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NONPARAMETRIC ESTIMATION OF ACTUARIAL PRESENT VALUE OF DEFERRED LIFE ANNUITY

The paper deals with the estimation problem of the actuarial present value of the deferred life annuity. The nonparametric estimator of the deferred life annuity was constructed. We found the principal term of the asymptotic mean squared error (MSE) of the proposed estimator and proved its asymptotic normality. The simulations show that the empirical MSE of the annuity estimator decreases when the sample size increases.
Keywords: nonparametric estimation; deferred life annuity; mean squared error; asymptotic normality.

Let x be the age of an individual and at the moment $t = 0$ payments start. The idea of the r-year deferred life annuity in accordance with [1. P. 174] is this: from the moment $t + r = r$, an individual starts receiving money once a year, which we take as a monetary unit, and payments are made only during the lifetime of an individual. It is known that the deferred life annuity is associated with the appropriate type of insurance. Thus, the average total cost of the present continuous r-year deferred life annuity is given by the following formula (see [1. P. 184]):

$$ {}_{r|}\bar{a}_x(\delta) = \frac{1 - {}_{r|}\bar{A}_x}{\delta}, $$

where ${}_{r|}\bar{A}_x = \int\limits_r^\infty e^{-\delta t} f_x(t)dt$ is the net premium (the expectation of the present value of an insured unit sum for the deferred life insurance at age x), δ is a force of interest, $f_x(t) = \frac{f(x+t)}{S(x)}$ is a probability density of future lifetime of an individual $T_x = X - x$ [1. P. 62], $f(x)$ is a probability density of lifetime of an individual X, $S(x) = P(X > x)$ is a survival function. Introduce the random variable

$$ z(x) = \frac{1 - e^{-\delta T_x}}{\delta}, T_x > r. \tag{1} $$

Then, by averaging $z(x)$ (1), we get the formula of the deferred life annuity (see [2–4]):

$$ {}_{r|}\bar{a}_x(\delta) = E(z) = \frac{1}{\delta}\left(1 - \frac{\Phi(x,\delta,r)}{S(x)}\right), \tag{2} $$

where E is the symbol of the mathematical expectation,

$$ \Phi(x,\delta,r) = e^{\delta x} \int\limits_{x+r}^{\infty} e^{-\delta t} dF(t), $$

$F(x) = P\left(X \le x\right) = 1 - S(x)$ is a distribution function.

Note that the whole life annuity $\bar{a}_x(\delta)$ [2] is the special case of the deferred life annuity (2) at $r = 0$.

1. Construction of the Deferred Annuity Estimator

Assume that we have a random sample $X_1, ..., X_N$ of N individuals' lifetimes. Using the empirical survival function

$$S_N(x) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{I}(X_i > x),$$

where I(A) is the indicator of an event A, obtain the following estimator of (2):

$$_{r|}\overline{a}_x^N(\delta) = \frac{1}{\delta}\left(1 - \frac{e^{\delta x}}{S_N(x)\cdot N}\sum_{i=1}^{N}\exp(-\delta X_i)\mathrm{I}(X_i > x+r)\right) = \frac{1}{\delta}\left(1 - \frac{\Phi_N(x,\delta,r)}{S_N(x)}\right), \quad (3)$$

$$\Phi_N(x,\delta,r) = \frac{e^{\delta x}}{N}\sum_{i=1}^{N}\exp(-\delta X_i)\mathrm{I}(X_i > x+r).$$

2. Bias and Mean Squared Error of the Estimator $_{r|}\overline{a}_x^N(\delta)$

Here we will obtain the principal term of the asymptotic MSE and the bias convergence rate of the estimator (3). Introduce the notation according to [5]: $t_N = (t_{1N}, t_{2N}, ..., t_{sN})^{\mathrm{T}}$ is an s-dimensional vector with the components $t_{jN} = t_{jN}(x) = t_{jN}(x; X_1, ..., X_N)$, $j = \overline{1,s}$, $x \in R^{\alpha}$, R^{α} is the α-dimensional Euclidean space; $H(t): R^s \to R^1$ is a function, where $t = t(x) = (t_1(x), ..., t_s(x))^{\mathrm{T}}$ is an s-dimensional bounded vector function; $\mathrm{N}_s(\mu, \sigma)$ is the s-dimensional normally distributed random variable with a mean vector and covariance matrix $\sigma = \sigma(x)$; $\nabla H(t) = (H_1(t), ..., H_s(t))^{\mathrm{T}}$, $H_j(t) = \left.\frac{\partial H(z)}{\partial z_j}\right|_{z=t}$, $j = \overline{1,s}$; $\Rightarrow$ is the symbol of convergence in distribution; $\|x\|$ is the Euclidean norm of a vector x; $\mathfrak{R}$ is the set of natural numbers.

Definition 1. The function $H(t): R^s \to R^1$ and the sequence $\{H(t_N)\}$ are said to belong to the class $\mathrm{N}_{\nu,s}(t;\gamma)$, provided that:

1) there exists an ε-neighborhood

$$\sigma = \left\{z : |z_i - t_i| < \varepsilon, i = \overline{1,s}\right\},$$

in which the function $H(z)$ and all its partial derivatives up to order ν are continuous and bounded;

2) for any values of variables $X_1, ..., X_N$ the sequence $\{H(t_N)\}$ is dominated by a numerical sequence $C_0 d_N^{\gamma}$, such that $d_N \uparrow \infty$, as $N \to \infty$, and $0 \le \gamma < \infty$.

Theorem 1 [5]. Let the conditions

1) $H(z)$, $\{H(t_N)\} \in \mathrm{N}_{2,s}(t;\gamma)$,

2) $E\|t_N - t\|^i = O\left(d_N^{-i/2}\right)$

hold for all $i \in \mathfrak{R}$. Then, for every $k \in \mathfrak{R}$,

$$\left|E\left[H(t_N) - H(t)\right]^k - E\left[\nabla H(t)\cdot(t_N - t)\right]^k\right| = O\left(d_N^{-(k+1)/2}\right). \quad (4)$$

If in formula (4) k = 1, we obtain the principal term of the bias for $H(t_n)$, and at k = 2, we have the principal term of the MSE.

Theorem 2. If $S(x) > 0$ and $S(t)$ is continuous at a point x, then

1) for the bias of (3), the following relation holds:

$$\left|b(_{r|}\overline{a}_x^N(\delta))\right| = \left|E(_{r|}\overline{a}_x^N(\delta) - {}_{r|}\overline{a}_x(\delta))\right| = O\left(N^{-1}\right);$$

2) the MSE of (3) is given by the formula

$$u^2(_{r|}\overline{a}_x^N(\delta)) = E(_{r|}\overline{a}_x^N(\delta) - {}_{r|}\overline{a}_x(\delta))^2 = \frac{\Phi(x,2\delta,r) - \Phi^2(x,\delta,r)/S(x)}{N\delta^2 S^2(x)} + O\left(N^{-3/2}\right).$$

Proof. For the estimator ${}_{r|}\bar{a}_x^N(\delta)$ (3) in the notation of Theorem 1, we have: $s=2$;

$$t_N=(t_{1N},t_{2N})^{\mathrm{T}}=(\Phi_N(x,\delta,r),S_N(x))^{\mathrm{T}};\quad d_N=N;\quad t=(t_1,t_2)^{\mathrm{T}}=(\Phi(x,\delta,r),S(x))^{\mathrm{T}};$$

$$H(t)=\frac{1}{\delta}\left(1-\frac{t_1}{t_2}\right)=\frac{1}{\delta}\left(1-\frac{\Phi(x,\delta,r)}{S(x)}\right)={}_{r|}\bar{a}_{\mathbf{x}}(\delta);\quad H(t_N)=\frac{1}{\delta}\left(1-\frac{\Phi_N(x,\delta,r)}{S_N(x)}\right)={}_{r|}\bar{a}_{\mathbf{x}}{}^N(\delta);$$

$$\nabla H(t)=\left(H_1(t),H_2(t)\right)^{\mathrm{T}}=\left(-\frac{1}{\delta S(x)},\frac{\Phi(x,\delta,r)}{\delta S^2(x)}\right)^{\mathrm{T}}\neq 0.$$

The sequence $\{H(t_N)\}$ satisfies the condition 1) of Theorem 1 with $C_0=\frac{1}{\delta}(1+e^{-\delta r})$, $\gamma=0.$ Indeed,

$$|H(t_N)|=\frac{1}{\delta}\left|1-\frac{\Phi_N(x,\delta,r)}{S_N(x)}\right|\le\frac{1}{\delta}\left(1+\frac{\Phi_N(x,\delta,r)}{S_N(x)}\right)\le\frac{1}{\delta}\left(1+\frac{e^{\delta x}\sum\limits_{i=1}^{N}\exp(-\delta X_i)\mathrm{I}(X_i>x+r)}{\sum\limits_{i=1}^{N}\mathrm{I}(X_i>x)}\right)\le$$

$$\le\frac{1}{\delta}\left(1+\frac{e^{\delta x}e^{-\delta(x+r)}\sum\limits_{i=1}^{N}\mathrm{I}(X_i>x+r)}{\sum\limits_{i=1}^{N}\mathrm{I}(X_i>x)}\right)\le\frac{1}{\delta}(1+e^{-\delta r}).$$

Further, the function $H(t)$ satisfies the condition 1) in view of $t_2=S(x)>0.$ Also, this function satisfies the condition 2) due to Lemma 3.1 [6], as for all $i\in\Re$ such inequalities hold: $E\{\mathrm{I}^i(X>x)\}=S(x)\le 1,$ $E\{e^{i\delta x}e^{-i\delta X}\mathrm{I}^i(X>x+r)\}\le e^{i\delta x}e^{-i\delta x}S(x+r)=S(x+r)\le 1$.

It is well known that $S_N(x)$ is the unbiased and consistent estimator of $S(x)$. Show that $\Phi_N(x,\delta,r)$ is the unbiased estimator of $\Phi(x,\delta,r)$ and calculate the variance of $\Phi_N(x,\delta,r)$:

$$E\Phi_N(x,\delta,r)=\frac{e^{\delta x}}{N}E\left\{\sum_{i=1}^{N}\exp(-\delta X_i)\mathrm{I}(X_i>x+r)\right\}=\Phi(x,\delta,r),$$

$$D\Phi_N(x,\delta,r)=\frac{e^{2\delta x}}{N^2}\sum_{i=1}^{N}D\left\{\mathrm{I}(X_i>x+r)e^{-\delta X_i}\right\}=\frac{1}{N}\left(\Phi(x,2\delta,r)-\Phi^2(x,\delta,r)\right).$$

Considering that $E(t_N-t)=0$ and all the conditions of Theorem 1 are fulfilled, in accordance with (4) we get the order of the bias of ${}_{r|}\bar{a}_x^N(\delta)$:

$$\left|E({}_{r|}\bar{a}_x^N(\delta)-{}_{r|}\bar{a}_x(\delta))-E\left[\nabla H(t)(t_N-t)\right]\right|=\left|E({}_{r|}\bar{a}_x^N(\delta)-{}_{r|}\bar{a}_x(\delta))\right|=O\left(N^{-1}\right).$$

Find the components of the covariance matrix $\sigma({}_{r|}\bar{a}_x(\delta))=\begin{bmatrix}\sigma_{11}\sigma_{12}\\ \sigma_{21}\sigma_{22}\end{bmatrix}$ for the statistics $\Phi_N(x,\delta,r)$ and $S_N(x)$:

$$\sigma_{11}=ND\{\Phi_N(x,\delta,r)\}=\Phi(x,2\delta,r)-\Phi^2(x,\delta,r);\quad \sigma_{22}=ND\{S_N(x)\}=S(X)(1-S(x));\quad \sigma_{12}=\sigma_{21}=$$

$$=N\operatorname{cov}(S_N(x),\Phi_N(x,\delta,r))=N\left(E\{S_N(x)\Phi_N(x,\delta,r)\}-E\{S_N(x)\}E\{\Phi_N(x,\delta,r)\}\right)=(1-S(x))\Phi(x,\delta,r).$$

Using the previous results on the bias and covariance matrix, we obtain

$$u^2({}_{r|}\bar{a}_x^N(\delta))=E\left[\nabla H(t)(t_N-t)\right]^2+O\left(N^{-3/2}\right)=H_1^2(t)\sigma_{11}+H_2^2(t)\sigma_{22}+2H_1(t)H_2(t)\sigma_{12}+O\left(N^{-3/2}\right)=$$

$$=\frac{\Phi(x,2\delta,r)-\Phi^2(x,\delta,r)/S(x)}{N\delta^2S^2(x)}+O\left(N^{-3/2}\right).\qquad(5)$$

The proof of Theorem 2 is completed.

3. Asymptotic Normality of the Estimator ${}_{r|}\bar{a}_x^N(\delta)$

We need the following two Theorems.

Theorem 3 [7, Appendix 5]. If $\xi_1, \xi_2, ..., \xi_N, ...$ is a sequence of independent and identically distributed s-dimensional vectors, $E\{\xi_k\} = 0$, $\sigma(x) = E\{\xi_k^{\mathrm{T}}\xi_k\}$, $t_N = \frac{1}{N}\sum_{k=1}^{N}\xi_k$, then, as $N \to \infty$, $\sqrt{N}t_N \Rightarrow \mathrm{N}_s(0, \sigma(x))$.

Theorem 4 [8]. Let $\sqrt{N} \cdot t_N \Rightarrow \mathrm{N}_s\{\mu, \sigma(x)\}$, $H(z)$ be differentiable at the point μ, $\nabla H(\mu) \neq 0$. Then

$$\sqrt{N}(H(t_N) - H(\mu)) \Rightarrow \mathrm{N}_1\left(\sum_{j=1}^{s} H_j(\mu)\mu_j,\ \sum_{p=1}^{s}\sum_{j=1}^{s} H_j(\mu)\sigma_{jp}H_p(\mu)\right).$$

Theorem 5. Under the conditions of Theorem 2

$$\sqrt{N}({}_{r|}\bar{a}_x^N(\delta) - {}_{r|}\bar{a}_x(\delta)) \Rightarrow \mathrm{N}_1\left(0, \frac{\Phi(x,2\delta,r) - \Phi^2(x,\delta,r)/S(x)}{\delta^2 S^2(x)}\right).$$

Proof. In the notation of Theorem 3, we have $s = 2$, $\sigma(x) = \sigma({}_{r|}\bar{a}_x(\delta))$. Thus,

$$\sqrt{N}\{(\Phi_N(x,\delta,r), S_N(x)) - (\Phi(x,\delta,r), S(x))\} \Rightarrow \mathrm{N}_2\left((0,0), \sigma\left({}_{r|}\bar{a}_x(\delta)\right)\right).$$

The function $H(z)$ is differentiable at the point $t = (\Phi(x,\delta), S(x))$ and $\nabla H(t) \neq 0$. Consequently, all the conditions of Theorem 4 hold, and using (5), we obtain the desired result.

The proof of Theorem 5 is completed.

4. Simulations

Introduce the denotation $I_x(a,b] = \begin{cases} 1, & if \quad x \in (a,b] \\ 0, & if \quad x \notin (a,b] \end{cases}$. Consider de Moivre's model, for which the individual's lifetime X is uniformly distributed in the interval (0, ω), where ω is a limiting age. For this model the probability density and survival function are defined by the following formulas:

$$f(x) = \frac{I_x(0,\omega)}{\omega}, \qquad S(x) = I_x(-\infty,\omega) - \frac{xI_x(0,\omega)}{\omega}. \tag{6}$$

Now, using (6), we obtain

$$f_x(t) = \frac{f(x+t)}{S(x)} = \frac{I_t(0,\omega - x)}{\omega - x},$$

$${}_{r|}\bar{a}_x(\delta) = \frac{1}{\delta}\left(1 - \int_r^{\infty} e^{-\delta t} f_x(t)dt\right) = \frac{1}{\delta}\left(1 - \frac{1}{\omega - x}\int_r^{\omega - x} e^{-\delta t}dt\right) = \frac{1}{\delta}\left(1 - \frac{e^{-\delta r} - e^{-\delta(\omega - x)}}{\delta(\omega - x)}\right). \tag{7}$$

The present value of the 5-years deferred annuity for a person at the age x = 45 years when $\omega = 100$ years, δ = 0,09531 (9,531%), and monthly payments in the size of 1000 rubles, is equal to

$$12000 \cdot {}_{5|}\bar{a}_{45}(0,09531) = 12000 \cdot 9{,}581854 = 114982 \text{ rubles.}$$

Note that for such δ the effective annual interest rate $i = e^{\delta} - 1 = 0{,}1$ (10%).

The simulations were carried out for de Moivre's model under the above presented conditions. The annuities and their estimators are presented in Fig. 1 for random samples $X_1, ..., X_N$ of the sizes N = 50, 100, 500, uniformly distributed in the interval (0, 100).

We will characterize the quality of estimators presented in Fig. 1 using the empirical MSE

$$G(N,r,\delta) = \frac{\sum_{x=0}^{95}({}_{r|}\bar{a}_x(\delta) - {}_{r|}\bar{a}_x^N(\delta))^2}{96}.$$

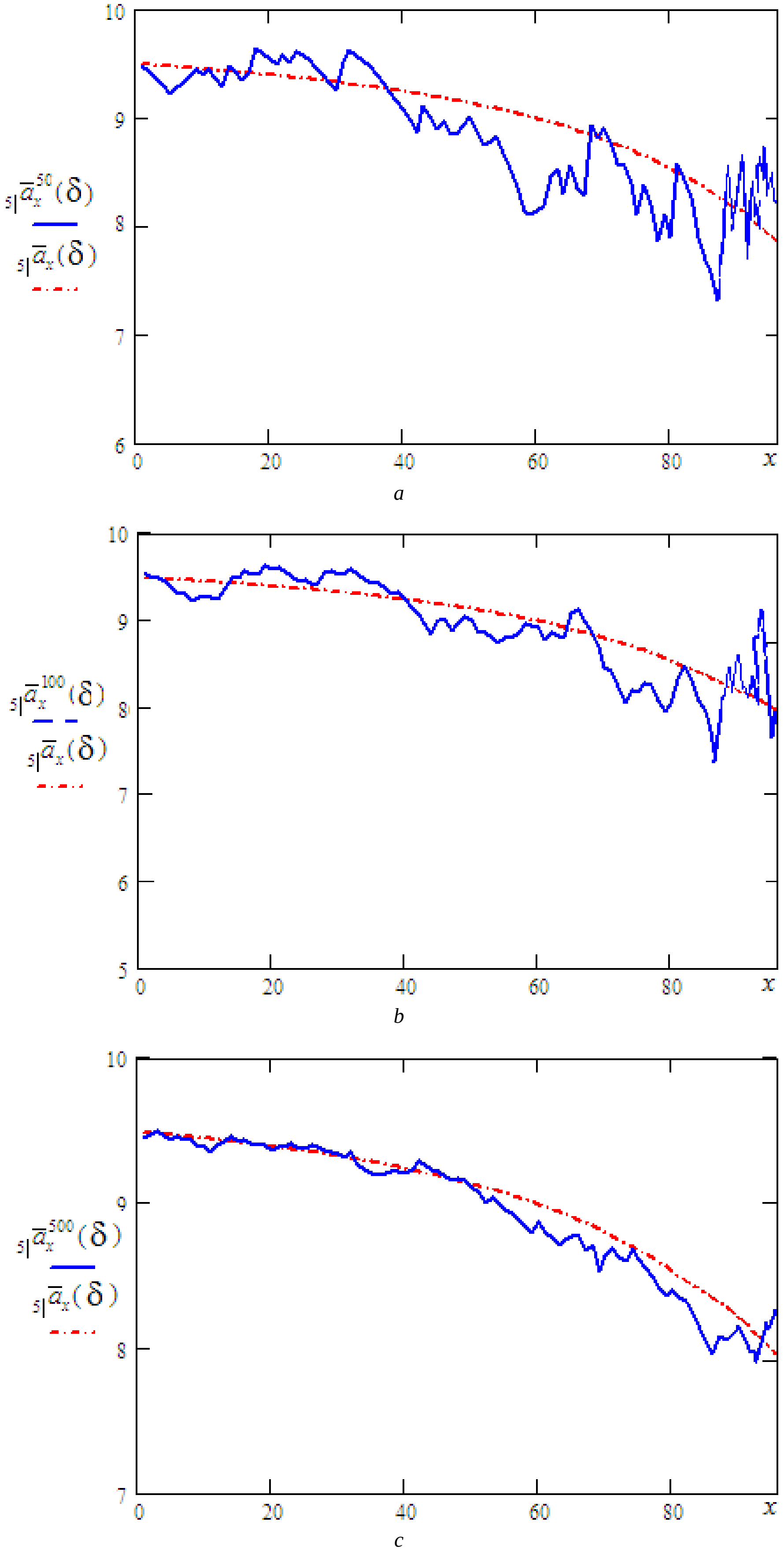

Fig. 1. Dependence on the age *x* of the 5-years deferred annuity ($\delta = 0{,}09531$) and its estimators for the sample sizes *N*: *a* – 50; *b* – 100, *c* – 500

The calculation results are given in Table I.

Table 1

Simulation results for the different sample sizes

N	25	50	100	250	500
$G(N; 5; 0{,}09531)$	1,632	0,815	0,413	0,117	0,052

So, according to Table 1, the quality of the deferred annuity estimators by the criterion $G(N; 5; 0{,}09531)$ is improving when the sample size N is increasing.

Conclusion

In the paper, we found the principal term of the asymptotic MSE of the estimator ${}_{r|}\bar{a}_x^N(\delta)$. Also, the following asymptotic properties of the estimator are proved: unbiasedness, consistency, and normality. Statistical modeling within the framework of de Moivre's model shows that the quality of estimation according to the empirical criterion $G(N, r, \delta)$ improves with the growth of the sample size. Note that the improved estimators of life annuities (3) can be obtained by substituting of empirical survival functions by the smooth empirical survival functions (cf. [9–24]) and using auxiliary information of the different type [25–33], for example, connected with random variables X, $T_x = X - x$, $T_x - r$.

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Губина О.В., Кошкин Г.М. НЕПАРАМЕТРИЧЕСКОЕ ОЦЕНИВАНИЕ АКТУАРНОЙ СОВРЕМЕННОЙ СТОИМОСТИ ОТСРОЧЕННОЙ РЕНТЫ. *Вестник Томского государственного университета. Управление, вычислительная техника и информатика*. 2019. № 46. С. 49–55

Рассматривается проблема оценивания актуарной современной стоимости отсроченной ренты. Синтезируется непараметрическая оценка отсроченной ренты. Находится главная часть асимптотической среднеквадратической ошибки оценки и ее предельное распределение. Моделирование показывает, что эмпирическая среднеквадратическая ошибка оценки ренты уменьшается с ростом объема выборки.

Ключевые слова: непараметрическое оценивание; отсроченная пожизненная рента; среднеквадратическая ошибка; асимптотическая нормальность.

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