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Robust extrapolation in discrete systems with random jump parameters and incomplete information

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Abstract

An algorithm for the synthesis of a robust extrapolator is considered, which determines the estimate of the state vector of a discrete linear system with random jump parameters described by a Markov chain with a finite number of states under incomplete information about the model and the observation channel. The transfer matrix of the extrapolator are invited to choose independent of the state of the jump process using the nonparametric smoothing procedure.

Keywords: discrete model, robust extrapolation, jump parameters, incomplete information, nonparametric smoothing.

Introduction

The problem of constructing estimates of extrapolation and filtering under incomplete information were considered in [3–6, 12, 13, 15]. In these papers, problems of estimating using recurrent algorithms of the Kalman type under the condition of the presence of unknown inputs in the model were considered, and the problem of estimation in the presence of an unknown vector in the observation channel was also considered in [16]. In [1, 2, 9–11, 14], estimation problems in systems with random jump parameters were considered. Such problems arise when building models of real processes with possible failures. In this paper, we consider the problem of synthesizing a robust extrapolator for a discrete object with random jump parameters with a finite number of states. Problems are considered for objects and observation channels with unknown parameters and unknown additive vectors. Using the procedure of non-parametric smoothing, the problem of synthesizing a robust extrapolator is solved, the transfer coefficients of which do not depend on the state of the jump process. The simulation results are given.

1 Statement Problem

We consider the discrete stochastic system, which is described by the equation

$$x(k+1) = (A_\gamma + \Delta A_\gamma)x(k) + B_\gamma f(k) + q_\gamma(k),\ x(0) = x_0, \tag{1}$$

and available observations are set as follows:

$$y(k) = (S_\gamma + \Delta S_\gamma)x(k) + H_\gamma \phi(k) + v_\gamma(k), \tag{2}$$

where $x(k) \in R^m$ is a state of the system; $\gamma = \gamma(k)$ is a jump parameter (Markov chain with n states $\gamma_1, \gamma_2, ..., \gamma_n$); $f(k)$, $\phi(k)$ are unknown vectors; x_0 is a random vector ($\bar{x}_0 = \mathsf{E}\{x_0\}$ and $N_{0,i} = \mathsf{E}\{(x_0 - \bar{x}_0) \times (x_0 - \bar{x}_0)^T / \gamma = \gamma_i\}, i = \overline{1,n}$); $y(k) \in R^l$ is the observation vector; A_γ, B_γ, S_γ, H_γ are given matrices; ΔA_γ, ΔS_γ are unknown matrices; $q_\gamma(k)$, $v_\gamma(k)$ are independent random sequences with the following characteristics: $\mathsf{E}\{q_\gamma(k)\} = 0$, $\mathsf{E}\{v_\gamma(k)\} = 0$, $\mathsf{E}\left\{q_\gamma(k) q_\gamma^T(j)\right\} = Q_\gamma \delta_{kj}$, $\mathsf{E}\left\{v_\gamma(k) v_\gamma^T(j)\right\} = V_\gamma \delta_{kj}$ (E denotes expectation and T denotes transposition of a matrix, δ_{kj} is the Kronecker symbol).

The probability $p_i(k) = P\{\gamma(k) = \gamma_i\}$, $i = \overline{1,n}$, satisfies the equation

$$p_i(k+1) = \sum_{j=1}^{n} p_{ij} p_j(k),\ p_i(0) = p_{i,0}, \tag{3}$$

where p_{ij} is the probability of transition from the state i to the state j in one step, $p_{i,0}$ is the initial probability of the i-th state. According to the information received at the k-th step, it is required to find estimates of extrapolation $\widehat{x}(k+1)$ based on the minimization of the following criterion:

$$\begin{aligned} J[0;T_f] = \frac{1}{T_f} \mathsf{E}\{[\sum_{k=0}^{T_f} \sum_{i=1}^{n} p_i(k) e^T(k) R_i(k) e(k) + \\ + \sum_{i=1}^{n} p_i(T_f) e^T(T_f) L_i(T_f) e(T_f)] / \gamma(0) = \gamma_0\}, \end{aligned} \tag{4}$$

where $e(k) = x(k) - \widehat{x}(k)$, $R_i(k) > 0$, $L_i(k) > 0$ are weight matrices, γ_0 is the initial value of jump parameter γ.

2 Optimization of the Criterion

We present the system (1) in the following form:

$$x(k+1) = A_\gamma x(k) + r(k) + q_\gamma(k),\ x(0) = x_0, \tag{5}$$

where $r(k) = \Delta A_\gamma x(k) + B_\gamma f(k)$ is an unknown input. The channel of observations we will present as

$$y(k) = S_\gamma x(k) + \varphi(k) + v_\gamma(k), \tag{6}$$

where $\varphi(k) = \Delta S_\gamma x(k) + H_\gamma \phi(k)$ is unknown vector in the observation channel.

To solve the problem of synthesizing a robust extrapolator, we use the separation principle. This means that we first construct estimates of the vector under the assumption that the vectors $r(k)$ and $\varphi(k)$ are known. For this, we use the recurrent algorithm of Kalman extrapolation [1]

$$\widehat{x}(k+1) = A_\gamma \widehat{x}(k) + r(k) + K(k)(y(k) - S_\gamma \widehat{x}(k) - \varphi(k)),\ \widehat{x}(0) = \bar{x}_0. \tag{7}$$

In (7), we will look for a matrix $K(k)$ independent of $\gamma(k)$ that will ensure the extrapolator robustness with respect to the error of jump parameter $\gamma(k)$. Then, the estimates of the vectors $\widehat{r}(k)$ and $\widehat{\varphi}(k)$ are constructed under the assumption that the prediction estimate of the state vector $\widehat{x}(k)$ is known.

We introduce the notation for the matrices $Q_\gamma, V_\gamma, R_\gamma, N_\gamma, L_\gamma, A_\gamma, S_\gamma$ at $\gamma = \gamma_i$ as $Q_i, V_i, R_i, N_i, L_i, A_i, S_i$, respectively ($i = \overline{1, n}$). Consider a theorem, in which we construct an algorithm determining the matrix $K(k)$ for extrapolator (7) based on optimization of criterion (4).

Theorem. Let there exist positive definite matrices N_i and L_i, which are the solution of a two-point boundary value problem:

$$N_i(k+1) = (A_i - K(k)S_i)(\sum_{j=1}^{n} p_{i,j} N_j(k))(A_i - K(k)S_i)^T + Q_i + \\ + K(k)V_i K(k)^T,\ N_i(0) = N_0, \tag{8}$$

$$L_i(k) = (A_i - K(k)S_i)^T(\sum_{j=1}^{n} p_{i,j} L_j(k+1))(A_i - K(k)S_i)^T + R_i,\ L_i(T_f) = L_{T,i}. \tag{9}$$

Then, the vector $\mathrm{ct}(K(k))$, composed of transpose rows of the matrix $K(k)$ and providing the minimum of criterion (4), is determined by the formula:

$$\mathrm{ct}(K(k)) = (\sum_{i=1}^{n} p_i(k+1)[L_i(k+1) \otimes S_i(\sum_{j=1}^{n} p_{i,j} N_j(k))S_i^T + \\ + L_i(k+1) \otimes V_i])^{-1} \mathrm{ct}(\sum_{i=1}^{n} p_i(k+1) L_i(k+1) A_i (\sum_{j=1}^{n} p_{i,j} N_j(k) S_i^T)). \tag{10}$$

In (10), the symbol $\otimes$ denotes the Kronecker product.

Proof. Let's present the criterion (4) as a sum

$$J[0; T_f] = \sum_{i=1}^{n} J_i[k, T_f],\ k = \overline{0, T_f}. \tag{11}$$

In (11) $J_i[k, T_f]$ is determined by the formula

$$J_i[k; T_f] = \sum_{\xi=k}^{T_f - 1} \mathrm{tr}\, p_i(\xi) N_i(\xi) R_i(\xi) + \mathrm{tr}\, p_i(T_f) N_i(T_f) L_i(T_f), \tag{12}$$

where tr is the trace of a matrix.

Introduce the following Lyapunov function:

$$W(k, N_i(k)) = \mathrm{tr}\, p_i(k) N_i(k) R_i(k) + \mathrm{tr} \sum_{t=k}^{T_f} p_i(t) \bar{\Psi}_i(t) L_i(t), \tag{13}$$

where $L_i(t)$ is determined by equation (9), $\bar{\Psi}_i(t) = Q_i + K(t)V_i K(t)^T + \Psi_i(t)$ ($\Psi_i(t)$ is some positive definite matrix).

Summing over $k = \overline{t, T_f - 1}$, the finite differences of the function $W(k, N_i(k))$, taking into account formula (9), we obtain:

$$\sum_{k=t}^{T_f} \Delta W(k, N_I(k)) = \sum_{k=t}^{T_f-1} [W(k+1, N_i(k+1) - W(k, N_i(k))] =$$
$$= \sum_{k=t}^{T_f-1} \mathrm{tr}[p_i(k+1)N_i(k+1)L_i(k+1) - p_i(k)N_i(k)L_i(k) - p_i(k)\bar{\Psi}_i(k)L_i(k)]. \quad (14)$$

On the other hand, this expression can be represented as

$$\sum_{k=t}^{T_f} \Delta W(k, N_i(k)) = W(t+1, N_i(t+1)) - W(t, N_i(t)) + ...+$$
$$+ W(T_f, N_i(T_f)) - W(T_f - 1, N_i(T_f - 1) = \quad (15)$$
$$= \mathrm{tr}\, p_i(T_f)N_i(T_f)L_i(T_f) - \mathrm{tr}\, p_i(t)N_i(t)L_i(t) - \mathrm{tr} \sum_{\xi=t}^{T_f-1} p_i(\xi)\bar{\Psi}_i(\xi)L_i(\xi).$$

Add into (12) the difference of the right parts of (14) and (15). Considering that this difference is zero, criterion (11) takes the form:

$$J[0; T_f] = \sum_{i=1}^{n}\{\sum_{\xi=k}^{T_f-1} \mathrm{tr} p_i(\xi)N_i(\xi)R_i(\xi) - \sum_{\xi=k+1}^{T_f-1} \mathrm{tr} p_i(\xi)N_i(\xi)L_i(\xi)+$$
$$+ \sum_{\xi=k}^{T_f-1} \mathrm{tr}\, p_i(\xi+1)[(A_i - K(\xi)S_i)(\sum_{j=1}^{n} p_{i,j}N_j(\xi))(A_i - K(\xi)S_i)^T + \quad (16)$$
$$+ Q_i + K(\xi)V_iK(\xi)^T]L_i(\xi+1)\}.$$

Now, calculate the derivatives:

$$\frac{dJ}{dK(k)} = \sum_{\xi=k}^{T_f-1} \sum_{i=1}^{n} \mathrm{tr}[-L_i(\xi+1)p_i(\xi+1)A_i(\sum_{j=1}^{n} p_{i,j}N_j(\xi))S_i^T -$$
$$-p_i(\xi+1)L_i(\xi+1)A_i(\sum_{j=1}^{n} p_{i,j}N_j(\xi))S_i^T + p_i(\xi+1)L_i(\xi+1)K(\xi)S_i \times \quad (17)$$
$$\times(\sum_{j=1}^{n} p_{i,j}N_j(\xi))S_i^T + L_i(\xi+1)p_i(\xi+1)K(\xi)S_i(\sum_{j=1}^{n} p_{i,j}N_j(\xi))S_i^T +$$
$$+p_i(\xi+1)L_i(\xi+1)K(\xi)V_i + L_i(\xi+1)p_i(\xi+1)K(\xi)V_i].$$

Equating this derivative to zero, assuming that each summand of summation with respect to i is zero, and using the Kronecker product operation [7], we obtain formula (10) determining the matrix $K(k)$.

Calculate the finite difference of the Lyapunov function:

$$\begin{aligned}
&\Delta W(k, N_i(k) = W(k+1, N_i(k+1)) - W(k, N_i(k)) = \\
&=\mathrm{tr} p_i(k+1)N_i(k+1)R_i(k+1)+ \\
&+\mathrm{tr}\sum_{t=k+1}^{T_f} p_i(t)[Q_i + K(t)V_iK(k)^T + \Psi_i(t)]L_i(t)- \\
&-\mathrm{tr} p_i(k)N_i(k)R_i(k) - \mathrm{tr}\sum_{t=k}^{T_f} p_i(t)[Q_i + K(t)V_iK(k)^T + \Psi_i(t)]L_i(t) = \\
&= \mathrm{tr} p_i(k+1)N_i(k+1)R_i(k+1) - \mathrm{tr} p_i(k)N_i(k)R_i(k)- \\
&-p_i(k)[Q_i + K(k)V_iK(k)^T + \Psi_i(k)]L_i(k).
\end{aligned} \tag{18}$$

Since the matrices N_i, L_i are positively determined by Theorem conditions, and the matrix $\Psi_i(t) > 0$ is given arbitrarily, it is obvious that these matrices can be chosen such that the final difference (18) becomes negative. This condition guarantees the Lyapunov stability of the extrapolator dynamic. Theorem is proved.

To construct the prediction estimate, we use the Kalman extrapolator

$$\widehat{x}(k+1) = A_\gamma \widehat{x}(k) + \widehat{r}(k) + K(k)(y(k) - S_\gamma \widehat{x}(k) - \widehat{\varphi}(k)),\ \widehat{x}(0) = \bar{x}_0, \tag{19}$$

where $K(k)$ is the transfer matrix depending on k and independing of jump parameter $\gamma(k)$.

3 Synthesis of the Stationary Extrapolator

In this case, the matrix of transfer coefficients $K(k)$ in Kalman extrapolator (19) will be constant, and the criterion will take the form:

$$J[0;\infty] = \lim_{T_f\to\infty} \frac{1}{T_f} \sum_{k=1}^{T_f} \sum_{i=1}^{n} \mathrm{tr}\bar{p}_i N_i(k) R_i(k). \tag{20}$$

The two-point boundary value problem is transformed into the following matrix equations:

$$N_i = (A_i - KS_i)(\sum_{j=1}^{n} p_{i,j}N_j)(A_i - KS_i)^T + Q_i + KV_iK^T, \tag{21}$$

$$L_i = (A_i - KS_i)^T(\sum_{j=1}^{n} p_{i,j}L_j)(A_i - KS_i)^T + R_i, \tag{22}$$

$$\mathrm{ct}(K) = (\sum_{i=1}^{n} \bar{p}_i[L_i \otimes S_i(\sum_{j=1}^{n} p_{i,j}N_j)S_i^T + L_i \otimes V_i])^{-1}\mathrm{ct}(\sum_{i=1}^{n} \bar{p}_i L_i A_i(\sum_{j=1}^{n} p_{i,j}N_jS_i^T)), \tag{23}$$

where $\bar{p}_i$ are steady-state probabilities (solution of system (3) as $k \to \infty$).

Thus, to synthesize a stationary extrapolator, it is necessary to solve the system of matrix equations (21)–(23).

Note that if there are positively defined solutions N_i, L_i $(i=\overline{1,n})$ of matrix equations (21)–(23), then using the equation (22) and condition $R_i > 0$ (see Theorem 1.6 [8]), it follows that a stationary extrapolator with jump parameters will be stochastic stable.

4 Estimates of Unknown Vectors

As estimates of unknown vectors, various algorithms can be used [3–5]. When using the LSM estimates, finding $\widehat{\varphi}(k)$ and $\widehat{r}(k)$ is based on minimizing of the following criteria:

$$J_1 = \sum_{i=1}^{k}(\left\|y(t) - S_t\widehat{x}(t)\right\|^2_{W_1} + \left\|\varphi(t-1)\right\|^2_{\bar{W}_1}), \tag{24}$$

$$J_2 = \sum_{i=1}^{k}(\left\|y(t) - \widehat{\varphi}(t) - S_\gamma A_\gamma \widehat{x}(t-1)\right\|^2_{W_2} + \left\|r(t-1)\right\|^2_{\bar{W}_2}), \tag{25}$$

where W_1, $\bar{W}_1$, W_2, $\bar{W}_2$ are weight matrices. At each step, the estimates of $\widehat{\varphi}(k)$ and $\widehat{r}(k)$ are constructed sequentially, first minimizing the criterion (24), then (25). In constructing vector estimates $\widehat{r}(k)$, based on the criterion (25), are used $\widehat{\varphi}(k)$ vector estimates obtained via the minimization of the criterion (24). Then, estimates of unknown vectors by the LSM estimates are determined as follows:

$$\widehat{\varphi}^{\langle LSM\rangle}(k) = [S_\gamma^T W_1 S_\gamma + \bar{W}_1]^{-1} S_\gamma^T W_1 \{y(k) - S_\gamma \widehat{x}(k)\}, \tag{26}$$

$$\widehat{r}^{\langle LSM\rangle}(k) = [S_\gamma^T W_2 S_\gamma + \bar{W}_2]^{-1} S_\gamma^T W_2 \{y(k) - \widehat{\varphi}(k) - S_\gamma A_\gamma \widehat{x}(k-1)\}. \tag{27}$$

By analogy with [6], using estimates (26) and (27), we construct prediction estimates by making use of the technique of nonparametric smoothing:

$$\widehat{\varphi}^{\langle NP\rangle}(k) = [S_\gamma^T W_1 S_\gamma + \bar{W}_1]^{-1} S_\gamma^T W_1 \Omega(k), \tag{28}$$

$$\widehat{r}^{\langle NP\rangle}(k) = [S_\gamma^T W_2 S_\gamma + \bar{W}_2]^{-1} S_\gamma^T W_2 \bar{\Omega}(k). \tag{29}$$

In (28) and (29) the components of the vectors Ω and $\bar{\Omega}$ are determined by the formulas:

$$\Omega_j(k) = \frac{\sum_{i=1}^{k} \frac{[y(i) - S_\gamma(\widehat{x}(i))]_j}{\mu_j} G(\frac{k-i+1}{\mu_j})}{\sum_{i=1}^{k} \frac{1}{\mu_j} G(\frac{k-i+1}{\mu_j})}\ (j=\overline{1,n}), \tag{30}$$

$$\bar{\Omega}_s(k) = \frac{\sum_{i=1}^{k} \frac{[y(i) - \widehat{\varphi}(i) - S_\gamma A_\gamma(\widehat{x}(i-1))]_s}{\bar{\mu}_s} G(\frac{k-i+1}{\bar{\mu}_s})}{\sum_{i=1}^{k} \frac{1}{\bar{\mu}_s} G(\frac{k-i+1}{\bar{\mu}_s})}\ (s=\overline{1,n}), \tag{31}$$

where in relations (30), (31) $G(\cdot)$ is a kernel function, μ_j and $\bar{\mu}_s$ are bandwidth parameters.

5 Simulation Results

The simulation was performed for the following data:

$$A_1 = \begin{pmatrix} 0.85 & 0.1 \\ -0.05 & 0.94 \end{pmatrix},\ A_2 = \begin{pmatrix} 0.89 & 0.05 \\ -0.02 & 0.45 \end{pmatrix},\ B_1 = B_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$Q_1 = Q_2 = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.03 \end{pmatrix},\ \Delta A_1 = \begin{pmatrix} 0.05 & 0.01 \\ 0.02 & -0.11 \end{pmatrix},\ \Delta A_2 = \begin{pmatrix} 0.06 & 0.012 \\ -0.01 & 0.05 \end{pmatrix},$$

$$S_1 = S_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \Delta S_1 = \begin{pmatrix} 0.01 & 0 \\ 0 & 0 \end{pmatrix},\ \Delta S_2 = \begin{pmatrix} 0.008 & 0 \\ 0 & 0 \end{pmatrix},$$

$$V_1 = V_2 = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.15 \end{pmatrix},\ H_1 = H_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ P = \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix},$$

$$R_1 = R_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.15 \end{pmatrix},\ W_1 = W_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \bar{W}_1 = \bar{W}_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix},$$

$$f(k) = \begin{pmatrix} 0.1 + 0.1\sin(0.1k) \\ 0.1 + 0.12\sin(0.15k) \end{pmatrix},\ \phi(k) = \begin{pmatrix} 0.3 \\ 0.25 \end{pmatrix}.$$

We use the Gaussian kernels

$$G(u) = \frac{exp(\frac{-u^2}{2})}{\sqrt{2\pi}}.$$

Fig.1 shows the graphs of the values of the jump parameter γ.

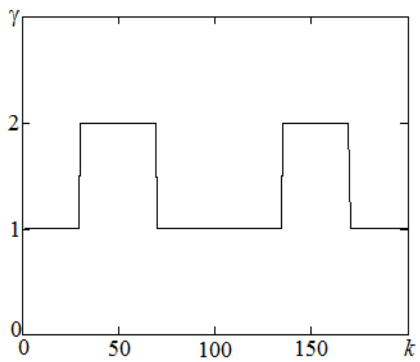

Figure 1: The Values of the Jump Parameter γ

The transfer matrix K of the extrapolator is determined from the solution of matrix equations (21)–(23).

The root-mean-square errors were calculated as follows:

$$\sigma_i(N) = \sqrt{\frac{\sum_{k=1}^{N}(x_i(k) - \widehat{x}_i(k))^2}{N-1}}\ (i = \overline{1,2}).$$

Table 1: Root-Mean-Square Errors for Estimating Accuracy of State ($\sigma_i(200)$)

Coordinate number (i)	LSM	Nonparametric smoothing
1	0.537	0.407
2	0.539	0.479

The corresponding values of the errors of extrapolation of the state vector (see Table 1) were obtained using estimates of unknown vectors of the form (26)–(31).

As can be seen from Table 1, the estimation algorithm with nonparametric smoothing of unknown additive vectors in the object model and in the model of the observation channel allows us to increase the extrapolation accuracy for discrete models with jump parameters.

Conclusions

The solution of the problem of synthesizing stationary and non-stationary robust extrapolators for a linear discrete models with random Markov jump parameters under incomplete information was obtained. The simulation results showed that the application of the robust extrapolation algorithm using the non-parametric smoothing procedure allows one to increase the prediction accuracy.

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