKOV SWITCHING AND PRIORITIES.....	284
37. Morozov V.P., Alikin K.A.	
SCALING ERROR SUPPRESSION IN SMALL SIGNAL PREAMPLIFIERS FOR VIBRATION MONITORING NETWORKS.....	290
38. Vanin A.B., Bogatyrev V.A., Bogatyrev S.V.	
DATA MIGRATION RATE OF THE CRUSH-BASED DISTRIBUTED OBJECT STORAGE WITH DYNAMIC TOPOLOGY.....	297
39. Vas Á, Tóth L.	
COMPARISON OF DIFFERENT METHODS FOR SMOOTHING INITIAL 2D DATA OF THE DSN-PC SYSTEM'S WEATHER PREDICTION ALGORITHM.....	305
40. Tsitovich I.I.	
GROUP POLLING METHOD FOR SENSORS DETECTING IN UNSYNCHRONIZED STRUCTURED WIRELESS MONITORING NETWORKS.....	315
41. Nekrasova R.S.	
REGENERATIVE ESTIMATION OF M/G/2-TYPE SYSTEM WITH SIMULTANEOUS SERVICE AND SPEED SCALING.....	323

42. Vorobiev V.M., Dyagilev R.A. НЕЙРОСЕТЬ В СОСТАВЕ СТАНЦИИ СЕЙСМИЧЕСКОГО МОНИТОРИНГА.....	332
43. Rudenkova M.A., Khayou H., Abrosimov L.I. A METHODOLOGY FOR ADAPTING WIRELESS CHANNEL RESOURCES TO THE LOAD BY SWITCHING BETWEEN MEDIUM ACCESS PROTOCOLS.....	338
44. Mamonov A.A., Varlamov R.A., Salpagarov S.I. DISTRIBUTION OF COMPUTING LOAD BY USING A P2P NETWORK.....	347
45. Sazonov D.D., Kiricheck R.V. IDENTIFICATION OF DEVICES IN A MESH NETWORKS BASED ON DIGITAL OBJECT ARCHITECTURE.....	355
46. Moshnikov A. EVALUATION OF NETWORK RELIABILITY AND ELEMENT IMPORTANCE METRICS USING THE R SOFTWARE PACKAGE.....	364
47. Alexandrov A., Monov V. SARSA BASED METHOD FOR WSN TRANSMISSION POWER MANAGEMENT.....	372
48. Nazarov A.A., Rozhkova S.V., Titarenko E.Yu. ASYMPTOTIC ANALYSIS OF $M^{[n]}/M/1$ RQ-SYSTEM WITH FEEDBACK AND BATCH POISSON ARRIVAL.....	380
49. Dorokhin S.V. SYNCHRONISATION OF ISS-OFDM SIGNALS.....	388
50. Nikolsky I.M., Furmanov K.K. ON EFFECTIVENESS OF MESSAGE RETRANSMISSION IN WIRELESS SENSOR NETWORKS.....	396
51. Efremova E.V., Kuzmin L.V. APPROACH TO INDOOR DISTANCE MEASUREMENT IN WIRELESS SENSOR NETWORKS BY MEANS OF ULTRA-WIDE-BAND CHAOTIC RADIO PULSES.....	404
52. Meykhanadzhyan L.A., Zaryadov I.S., Milovanova T.A. STATIONARY CHARACTERISTICS OF THE TWO-NODE TANDEM QUEUEING SYSTEM WITH POISSON ARRIVALS AND GENERAL RENOVATION.....	413
53. Korolkova A.V., Kulyabov D.S., Hnatič M. THE MULTI-MODEL APPROACH TO THE STUDY OF COMPLEX SYSTEMS USING THE EXAMPLE OF THE RED ACTIVE QUEUE MANAGEMENT ALGORITHM.....	418
54. Rogozin S.S. SIMULATION A MODIFIED ERLANG SYSTEM WITH PRIORITY CUSTOMERS.....	426

55. Apreutesey A.M.Y., Korolkova A.V., Kulyabov D.S. ВОЗМОЖНОСТИ ГИБРИДНОГО МОДЕЛИРОВАНИЯ СИСТЕМ С УПРАВЛЕНИЕМ НА ЯЗЫКАХ MODELICA И JULIA.....	433
56. Suranga Sampath M.I.G. TRANSIENT ANALYSIS OF AN M/M/1/N QUEUE WITH BALKING, CATASTROPHES, SERVER FAILURES AND REPAIRS.....	441
57. Borodina A.V., Tishenko V.A. ON ALGORITHMS FOR EFFECTIVE SPEED-UP SIMULATION OF RELIABILITY MODELS.....	449
58. Kulik V.A., Pham V.D., Kirichek R.V. MODELS AND METHODS OF USAGE OF THE HETEROGENEOUS GATEWAYS IN THE MESH LPWAN NETWORKS	458
59. Kulik V.A., Pham V.D., Kirichek R.V. ПРИМЕНЕНИЕ ГЕТЕРОГЕННЫХ ШЛЮЗОВ В ЯЧЕИСТЫХ СЕТЯХ LPWAN.....	466
60. Pham V.D., Le D.T., Kirichek R.V. ИССЛЕДОВАНИЕ ПРОТОКОЛОВ МАРШРУТИЗАЦИИ ДЛЯ ЯЧЕИСТОЙ СЕТИ ДАЛЬНЕГО РАДИУСА ДЕЙСТВИЯ.....	474
61. Pham V.D., Vorozheikina O.I., Grishin I.V., Okuneva D.V., Kirichek R.V. МЕТОД ОПРЕДЕЛЕНИЯ КООРДИНАТ УЗЛОВ В БЕСПРОВОДНОЙ СЕНСОРНОЙ СЕТИ С ЯЧЕИСТОЙ ТОПОЛОГИЕЙ.....	482
62. Pham V.D., Le D.T., Kirichek R.V. ИССЛЕДОВАНИЕ ИСПОЛЬЗОВАНИЯ ПРОТОКОЛА AODV В ЯЧЕИСТОЙ СЕТИ LORA.....	490
63. Pham V.D., Le D.T., Kirichek R.V. A STUDY OF USING AODV PROTOCOL IN LORA MESH NETWORK.....	499
64. Petrov P.D., Kostadinov G.B., Zhivkov P.R., Velichkova V.I., Balabanov T.D. APPROXIMATE SEQUENCING OF VIRTUAL REELS WITH GENETIC ALGORITHMS.....	507
65. Dimitrov B., Rykov V., Esa S. ON DIFFERENT APPROACHES TO STUDY A DOUBLE REDUNDANT RENEWABLE SYSTEM UNDER MARSHALL-OLKIN FAILURE MODEL....	515
66. Zverkina G.A. ERGODICITY OF GENERALIZED MARKOV MODULATED POISSON PROCESSES.....	523
67. Nazarov A.A., Phung-Duc T., Paul S.V., Lizyura O.D. ASYMPTOTIC-DIFFUSION ANALYSIS OF MULTISERVER RETRIAL QUEUE WITH TWO-WAY COMMUNICATION.....	531
68. Nazarov A.A., Paul S.V., Klyuchnikova P.N. ИССЛЕДОВАНИЕ ЦИКЛИЧЕСКОЙ СИСТЕМЫ С ПОВТОРНЫМИ ВЫЗОВАМИ.....	540
69. Khayou H., Rudenkova M.A., Abrosimov L.I. AN ALGEBRAIC APPROACH TO LOOP FREE ROUTING.....	548

70. Goldstein B.S., Fitsov V.V.	
THE MATHEMATICAL MODEL OF FRONT-END CALCULATING IN DPI SYSTEM.....	563
71. Khalina V., Prosvirov V., Gaidamaka Yu., Pokornyy J., Hosek J., Samouylov K.	
SIMULATION-BASED ANALYSIS OF MOBILITY MODELS FOR WIRELESS UAV-TO-X NETWORKS.....	571
72. Gerdt V.P., Kotkova E.A.	
ON THE QUANTUM TELEPORTATION OF BELL STATES PERFORMED ON 5-QUBIT IBM Q COMPUTERS.....	579
73. Kulik V.A., Gallyamov D.A., Kirichek R.V.	
ПОДХОДЫ К ОПРЕДЕЛЕНИЮ ПРИОРИТЕТОВ ОБСЛУЖИВАНИЯ СЕТЕВОГО ТРАФИКА ДЛЯ ГЕТЕРОГЕННЫХ ШЛЮЗОВ ПРОМЫШЛЕННОГО ИНТЕРНЕТА ВЕЩЕЙ.....	584
74. Chukhno N., Chukhno O., Araniti G., Iera A., Molinaro A., Pizzi S.	
DELIVERING MULTICAST TRAFFIC IN MMWAVE SYSTEMS: CHALLENGES AND PERFORMANCE ANALYSIS.....	590
75. Anilkumar M.P., Jose K.P.	
COMPARISON OF DIFFERENT LEVELS OF LOCAL PURCHASE QUANTITIES IN A GEO/GEO/I PRODUCTION INVENTORY SYSTEM.....	599
76. Simonov A.S., Brekhov O.M.	
ARCHITECTURE AND FUNCTIONALITY OF THE COLLECTIVE OPERATIONS SUBNET OF THE ANGARA INTERCONNECT.....	607
77. Tsarev A., Abaev P.	
MATHEMATICAL MODEL FOR HORIZONTAL ON-DEMAND VEPC SCALABILITY IN SDN-BASED ENVIRONMENT.....	620
78. Zhukova K.	
ESTIMATING THE OVERFLOW PROBABILITY IN SINGLE-SERVER RETRIAL SYSTEM WITH TWO CLASSES OF CUSTOMERS.....	632
79. Polin E.P., Moiseeva S.P., Moiseev A.N.	
ИССЛЕДОВАНИЕ БЕСКОНЕЧНОЛИНЕЙНОЙ СМО С ИНТЕНСИВНОСТЬЮ ВХОДЯЩЕГО ПОТОКА, ЗАВИСЯЩЕЙ ОТ СОСТОЯНИЯ СИСТЕМЫ.....	638
80. Tesfay A.A., Simon E.P., Clavier L.	
MULTI-USER DETECTION TO IMPROVE DOWNLINK COMMUNICATION OF CSS-BASED LORA-LIKE NETWORKS.....	645
81. Mathew N., Joshua V.C., Krishnamoorthy A.	
A QUEUEING INVENTORY SYSTEM WITH TWO CHANNELS OF SERVICE.....	653
82. Shorokhov S.G.	
ON WIRELESS CHANNEL MODELING WITH K DISTRIBUTION.....	662
83. Razumchik R.V.	
STATIONARY WAITING TIME DISTRIBUTION IN THE INFINITE-CAPACITY TWO-QUEUE SINGLE-SERVER RESEQUENCING SYSTEM WITH HOQ-LIFO- LIFO POLICY OPERATING IN RANDOM ENVIRONMENT.....	670

84. Vladimirov S., Vishnevsky V., Larionov A., Kirichek R. CONCEPT OF UFP BASED WBAN DATA ACQUISITION NETWORK.....	677
85. Kochetkov D.M., Kochetkova I.A., Makeeva E.D. ВЛИЯНИЕ ТЕХНОЛОГИЙ 5G НА РАЗВИТИЕ ЦИФРОВЫХ ЭКОСИСТЕМ УМНЫХ ГОРОДОВ: НАУКОМЕТРИЧЕСКИЙ И ПАТЕНТНЫЙ АНАЛИЗ.....	685
86. Makolkina M., Shipota N., Koucheryavy A. DEVELOPMENT AND INVESTIGATION OF MODEL NETWORK IMT2020 WITH THE USE OF MEC AND VOICE ASSISTANT TECHNOLOGIES.....	696
87. Kochetkov D.M., Almaganbetov M.O. 5G: ПАТЕНТНЫЙ ЛАНДШАФТ.....	706
88. Vishnevsky V., Rykov V., Finkelstein M. ПРОФИЛАКТИЧЕСКОЕ ОБСЛУЖИВАНИЕ ПРИВЯЗНОГО МОДУЛЯ ВЫСОТНОЙ ТЕЛЕКОММУНИКАЦИОННОЙ ПЛАТФОРМЫ.....	712
89. Yermakov A.S., Shukmanova A.A., Seilova N.A. THE MARKOV MODEL FOR A MULTIPHASE SECURITY SYSTEM WITH THE PARTIAL CONCURRENT SERVICE.....	724
90. Golovinov E.E., Aminev D.A., Tatunov S.Yu., Polesskiy S.N., Kozyrev D.V. ОЦЕНКА КОМПЛЕКТОВ ЗИП ДЛЯ РАСПРЕДЕЛЁННОЙ КОММУНИКАЦИОННОЙ СЕТИ МЕТЕОСТАНЦИЙ МИНИМАЛЬНОЙ КОНФИГУРАЦИИ.....	733
91. Rassadin Yu., Dushin S. БЕСПРОВОДНАЯ СЕНСОРНАЯ СЕТЬ ДЛЯ ИНТЕНСИВНОГО СБОРА ДААННЫХ НА ОСНОВЕ ТЕХНОЛОГИИ LORAWAN И РАСПРЕДЕЛЕННОГО АЛГОРИТМА СЖАТИЯ ИНФОРМАЦИИ.....	743
92. Melnikov S.Yu., Samouylov K.E. CESARO-HEREDITY PROPERTY IN THE SHIFT REGISTER FAMILY.....	751
93. Rassadin Yu., Dushin S. WIRELESS SENSOR NETWORK FOR INTENSIVE DATA COLLECTION BASED ON LORAWAN TECHNOLOGY AND DISTRIBUTED DATA COMPRESSION ALGORITHM.....	764
94. Vishnevsky V., Dinh T.D., Vybornova A., Kirichek R. FLYING NETWORK FOR EMERGENCY USING TETHERED MULTICOPTERS.....	771

UDC: 519.872

Asymptotic-Diffusion Analysis of Multiserver Retrial Queue with Two-Way Communication

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Abstract

In this paper, we consider multiserver retrial queue with two-way communication. Incoming calls arrive according to the Poisson process and reserve the server for an exponentially distributed time. If all of the servers are busy the incoming call joins the orbit and makes a delay for an exponentially distributed time before the next attempt to occupy the server. Idle servers also make outgoing calls following a distinct exponential distribution. Using the asymptotic-diffusion analysis method we derive the approximation for the stationary probability distribution of the number of calls in the orbit.

Keywords: multiserver retrial queue, two-way communication, incoming call, outgoing call, asymptotic-diffusion analysis, diffusion approximation

1. Introduction

Currently, more and more services are partially or fully working in a call-center mode. Banks use call centers to advise customers and to advertise their services; online stores utilize call centers to refine and confirm orders, and call centers also exist independently to conduct social surveys.

A lot of research papers are devoted to modeling call centers. Papers [1, 2, 3, 4] are devoted to the quality of customer service in telephone services. In [5], the authors present a study of incoming processes models in real call-centers. A statistical analysis of the work of call-centers taking into account various aspects of the functioning of the service is presented in the study [6].

Recently, retrial queues with two-way communication have been used as a model of a blended call-center. The most detailed studies on retrial queues can be found

The publication has been prepared with the support of RFBR according to the research project No.18-01-00277.

in [7, 8]. The main feature of model with two-way communication is that during idle time, the server makes outgoing calls and serves them along with incoming calls. As an outgoing call we refer to an operator's call to a client, or any other type of alternative operator's activities rather than serving incoming calls. Thus, retrial queueing models with two-way communication are flexible and allow you to simulate most modern telephone services. In [9] the model of multiserver retrial queue with two-way communication was proposed and numerical analysis was presented.

In this paper, we consider the same model as in [9] but rather than numerical analysis of the stationary distribution, our focus is to obtain the diffusion limit of the underlying time-dependent Markov process [10]. As a byproduct, the limiting results are then used to approximate the probability distribution of the number of customers in the orbit in the stationary state.

The rest of our paper is organized as follows. Section 2 is devoted to the presentation of the model and preliminary analysis of the underlying Markov chain. Our main results are in Section 3 where we present the asymptotic-diffusion analysis to obtain the diffusion limit of the underlying Markov chain. In Section 4 we describe an algorithm for constructing a probability distribution approximation of the system state. Section 5 shows the diffusion approximation accuracy for several values of system parameters. Finally, Section 6 is devoted to some concluding remarks.

2. Mathematical model

We consider multiserver retrial queue with two-way communication. The input process is a stationary Poisson process with rate λ . Service times of incoming calls are exponentially distributed with rate μ_1 . Calls that find servers fully occupied join the orbit and reattempt to access the server after an exponentially distributed delay with rate σ . When the server is idle it makes an outgoing calls with rate α and provides the service for an exponentially distributed time with rate μ_2 . We denote N is the number of servers in the system.

Let $n_1(t), n_2(t)$ denote the number of servers busy serving incoming and outgoing calls at the time t , respectively. Also $i(t)$ is a number of calls in the orbit. Thus, we can see that three-dimensional random process $\{n_1(t), n_2(t), i(t)\}$ is a continuous time Markov chain.

Let $P(n_1, n_2, i, t) = P\{n_1(t) = n_1, n_2(t) = n_2, i(t) = i\}$ denotes the probability distribution of the process $\{n_1(t), n_2(t), i(t)\}$, which is the solution of Kolmogorov's system of equations

$$\begin{aligned}\frac{\partial P(0, 0, i, t)}{\partial t} &= -(\lambda + i\sigma + N\alpha)P(0, 0, i, t) + \mu_1 P(1, 0, i, t) + \mu_2 P(0, 1, i, t), \\ \frac{\partial P(n_1, n_2, i, t)}{\partial t} &= -(\lambda + i\sigma + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2)P(n_1, n_2, i, t) + \\ &\quad + (\lambda + i\sigma + (N - n_1 - n_2)\alpha)P(n_1, n_2, i-1, t) + \\ &\quad + (n_1\mu_1 + n_2\mu_2)P(n_1, n_2, i+1, t) + \\ &\quad + \alpha P(n_1, n_2, i, t-1) + \alpha P(n_1, n_2, i+1, t) - \alpha P(n_1, n_2, i, t).\end{aligned}$$

$$\begin{aligned}
& +\lambda P(n_1 - 1, n_2, i, t) + (i + 1)\sigma P(n_1 - 1, n_2, i + 1, t) + \\
& +(N - n_1 - n_2 + 1)\alpha P(n_1, n_2 - 1, i, t) + (n_1 + 1)\mu_1 P(n_1 + 1, n_2, i, t) + \\
& +(n_2 + 1)\mu_2 P(n_1, n_2 + 1, i, t), \quad 0 < n_1 + n_2 < N, \\
\frac{\partial P(n_1, n_2, i, t)}{\partial t} = & -(\lambda + n_1\mu_1 + n_2\mu_2)P(n_1, n_2, i, t) + \lambda P(n_1 - 1, n_2, i, t) + \\
& +\lambda P(n_1, n_2, i - 1, t) + (i + 1)\sigma P(n_1 - 1, n_2, i + 1, t) + \\
& +\alpha P(n_1, n_2 - 1, i, t), \quad n_1 + n_2 = N.
\end{aligned} \tag{1}$$

Then we transform the system (1) into system for partial characteristic functions

$$H(n_1, n_2, u, t) = \sum_{i=0}^{\infty} e^{ju_i} P(n_1, n_2, i, t), \text{ where } j = \sqrt{-1}$$

$$\begin{aligned}
\frac{\partial H(n_1, n_2, u, t)}{\partial t} = & -(\lambda + N\alpha)H(n_1, n_2, u, t) + j\sigma \frac{\partial H(n_1, n_2, u, t)}{\partial u} + \\
& +\mu_1 H(n_1 + 1, n_2, u, t) + \mu_2 H(n_1, n_2 + 1, u, t), \quad n_1 + n_2 = 0, \\
\frac{\partial H(n_1, n_2, u, t)}{\partial t} = & -(\lambda + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2)H(n_1, n_2, u, t) + \\
& +j\sigma \frac{\partial H(n_1, n_2, u, t)}{\partial u} + \lambda H(n_1 - 1, n_2, u, t) - j\sigma e^{-ju} \frac{\partial H(n_1 - 1, n_2, u, t)}{\partial u} + \\
& +(N - n_1 - n_2 + 1)\alpha H(n_1, n_2 - 1, u, t) + (n_1 + 1)\mu_1 H(n_1 + 1, n_2, u, t) + \\
& +(n_2 + 1)\mu_2 H(n_1, n_2 + 1, u, t), \quad 0 < n_1 + n_2 < N, \\
\frac{\partial H(n_1, n_2, u, t)}{\partial t} = & -(\lambda + n_1\mu_1 + n_2\mu_2)H(n_1, n_2, u, t) + \lambda H(n_1 - 1, n_2, u, t) + \\
& +\lambda e^{ju} H(n_1, n_2, u, t) - j\sigma e^{-ju} \frac{\partial H(n_1 - 1, n_2, u, t)}{\partial u} + \\
& +\alpha H(n_1, n_2 - 1, u, t), \quad n_1 + n_2 = N.
\end{aligned} \tag{2}$$

We denote $\mathbf{H}(u, t)$ is a matrix of functions $H(n_1, n_2, u, t)$ and rewrite the system (2) in form of

$$\frac{\partial \mathbf{H}(u, t)}{\partial t} = (\mathbf{A} + \lambda e^{ju} \mathbf{B}) \mathbf{H}(u, t) + j\sigma (\mathbf{I}_0 - e^{-ju} \mathbf{I}_1) \frac{\partial \mathbf{H}(u, t)}{\partial u}, \tag{3}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{I}_0$, $\mathbf{I}_1$ are operators, which set in the following form

$$\mathbf{A}\mathbf{H}(u, t) = \begin{cases} -(\lambda + N\alpha)H(n_1, n_2, u, t) + \mu_1 H(n_1 + 1, n_2, u, t) + \\ + \mu_2 H(n_1, n_2 + 1, u, t), & n_1 + n_2 = 0, \\ -(\lambda + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2)H(n_1, n_2, u, t) + \\ + (N - n_1 - (n_2 - 1))\alpha H(n_1, n_2 - 1, u, t) + \\ + \lambda H(n_1 - 1, n_2, u, t) + (n_1 + 1)\mu_1 H(n_1 + 1, n_2, u, t) + \\ + (n_2 + 1)\mu_2 H(n_1, n_2 + 1, u, t), & 0 < n_1 + n_2 < N, \\ -(\lambda + n_1\mu_1 + n_2\mu_2)H(n_1, n_2, u, t) + \lambda H(n_1 - 1, n_2, u, t) + \\ + \alpha H(n_1, n_2 - 1, u, t), & n_1 + n_2 = N, \end{cases} \quad (4)$$

$$\mathbf{B}\mathbf{H}(u, t) = \begin{cases} 0, & n_1 + n_2 < N, \\ H(n_1, n_2, u, t), & n_1 + n_2 = N, \end{cases} \quad (5)$$

$$\mathbf{I}_0\mathbf{H}(u, t) = \begin{cases} H(n_1, n_2, u, t), & n_1 + n_2 < N, \\ 0, & n_1 + n_2 = N, \end{cases} \quad (6)$$

$$\mathbf{I}_1\mathbf{H}(u, t) = \begin{cases} 0, & n_1 + n_2 = 0, \\ H(n_1 - 1, n_2, u, t), & n_1 + n_2 > 0. \end{cases} \quad (7)$$

We denote $\mathbf{E}$ as an operator that summarizing over all available values of n_1 , n_2 and present the additional equation that we need to provide analysis

$$\mathbf{E} \frac{\partial \mathbf{H}(u, t)}{\partial t} = \mathbf{E}(\mathbf{A} + \lambda e^{ju} \mathbf{B})\mathbf{H}(u, t) + j\sigma \mathbf{E}(\mathbf{I}_0 - e^{-ju} \mathbf{I}_1) \frac{\partial \mathbf{H}(u, t)}{\partial u}. \quad (8)$$

We also note that

$$\mathbf{E}(\mathbf{A} + \lambda \mathbf{B}) = 0, \quad \mathbf{E}(\mathbf{I}_0 - \mathbf{I}_1) = 0. \quad (9)$$

3. Asymptotic-Diffusion Analysis

In operator equations (3) and (8) we introduce the following notations

$$\sigma = \varepsilon, \quad u = \varepsilon w, \quad \tau = \varepsilon t, \quad \mathbf{H}(u, t) = \mathbf{F}(w, \tau, \varepsilon),$$

to obtain the equations

$$\varepsilon \frac{\partial \mathbf{F}(w, \tau, \varepsilon)}{\partial \tau} = (\mathbf{A} + \lambda e^{jw\varepsilon} \mathbf{B})\mathbf{F}(w, \tau, \varepsilon) + j(\mathbf{I}_0 - e^{-jw\varepsilon} \mathbf{I}_1) \frac{\partial \mathbf{F}(w, \tau, \varepsilon)}{\partial w}, \quad (10)$$

$$\varepsilon \mathbf{E} \frac{\partial \mathbf{F}(w, \tau, \varepsilon)}{\partial \tau} = \mathbf{E}(\mathbf{A} + \lambda e^{jw\varepsilon} \mathbf{B}) \mathbf{F}(w, \tau, \varepsilon) + j \mathbf{E}(\mathbf{I}_0 - e^{-jw\varepsilon} \mathbf{I}_1) \frac{\partial \mathbf{F}(w, \tau, \varepsilon)}{\partial w}. \quad (11)$$

We solve the system (11) taking the limit as $\varepsilon \rightarrow 0$ and present the result in following theorem.

Theorem 1. In considered retrial queue, the stationary probability distribution $R(n_1, n_2)$ of the two-dimensional process $\{n_1(t), n_2(t)\}$ is a solution of the system of operator equations

$$\begin{aligned} (\mathbf{A} + \lambda \mathbf{B} - x(\mathbf{I}_0 - \mathbf{I}_1)) \mathbf{R} &= 0, \\ \mathbf{E} \mathbf{R} &= 1, \end{aligned} \quad (12)$$

where $\mathbf{R}$ is a matrix of probabilities $R(n_1, n_2)$, function $x(\tau)$ is a solution of differential equation

$$x'(\tau) = \mathbf{E} [\lambda \mathbf{B} - x(\tau) \mathbf{I}_1] \mathbf{R}.$$

Denoting

$$a(x) = \mathbf{E} [\lambda \mathbf{B} - x \mathbf{I}_1] \mathbf{R} \quad (13)$$

and making the following replacements in the operator equations (3) and (8)

$$\mathbf{H}(u, t) = e^{j \frac{u}{\sigma} x(\sigma t)} \mathbf{H}^{(2)}(u, t), \quad (14)$$

we have

$$\begin{aligned} & \frac{\partial \mathbf{H}^{(2)}(u, t)}{\partial t} + j u x'(\sigma t) \mathbf{H}^{(2)}(u, t) = \\ & = (\mathbf{A} + \lambda e^{ju} \mathbf{B} - x(\sigma t)(\mathbf{I}_0 - e^{-ju} \mathbf{I}_1)) \mathbf{H}^{(2)}(u, t) + j \sigma (\mathbf{I}_0 - e^{-ju} \mathbf{I}_1) \frac{\partial \mathbf{H}^{(2)}(u, t)}{\partial u}, \end{aligned} \quad (15)$$

$$\begin{aligned} & \mathbf{E} \frac{\partial \mathbf{H}^{(2)}(u, t)}{\partial t} + j u x'(\sigma t) \mathbf{E} \mathbf{H}^{(2)}(u, t) = \\ & = \mathbf{E}(\mathbf{A} + \lambda e^{ju} \mathbf{B} - x(\sigma t)(\mathbf{I}_0 - e^{-ju} \mathbf{I}_1)) \mathbf{H}^{(2)}(u, t) + j \sigma \mathbf{E}(\mathbf{I}_0 - e^{-ju} \mathbf{I}_1) \frac{\partial \mathbf{H}^{(2)}(u, t)}{\partial u}. \end{aligned} \quad (16)$$

In operator equations (15) and (16) we introduce the following notations

$$\sigma = \varepsilon^2, \quad \tau = \varepsilon^2 t, \quad u = \varepsilon w, \quad \mathbf{H}^{(2)}(u, t) = \mathbf{F}^{(2)}(w, \tau, \varepsilon), \quad (17)$$

to obtain the equations

$$\begin{aligned} & \varepsilon^2 \frac{\partial \mathbf{F}^{(2)}(w, \tau, \varepsilon)}{\partial \tau} + j w \varepsilon a(x) \mathbf{F}^{(2)}(w, \tau, \varepsilon) = \\ & = (\mathbf{A} + \lambda e^{jw\varepsilon} \mathbf{B} - x(\mathbf{I}_0 - e^{-jw\varepsilon} \mathbf{I}_1)) \mathbf{F}^{(2)}(w, \tau, \varepsilon) + \end{aligned}$$

$$+j\varepsilon(\mathbf{I}_0 - e^{-jw\varepsilon}\mathbf{I}_1)\frac{\partial \mathbf{F}^{(2)}(w, \tau, \varepsilon)}{\partial w}, \quad (18)$$

$$\begin{aligned} & \varepsilon^2 \mathbf{E} \frac{\partial \mathbf{F}^{(2)}(w, \tau, \varepsilon)}{\partial \tau} + jw\varepsilon a(x) \mathbf{E} \mathbf{F}^{(2)}(w, \tau, \varepsilon) = \\ & = \mathbf{E}(\mathbf{A} + \lambda e^{jw\varepsilon} \mathbf{B} - x(\mathbf{I}_0 - e^{-jw\varepsilon} \mathbf{I}_1)) \mathbf{F}^{(2)}(w, \tau, \varepsilon) + \\ & + j\varepsilon \mathbf{E}(\mathbf{I}_0 - e^{-jw\varepsilon} \mathbf{I}_1) \frac{\partial \mathbf{F}^{(2)}(w, \tau, \varepsilon)}{\partial w}. \end{aligned} \quad (19)$$

Solving the equations (18) and (19) taking the limit as $\varepsilon \rightarrow 0$ we present the following theorem.

Theorem 2. Probability density of diffusion limit $z(\tau)$ of the number of calls in the orbit given as follows

$$\Pi(z) = \frac{C}{b(z)} \exp \left\{ \frac{2}{\sigma} \int_0^z \frac{a(x)}{b(x)} dx \right\}, \quad (20)$$

where C is a normalization factor, function $a(x)$ is defined by (13), function $b(x)$ has the following form

$$b(x) = a(x) + 2[\mathbf{E}(\lambda \mathbf{B} - x \mathbf{I}_1) \mathbf{g} + x \mathbf{E} \mathbf{I}_1 \mathbf{R}]. \quad (21)$$

Here $\mathbf{g}$ is the matrix of additional values. It has the same dimension as $\mathbf{R}$ and appears as the solution of the system of operator equations

$$\begin{aligned} & (\mathbf{A} + \lambda \mathbf{B} - x(\mathbf{I}_0 - \mathbf{I}_1)) \mathbf{g} = a(x) \mathbf{R} - (\lambda \mathbf{B} - x \mathbf{I}_1) \mathbf{R}, \\ & \mathbf{E} \mathbf{g} = 0. \end{aligned} \quad (22)$$

From the obtained probability density $\Pi(z)$ we build the approximation of the probability distribution of the number of calls in the orbit using expression

$$PD(i) = \frac{\Pi(i\sigma)}{\sum_{n=0}^{\infty} \Pi(n\sigma)}. \quad (23)$$

4. Algorithm of Calculating Drift and Diffusion Coefficients

To obtain the function $a(x)$ we need to calculate the elements of the matrix $\mathbf{R}$. We rewrite the system of operator equations (12) in scalar form

$$-(\lambda + N\alpha + x(\tau))R(n_1, n_2) + \mu_1 R(n_1 + 1, n_2) + \mu_2 R(n_1, n_2 + 1) = 0, \quad n_1 + n_2 = 0,$$

$$\begin{aligned}
& -(\lambda + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2 + x(\tau))R(n_1, n_2) + \\
& + (N - n_1 - (n_2 - 1))\alpha R(n_1, n_2 - 1) + (\lambda + x(\tau))R(n_1 - 1, n_2) + \\
& + (n_1 + 1)\mu_1 R(n_1 + 1, n_2) + (n_2 + 1)\mu_2 R(n_1, n_2 + 1) = 0, \quad 0 < n_1 + n_2 < N, \\
& -(n_1\mu_1 + n_2\mu_2)R(n_1, n_2) + (\lambda + x(\tau))R(n_1 - 1, n_2) + \alpha R(n_1, n_2 - 1) = 0, \quad n_1 + n_2 = N.
\end{aligned}$$

$$\sum_{n_1=0}^N \sum_{n_2=0}^{N-n_1} R(n_1, n_2) = 1. \quad (24)$$

We transform the system of equations (24) to the system of linear algebraic equations renumbering the elements of the matrix $\mathbf{R}$ in the following way

$$(n_1, n_2) \rightarrow 2n_1 + n_2 + \frac{(n_1 + n_2)^2 - (n_1 + n_2)}{2}. \quad (25)$$

Then we obtain the system of equations

$$\tilde{\mathbf{R}}\mathbf{D}(x) = 0, \quad \tilde{\mathbf{R}}\mathbf{e} = 1, \quad (26)$$

where $\tilde{\mathbf{R}}$ is a vector of probabilities $R(n_1, n_2)$, $\mathbf{e}$ is a unit vector, the matrix $\mathbf{D}(x)$ is a matrix of the system with renumbered elements.

Solving the system of equations (26) we can express the function $a(x)$ as follows

$$a(x) = (\lambda + x)\tilde{\mathbf{R}}\mathbf{e}_1 - x, \quad (27)$$

where $\mathbf{e}_1$ is a vector, where the last $N + 1$ elements are ones and the other elements are zeroes.

To derive the function $b(x)$ we need to calculate the elements of the matrix $\mathbf{g}$. We rewrite the system of operator equations (22) in scalar form

$$\begin{aligned}
& -(\lambda + N\alpha + x(\tau))g(n_1, n_2) + \mu_1 g(n_1 + 1, n_2) + \mu_2 g(n_1, n_2 + 1) = a(x)R(n_1, n_2), \\
& n_1 + n_2 = 0, \\
& -(\lambda + (N - n_1 - n_2)\alpha + n_1\mu_1 + n_2\mu_2 + x(\tau))g(n_1, n_2) + (N - n_1 - (n_2 - 1))\alpha g(n_1, n_2 - 1) + \\
& + (\lambda + x(\tau))g(n_1 - 1, n_2) + (n_1 + 1)\mu_1 g(n_1 + 1, n_2) + (n_2 + 1)\mu_2 g(n_1, n_2 + 1) = \\
& = a(x)R(n_1, n_2) + x(\tau)R(n_1 - 1, n_2), \quad 0 < n_1 + n_2 < N, \\
& -(n_1\mu_1 + n_2\mu_2)g(n_1, n_2) + (\lambda + x(\tau))g(n_1 - 1, n_2) + \alpha g(n_1, n_2 - 1) = \\
& = (a(x) - \lambda)R(n_1, n_2) + x(\tau)R(n_1 - 1, n_2), \quad n_1 + n_2 = N.
\end{aligned}$$

$$\sum_{n_1=0}^N \sum_{n_2=0}^{N-n_1} g(n_1, n_2) = 0. \quad (28)$$

We transform the system of the operator equations (22) to the system of linear algebraic equations renumbering the elements of the matrix $\mathbf{g}$ using (25). Thus, we obtain the system of equations

$$\tilde{\mathbf{g}}\mathbf{D}(x) = \mathbf{d}(x), \quad \tilde{\mathbf{g}}\mathbf{e} = 0, \quad (29)$$

where $\tilde{\mathbf{g}}$ is a vector of elements of the matrix $\mathbf{g}$ with renumbered elements, vector $\mathbf{d}(x)$ is a vector of right parts of the system (28) with renumbered elements.

Solving the system of equations (29) we can express the function $b(x)$ as follows

$$b(x) = a(x) + 2[(\lambda + x)\tilde{\mathbf{g}}\mathbf{e}_1 + x(1 - \tilde{\mathbf{R}}\mathbf{e}_1)]. \quad (30)$$

5. Numerical Examples

We fix the number of servers in the system $N = 5$. We assume that the rates of service times are $\mu_1 = 1$, $\mu_2 = 2$. Outgoing calls rate is $\alpha = 1$.

The accuracy of approximation we will determine using Kolmogorov range

$$\Delta = \max_{0 \leq i \leq \infty} \left| \sum_{n=0}^i (P(n) - PD(n)) \right|, \quad (31)$$

where $P(n)$ is the probability distribution of the number of calls in the orbit, obtained with simulation, $PD(n)$ is a diffusion approximation defined by (23). Calculation of Δ we provide while both $P(n)$ and $PD(n)$ are not equal to zero.

We assume that the approximation is acceptable when its accuracy $\Delta < 0.05$. Table 1 depicts the accuracy of the diffusion approximation $PD(n)$ depending on parameters σ and ρ , where ρ characterizes the system load $\rho = \lambda/N\mu_1$.

Δ	$\sigma = 5$	$\sigma = 2$	$\sigma = 1$	$\sigma = 0.5$	$\sigma = 0.2$	$\sigma = 0.1$
$\rho = 0.6$	0,04	0,039	0,039	0,022	0,015	0,023
$\rho = 0.7$	0,07	0,063	0,051	0,036	0,031	0,032
$\rho = 0.8$	0,09	0,078	0,064	0,049	0,039	0,041
$\rho = 0.9$	0,081	0,069	0,065	0,053	0,044	0,05

Table 1. Kolmogorov range

6. Conclusion

We have considered multiserver retrial queue with two-way communication. Using asymptotic-diffusion analysis method we have built the diffusion process the distribution density of which we used to construct the approximation of the number of calls in the orbit.

We have presented numerical experiments where we have estimated the accuracy of the approximation by comparing to simulation.

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Номер госрегистрации 0322002892 в НТЦ «Информрегистр».

Научное электронное издание

**Распределенные компьютерные и
телекоммуникационные сети: управление,
вычисление, связь (DCCN-2020)**

МАТЕРИАЛЫ XXIII МЕЖДУНАРОДНОЙ НАУЧНОЙ КОНФЕРЕНЦИИ

(14–18 СЕНТЯБРЯ 2020 г., МОСКВА)

Под общей редакцией д.т.н. В.М. Вишневского, д.т.н. К.Е. Самуйлова

Составитель: к.ф.-м.н. Козырев Дмитрий Владимирович

Локальное электронное издание

Номер госрегистрации 0322002892 в НТЦ «Информрегистр»

Мин. системные требования:

Pentium 4, Internet Explorer, Acrobat reader 4.0 и выше

Дата подписания к использованию: 01.09.2020

1 электронно-оптический диск (CD-R), 44,9 Мб, Тираж 210 экз.

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