

A transient free convection study with temperature-dependent viscosity in a square cavity with a local heat source

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Abstract. Unsteady natural convection inside of a differentially-heated square enclosure filled with a fluid of temperature-dependent viscosity has been numerically studied. A mathematical model formulated in the dimensionless stream function and vorticity has been solved by a finite difference method of the second order accuracy. The effect of dimensionless time and Prandtl number on streamlines and isotherms has been investigated for $Ra = 10^5$. The results clearly demonstrate an evolution of fluid flow and heat transfer in the case of variable viscosity fluid.

1. Introduction

Natural convection in enclosures takes place in many areas of modern life such as cooling of electronic devices, melting of new materials, solar collectors, new heat exchangers and so on [1]. Variable physical properties in these problems (variable viscosity, conductivity) can essentially affect the fluid flow and heat transfer [2-5]. Thus, Astanina et al. [2] have numerically studied natural convection with temperature-dependent viscosity in a porous cavity. They have demonstrated that an increase in the viscosity variation parameter leads to an intensification of the convective flow and heat transfer and a formation of a single-core convective cell while in the case of the clear fluid an increase in the viscosity variation parameter leads to a decrease in the heat transfer rate. Cordoba et al. [3] have analyzed numerically and experimentally laminar natural convection with temperature-dependent viscosity in a differentially-heated cubical cavity. It has been found that numerical and experimental flow patterns illustrate a stable convective flow without any oscillations that is not expected for high Rayleigh numbers. Also the used variable viscosity leads to an essential asymmetry in the flow and heat transfer. Umavathi and Ojjela [4] have investigated the effect of variable viscosity on convective heat transfer in a vertical rectangular duct. They have found that isotherms remain linear for all values of viscosity variation parameter while velocity contours are changed. Kumar and Mahulikar [5] have studied the effects of temperature-dependent viscosity on the flow in a microchannel. They have revealed four different flow regions.

The objective of the present paper is a numerical analysis of the fluid flow and heat transfer evolution in the case of temperature-dependent viscosity and the effect of Prandtl number on heat transfer enhancement.

2. Mathematical model and numerical method

The domain of interest is schematically shown in Figure 1. A square cavity filled with a clear fluid of temperature-dependent viscosity has isothermal vertical walls with temperature T_c , adiabatic horizontal walls and a local heat source of constant temperature T_h ($T_h > T_c$). The heat source is located



at the centre of the bottom wall. For the description of this problem the following assumptions have been introduced in the model: the Boussinesq approximation is valid, the fluid is Newtonian and the flow is laminar.

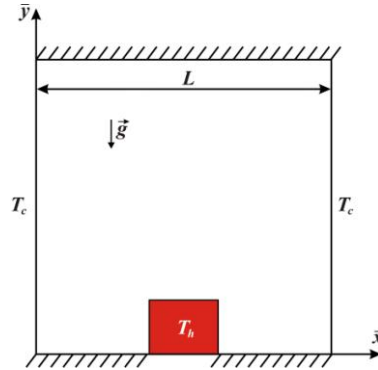


Figure 1. The domain of interest.

A mathematical model has been formulated in dimensionless variables ‘stream function – vorticity – temperature’ as follows:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega, \quad (1)$$

$$\begin{aligned} \frac{\partial \omega}{\partial \tau} + \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} = \sqrt{\frac{\text{Pr}}{\text{Ra}}} \left(\frac{\partial^2 \xi \omega}{\partial x^2} + \frac{\partial^2 \xi \omega}{\partial y^2} \right) + \frac{\partial \theta}{\partial x} + \\ + 2\sqrt{\frac{\text{Pr}}{\text{Ra}}} \left[\frac{\partial^2 \xi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} - 2 \frac{\partial^2 \xi}{\partial x \partial y} \frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \xi}{\partial y^2} \frac{\partial^2 \psi}{\partial x^2} \right], \end{aligned} \quad (2)$$

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{1}{\sqrt{\text{Ra} \cdot \text{Pr}}} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right). \quad (3)$$

Here x, y are the dimensionless Cartesian coordinates; ψ is the dimensionless stream function $u = \partial \psi / \partial y, v = -\partial \psi / \partial x$; ω is the dimensionless vorticity $\omega = \partial v / \partial x - \partial u / \partial y$; u and v are the dimensionless velocity components; τ is the dimensionless time; Ra is Rayleigh number; Pr is Prandtl number; ξ is the dimensionless variable viscosity $\xi = \exp -C\theta$; θ is the dimensionless temperature.

The initial and boundary conditions for the formulated governing equations (1) – (3) are as follows:

$$\tau = 0: \quad \psi = \omega = \theta = 0;$$

$$\tau > 0: \quad \psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial x^2}, \quad \theta = -0.5 \quad \text{on } x = 0 \text{ and } x = 1;$$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial y^2}, \quad \frac{\partial \theta}{\partial y} = 0 \quad \text{on } y = 0 \text{ and } y = 1;$$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial \mathbf{n}^2}, \quad \theta = 0.5 \quad \text{on heat source surface.}$$

The governing equations (1) – (3) with corresponding initial and boundary conditions have been solved using finite difference method of the second order accuracy. Detailed description of the

developed numerical method has been done earlier in [2, 6]. The developed computational code has been verified for several benchmark problems [2, 6].

3. Results and discussion

The numerical analysis has been conducted for the following values of the governing parameters: $Ra = 10^5$, $Pr = 7-700$; $0 \leq \tau \leq 1000$; $C = 2$. Figure 2 shows the evolution of streamlines and isotherms at $Pr = 7$.

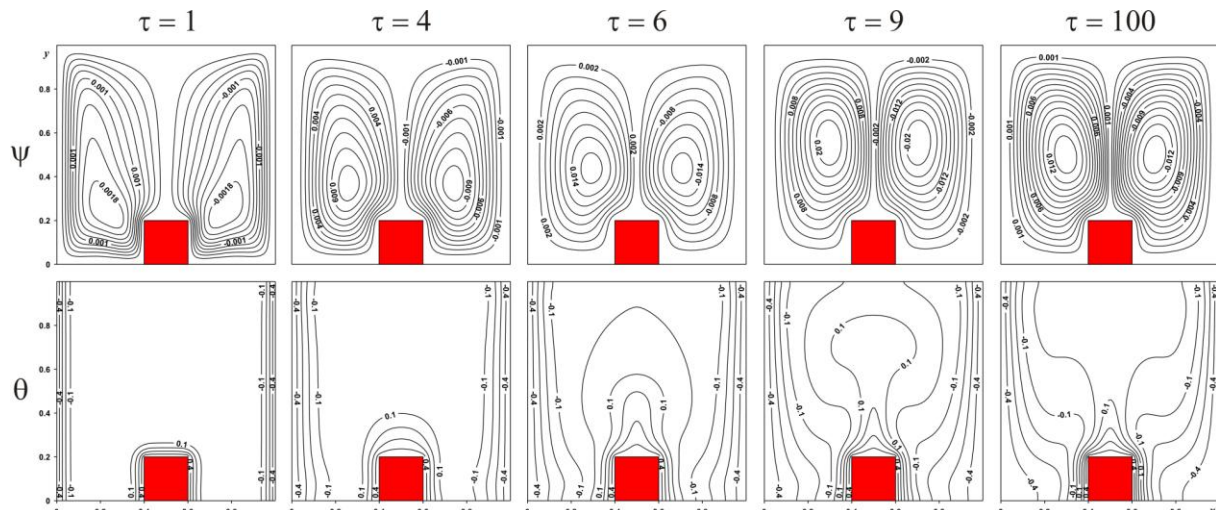


Figure 2. Streamlines ψ and isotherms θ for $Pr = 7$ and different values of the dimensionless time.

It should be noted that regardless of the dimensionless time values two convective cells form inside of the cavity. An appearance of these vortices is due to the presence of the centered heat source of constant temperature. These two circulations illustrate a formation of the ascending hot flow in the central part of the cavity over the heat source and two descending cold flows close to the vertical walls. At the beginning time level ($\tau = 1$) the dominating heat transfer mechanism is heat conduction. Therefore one can find that isotherms are parallel to the vertical isothermal walls and heat source surfaces. An increase in time leads to an intensification of the convective flow and a displacement of the convective cores along the positive direction of the vertical axis. It is worth noting a formation of thermal plume over the heater for $\tau \geq 4$. Also heating of the cavity leads to a non-monotonic variation of the flow rate taking into account the maximum values of $|\psi|$. Such situation can be explained by more intensive heating of the domain of interest and as a result one can find a decrease in the temperature gradient that illustrates attenuation of the convective flow. The time moment $\tau = 100$ characterizes a steady-state regime because for $\tau > 100$ there are not any variations of streamlines and isotherms.

The effect of the dimensionless time and Prandtl number on the average Nusselt number along the heat source surface is presented in Figure 3. The average Nusselt number has been defined as

$$Nu_{avg} = \frac{1}{3} \left[\frac{L}{h} \int_0^{h/L} \left(-\frac{\partial \Theta}{\partial X} \right)_{X=L-l/2L} dY + \frac{L}{l} \int_{L-l/2L}^{L+l/2L} \left(-\frac{\partial \Theta}{\partial Y} \right)_{Y=h/L} dX + \frac{L}{h} \int_0^{h/L} \left(-\frac{\partial \Theta}{\partial X} \right)_{X=L+l/2L} dY \right], \text{ where } h \text{ is a}$$

height of the heat source, l is a length of the heat source.

Taking into account Figure 3 one can find that an increase in Prandtl number leads to both weak intensification of the heat transfer at a steady-state regime and an increase in the time for reaching the steady-state regime.

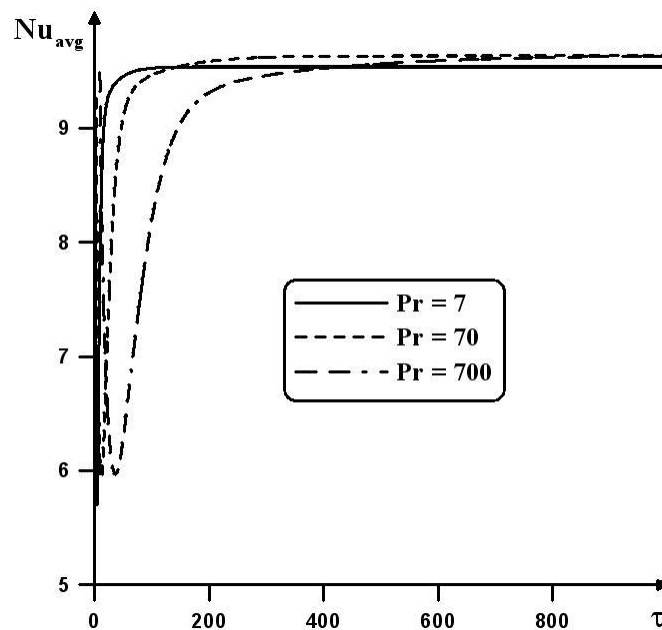


Figure 3. Variation of the average Nusselt number along the heat source surface with dimensionless time and Prandtl number.

4. Conclusion

The numerical analysis of natural convection with temperature-dependent viscosity in a square cavity having isothermal vertical walls and a heater located on the bottom wall has been carried out. The evolution of the streamlines and isotherms has been studied for $Ra = 10^5$, $Pr = 7$ and $C = 2$. It has been found that an increase in dimensionless time leads to an intensification of convective flow and displacement of the convective cores along the vertical axis. This mentioned behavior occurs during the first time level. After that one can find an attenuation of the convective flow due to a decrease in the temperature gradient. In addition, an increase in Prandtl number illustrates the convective heat transfer enhancement and an increase in the time for reaching the steady-state regime.

Acknowledgments

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