

Mathematical Simulation of the Influence of Surface Roughness and Mass Loss on Thermal Protection

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Abstract—A mathematical model of the thermochemical destruction of carbon fiber-reinforced plastic in the presence of surface roughness and composite ablation is refined based on known theoretical results. It is shown that mass loss through rough surface ambiguously influences the intensity of heat exchange processes in thermal protective material. Results of numerical calculations are compared with known data.

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INTRODUCTION

In a high-enthalpy flow, one of the factors that affects heat flux (the heat exchange coefficient) and finally determines the intensity of heat exchange is the state of the surface [1–4]. Ablating composite materials consisting of a filler and binder, glass and carbon fiber-reinforced plastics [4, 5], can have surface roughness in a wide range of $(0.05–0.5) \times 10^{-3}$ m. This value of roughness suffices to transform a laminar boundary layer into a turbulent one [1–3].

The creation of space applications has shown in [3, 6–8] that the influence of roughness should be taken into account for both ablating nozzles of bodies in the flow and ablating lateral surfaces with a relief surface structure. A relief surface structure is formed under ablation in a supersonic turbulent flow, which is confirmed by results [8].

The surface roughness can imply a considerable increase in surface friction and growth of the convective heat flux [6]. The growth of heat flux in a turbulent boundary layer (BL) due to roughness has a significant effect on mass loss of the thermal protective material and on the change of the nozzle shape [6–8].

PROBLEM STATEMENT

Since many experiments [2, 3, 6–8] demonstrate that the surface friction increases as the roughness height grows and the heat flux attains a maximum [6, 7], the Reynolds analogy is not likely to be applicable to rough surfaces. Let us make use of the results from [7], where a new approach to engineering calculations of heat fluxes toward rough surfaces in supersonic flows has been proposed:

$$\begin{aligned} \frac{St}{St_*} &= \frac{C_f/C_{f*}}{1 + \eta F}, \\ \frac{C_f}{C_{f*}} &= \log Re_K \left(0.365 \frac{h_w}{h_r} + 0.635 \right), \\ F &= Pr^{0.8} Re_K^{0.45} \left(\frac{C_f}{C_{f*}} \right)^{0.725} \frac{U_{\tau*}}{U_e}, \\ \frac{U_{\tau*}}{U_e} &= \left(\frac{C_{f*} \rho_e}{2 \rho_w} \right)^{0.5}, \quad Re_K = \frac{U_{\tau*} K}{\nu_*}, \\ St_* &= \frac{(\alpha/c_p)_*}{(\rho U)_e}, \\ St &= \frac{q_w}{(\rho U)_e (h_e - h_w)}, \quad Pr = \left(\frac{c_p \mu}{\lambda} \right)_e, \\ U_{\tau} &= \left(\frac{\tau_w}{\rho_w} \right)^{0.5}, \quad \nu = \frac{\mu}{\rho}, \quad \chi = \frac{c_p}{c_v}, \quad c_{pg} = b_1 + 2b_2 T, \\ h_r &= c_{pg} T_r, \quad T_r = T_{\infty} [1 + 0.5r(\chi - 1) M_{\infty}^2], \\ C_{f*} &= 0.0162(K/l)^{1/7}. \end{aligned} \quad (1)$$

Here Pr , M_{∞} , and St are the Prandtl, Mach, and Stanton numbers, respectively; Re_K is the Reynolds number for roughness; C_f is the local coefficient of friction; h is enthalpy; l is body length; U is speed; c_p , λ , α , and μ are the coefficients of specific heat, thermal conduction, heat exchange, and dynamic viscosity, respectively; ρ is density; r is the recovery factor; T is temperature; τ is tangential stress; χ is the adiabatic index; η , b_1 , and b_2 are constants; and K is the equivalent roughness height (ERH) [2, 7]. The indices e and w stand for the outer boundary of the BL and the

outer heated surface of the body, respectively; τ stands for dynamic velocity; g , for the gas phase; 0, for the initial condition (the absence of blow-in); ∞ , for parameters at infinity; and the asterisk, for a characteristic quantity (smooth surface).

The power law of resistance (the last formula in equalities (2)) has been obtained in [9] for a steady-state roughness, when the coefficient of total body resistance depends not on the Reynolds number but only on the local relative roughness.

It is assumed that the convective heat flux $q_w(t)$, which acts on the thermal protective material (TPM) for a certain time, is known; then, in view of [4, 7, 9, 10], from (1) and (2), we have

$$q_w = \frac{\alpha C_f / C_{f^*}}{c_p (1 + \eta F)} (h_e - h_w),$$

$$\frac{\alpha}{c_p} = \left(\frac{\alpha}{c_p} \right)_0 \left[1 - \frac{\gamma(\rho v)_w}{(\alpha/c_p)_0} \right], \tag{3}$$

where $(\rho v)_w$ is the total mass loss of the TPM and γ is the attenuation factor for the turbulent mode of flow in the BL.

In the study of convective heat exchange on a spherically blunted cone or on a hemisphere in a laminar–turbulent BL on a permeable wall, one can employ the results from [5, 11].

The physics of the process in the condensed phase (*c-phase*) is as follows [5, 10, 12]: under the action of a high-temperature flow, the temperature of the carbon fiber-reinforced plastic (CFRP) increases up to the decomposition temperature of the binding material (resin); thereafter, the reaction of pyrolysis of the thermosetting binder proceeds with the formation of pyrazole and carbon residue (coke), which is kept inside the matrix by reinforcing carbon fiber.

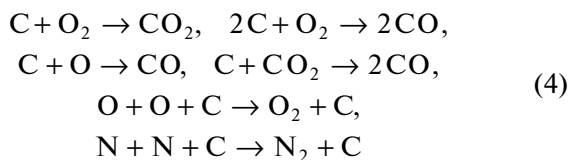
For $T_w > 800$ K, the carbonaceous surface is destructed as a result of the interaction with components of dissociated air [10, 12]. Gaseous pyrolysis products can be filtered toward the interface $y = 0$ and blown into the BL; along with carbon oxidation products, these can reduce the convective heat flux to the body.

We make the following assumptions in problem statement [10]:

(i) the Reynolds number in the incident hypersonic air flow is sufficiently large ($Re_\infty \gg 1$), and a BL is formed in the vicinity of the body surface;

(ii) the air at the outer boundary of the BL is in thermochemical equilibrium and constitutes a five-component mixture of O, O₂, N, N₂, and NO

(iii) at the TPM outer surface, the heterogeneous reactions



proceed for $T_w < 2200$ K.

The largest contribution to mass loss in ablation of the carbon surface in the temperature range $1000 \text{ K} < T \leq 2200 \text{ K}$ is due to the second reaction from (4); and, for $T > 2200 \text{ K}$, to the second and third reactions.

For higher temperatures $T > 2200 \text{ K}$, we can apply the results from [13], where basic peculiarities of the mechanism of mass loss of the carbon material are determined; these have been obtained using a complete thermochemical model.

Hereinafter, serial numbers are assigned to the components O, O₂, N, N₂, CO, and CO₂, which enter (4). Molar and mass rates of chemical reactions (4) are described in detail in [10, 12]; the expression for the mass loss rate

$$\begin{aligned} (\rho v)_{2w} &= (\Phi_4 \rho)_w \\ &= \left[\left(\frac{m_6}{m_2} - 1 \right) c_{2w} B_1 + \left(2 \frac{m_5}{m_2} - 1 \right) c_{2w} B_2 \right. \\ &\quad \left. + \left(\frac{m_5}{m_1} - 1 \right) c_{1w} B_3 + \left(2 \frac{m_5}{m_6} - 1 \right) c_{6w} B_4 \right], \tag{5} \\ B_i &= k_{iw} \exp \left(- \frac{E_{iw}}{RT_w} \right), \quad i = \overline{1, 4}, \\ \rho_w &= \frac{P_e m_w}{RT_w}, \quad m_w^{-1} = \sum_{\alpha=1}^6 \frac{c_{\alpha w}}{m_\alpha} \end{aligned}$$

is given in [10].

We note that, from the experimental data [14], a turbulent flow past the TPM was observed in the BL; the corresponding attenuation factor γ in heat flux (3) can be obtained in accordance with the Mugalev modification [4].

We write balance relations for mass concentrations of the components c_{iw} using the analogy of the processes of heat and mass exchange for diffusive fluxes [4, 12] to obtain

$$\begin{aligned} J_{iw} + (\rho v)_w c_{iw} &= R_{iw}, \quad i = \overline{1-6}, \\ J_{iw} &= \beta (c_{iw} - c_{ie}), \quad \beta = \alpha / c_p, \end{aligned}$$

where the total mass loss rate is as follows: $(\rho v)_w = (\rho v)_{1w} + (\rho v)_{2w}$; $(\rho v)_{1w}$ is the mass loss rate due to pyrolysis; $(\rho v)_{2w}$ is the mass loss rate due to heterogeneous chemical reactions (4); P_e is pressure; $\gamma = 0.19(m_e/m_w)^{0.35}$ [4], $m_e^{-1} = \sum_{i=1}^5 c_{ie}/m_{ie}$; β is the coefficient of mass exchange; m_i ($i = \overline{1-6}$) are molecular weights of the components; E_{iw} and k_{iw} , $i = \overline{1, 4}$,

are activation energies and pre-exponential factors in (5) for the reactions from (4), respectively; and the expressions for R_{iw} are given in [10].

It is believed that, for $T_w < 2200$ K, destruction products weakly dilute the air mixture in the BL. This allows us to accept assumptions (i)–(iii) in the BL.

In the mathematical statement of the problem, in the c-phase, we take into account the following:

(iv) the process of CFRP thermochemical destruction is one-dimensional;

(v) the filtered gas is homogeneous with a molecular mass close to that of the air mixture;

(vi) the porous medium is one-temperature; and

(vii) the gas flow inside burning pores is laminar and described by linear Darcy's law [4, 12].

The fourth assumption is sufficiently justified, since the size of the reaction zone for $T \geq 600$ K is of an order of 2.5×10^{-3} m and the thickness of the heated layer at $t_p = 10$ s (see Fig. 2 in [10]) is less than 4×10^{-3} m, which is considerably thinner than the initial TPM thickness $L_0 = 10^{-2}$ m.

The fifth assumption is employed to simplify the model, since the chemical composition of products of the filtered gas in the considered carbon-phenolic material (P5-13N) [14] under thermochemical destruction is unknown.

As a rule, the volumetric heat exchange coefficient A_v in a model of a two-temperature medium [15] (unlike an inert medium [4]) is determined with a significant error. According to [15], the temperature difference between a solid and gas phase for the most realistic values $A_v = 10^6 - 10^8$ W/(K m³) does not exceed 323–373 K, if the coke residue temperature is 1273–2273 K.

Finally, the transient and turbulent modes of gas filtering in pores do occur under CFRP thermochemical destruction [14]; these are described by nonlinear Darcy's law [4, 12]. However, for the material considered below [14], the viscosity and inertia coefficients in the case of a chemically reacting medium are unknown to the authors.

Mathematically, the problem reduces to solving a system of equations written in a moving system of coordinates [10, 12] that is related to the thermochemical destruction front. This system of equations is as follows:

$$c_p \left(\frac{\partial T}{\partial t} - \omega \frac{\partial T}{\partial y} \right) + c_{p5} \rho_5 \varphi_5 v \frac{\partial T}{\partial y} \quad (6)$$

$$= \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) - q_1 R_1 + q_2 R_2,$$

$$\frac{\partial \rho_5 \varphi_5}{\partial t} - \omega \frac{\partial \rho_5 \varphi_5}{\partial y} + \frac{\partial \rho_5 \varphi_5 v}{\partial y} \quad (7)$$

$$= (1 - \alpha_1) R_1 + (1 - \alpha_2) R_2,$$

$$\rho_1 \left(\frac{\partial \varphi_1}{\partial t} - \omega \frac{\partial \varphi_1}{\partial y} \right) = -k_1 \rho_1 \varphi_1 \exp \left(-\frac{E_1}{RT} \right), \quad (8)$$

$$\rho_2 \left(\frac{\partial \varphi_2}{\partial t} - \omega \frac{\partial \varphi_2}{\partial y} \right) = \alpha_1 k_1 \rho_1 \varphi_1 \exp \left(-\frac{E_1}{RT} \right) - k_2 \rho_2 \varphi_2 \exp \left(-\frac{E_2}{RT} \right), \quad (9)$$

$$\rho_3 \left(\frac{\partial \varphi_3}{\partial t} - \omega \frac{\partial \varphi_3}{\partial y} \right) = \alpha_2 k_2 \rho_2 \varphi_2 \exp \left(-\frac{E_2}{RT} \right), \quad (10)$$

$$\rho_4 \left(\frac{\partial \varphi_4}{\partial t} - \omega \frac{\partial \varphi_4}{\partial y} \right) = 0, \quad (11)$$

$$v = -\frac{z}{\mu} \frac{\partial P}{\partial y}, \quad z = \frac{z_* \varphi_5^3}{(1 - \varphi_5)^2}, \quad \varphi_5 = 1 - \sum_{i=1}^4 \varphi_i,$$

$$P = \frac{\rho_5 R T}{M_5}, \quad \lambda = \sum_{i=1}^5 \lambda_i \varphi_i, \quad c_p = \sum_{i=1}^4 c_{pi} \rho_i \varphi_i,$$

$$\lambda_5 = \lambda_{5.0} \sqrt{\frac{T}{T_0}}, \quad \mu = \mu_0 \sqrt{\frac{T}{T_0}}, \quad c_{p5} = c_{pg}, \quad (12)$$

$$\alpha_1 = \frac{v_2 M_{2s}}{v_1 M_{1s}}, \quad R_1 = k_1 \rho_1 \varphi_1 \exp \left(-\frac{E_1}{RT} \right),$$

$$R_2 = k_2 \rho_2 \varphi_2 \exp \left(-\frac{E_2}{RT} \right),$$

$$\alpha_2 = \frac{v_3'' M_{3s}}{v_2' M_{2s}},$$

$$\omega = \frac{(\rho v)_{2w}}{(\rho_1 \varphi_1 + \rho_2 \varphi_2 + \rho_3 \varphi_3 + \rho_4 \varphi_4)_w}, \quad (13)$$

$$(\rho v)_{1w} = -(\rho_5 v \varphi_5)_w.$$

The system of equations (6)–(11) should be solved with consideration for the following initial and boundary conditions

$$T|_{t=0} = T_0, \quad \rho_5|_{t=0} = \rho_{5.0}, \quad (14)$$

$$\varphi_i|_{t=0} = \varphi_{i,0}, \quad i = 1, \dots, 4;$$

$$q_w - (\rho v)_{1w} (h_w - h_g) - (\rho v)_{2w} (h_w - h_c) - \varphi_4 \varepsilon \sigma T_w^4 = -\lambda \frac{\partial T}{\partial y} \Big|_{y=0-s(t)}, \quad (15)$$

$$T|_{y=l} = T_0, \quad v|_{y=l} = 0, \quad \varphi_i|_{y=l} = \varphi_{i,0}, \quad i = 1, \dots, 4, \quad (16)$$

$$P_e = P_w|_{y=0-s(t)}, \quad P_e = \rho_e T_e R \sum_{i=1}^5 \frac{c_{ie}}{m_{ie}},$$

$$l = L_0 - s(t), \quad s(t) = \int_0^t \omega(\tau) d\tau, \quad (17)$$

$$h_w = \sum_{\alpha=1}^6 h_{\alpha} c_{\alpha w}, \quad h_g = \int_0^T c_{pg} dT,$$

where t is time; y is the spatial coordinate; E_i is activation energy; k_i , $i = 1, 2$, is the pre-exponential factor; M_5 is the molecular weight of gaseous pyrolysis products; P is the pressure in pores; q_i ($i = 1, 2$) are the heat effects of binder and pyrazole decomposition, respectively; R is the universal gas constant; $s(t)$ is the burn-up depth; v is the speed of gaseous products of binder decomposition; z is the coefficient of permeability of the CFRP; α_1 and α_2 are the reduced stoichiometric coefficients [12] from the kinetic pyrolysis scheme [10]; ε is the integral coefficient of radiation; φ_i , $i = 1, 4$, are volumetric fractions; φ_5 is porosity; σ is the Stefan–Boltzmann constant; and ω is the linear rate of CFRP thermochemical destruction. The index c stands for the carbon surface; the indices 1–5 in the c -phase correspond to the binder, pyrazole, coke, filler, and gas, respectively; and the index p marks the time moment of heat action termination.

CALCULATION METHODS, TESTING, AND INITIAL DATA

The system of equations (6)–(11) with initial and boundary conditions (14)–(17) was numerically solved using an implicit absolutely stable monotonic difference scheme [16]. The results from [10] were repeated to test the calculation program. The numerical method was tested for the basic variant (combined heat exchange regime). All other initial data being equal, the calculation was performed for different spatial steps $H_1 = 2 \times H$, $H_2 = H$, $H_3 = H/2$, and $H_4 = H/4$, where $H = 10^{-4}$ m. The TPM temperature and gas density ρ_5 along the body depth were registered at different time moments. In all the variants, the problem was solved with a varying time step that was chosen from a given accuracy identical for all the spatial steps. The difference in errors of the material temperature and gas density in the TPM ($\delta = \max[\delta_T, \delta_{\rho_5}]$) decreased: $\delta_1 = 11.2\%$, $\delta_2 = 6.3\%$, and $\delta_3 = 2.4\%$. The below results were obtained for the spatial step $H = 0.5 \times 10^{-4}$ m.

The thermokinetic constants k_{iw} and E_{iw} in (5) for reactions (4) are given in [12]; and, for k_i and E_i , $i = 1, 2$, in [5, 14]. The enthalpy of the carbon material in the condensed state h_c is calculated by the formulas from [17]. For the carbon-phenolic composite, the thermo-physical coefficients c_{p4} and λ_4 are taken from [18]; and the Reynolds number Re_K for the ERH is given in [7]. The coefficient of specific heat of the air at a constant pressure c_{pg} and c_{p5} from (2) and (13) is obtained by the interpolation formula from [19]. The permeability z of the ideal porous medium in Darcy’s law from (12) is determined by the known Kozeny formula [12].

The below results are obtained with $T_\infty = T_0 = 293$ K, $(\alpha/c_p)_0 = 0.132$ kg/(m² s), $\mu_0 = 1.81 \times 10^{-5}$ kg/(m s), $\rho_{5,0} = 1.29$ kg/m³, $\lambda_{5,0} = 0.0257$ W/(m K), $T_e = 3600$ K, $h_e = 1.449 \times 10^7$ J/kg, $\rho_e = 0.088$ kg/m³, $\mu_e = 1.74 \times 10^{-4}$ kg/(m s), $\lambda_e = 0.782$ W/(m K), $c_{pe} = 4024$ J/(kg K), $c_{p1} = c_{p2} = 1700$ J/(kg K), $c_{p3} = 1020$ J/(kg K), $\rho_1 = 1200$ kg/m³, $\rho_2 = 1100$ kg/m³, $\rho_3 = 1300$ kg/m³, $\rho_4 = 1400$ kg/m³, $\lambda_1 = \lambda_2 = 0.21$ W/(m K), $\lambda_3 = 0.041$ W/(m K), $R = 8.314$ J/(mol K), $M_5 = 29$ kg/kmol, $z_* = 5 \times 10^{-11}$ m², $L_0 = 10^{-2}$ m, $E_1 = 28783$ J/mol, $E_2 = 93990$ J/mol, $k_1 = 9.6 \times 10^4$ s⁻¹, $k_2 = 1.2 \times 10^5$ s⁻¹, $q_1 = 2 \times 10^5$ J/kg, $q_2 = 10^5$ J/kg, $m_1 = 16$ kg/kmol, $m_2 = 32$ kg/kmol, $m_3 = 14$ kg/kmol, $m_4 = 28$ kg/kmol, $m_5 = 28$ kg/kmol, $m_6 = 44$ kg/kmol, $K = (0.127 - 0.508) \times 10^{-3}$ m, $t_p = 10$ s, $Pr = 0.9$, $Re_K = 15 - 65$, $M_\infty = 5$, $\sigma = 5.67 \times 10^{-8}$ W/(m² K⁴), $\chi = 1.4$, $r = 0.9$, $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\varepsilon = 0.9$, $\varphi_{1,0} = 0.3$, $\varphi_{2,0} = 0.01$, $\varphi_{3,0} = 0.01$, $\varphi_{4,0} = 0.6$, $b_1 = 965.5$, $b_2 = 0.0735$, and $\eta = 0.52$.

RESULTS OF NUMERICAL SOLUTION AND THEIR ANALYSES

First, a flow past the CFRP with no pyrolysis and only accounting for heterogeneous reactions (4) is considered to obtain $(\rho v)_w = (\rho v)_{2w}$. Figure 1 depicts time-dependences of the surface temperature T_w for various values of the ERH friction coefficient.

Figure 2 displays ERH-dependences of the friction coefficient and Stanton number at $t = t_p$.

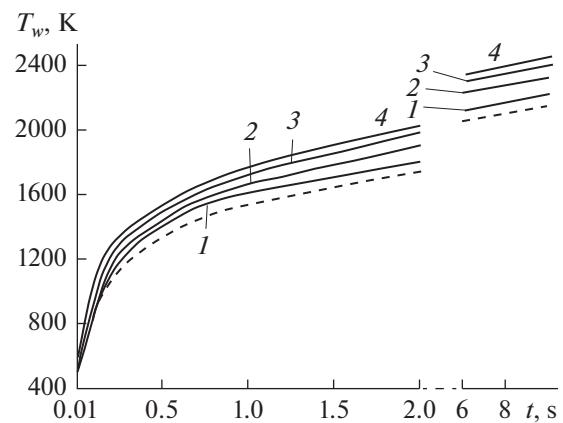


Fig. 1. Time-dependence of the surface temperature at different values of the ERH. Curves 1, 2, 3, and 4 correspond to $K = 1.27 \times 10^{-4}$ m, 2.54×10^{-4} m, 3.81×10^{-4} m, and 5.08×10^{-4} m, respectively. The dashed line stands for $K = 0$ with $(\rho v)_w = (\rho v)_{2w}$.

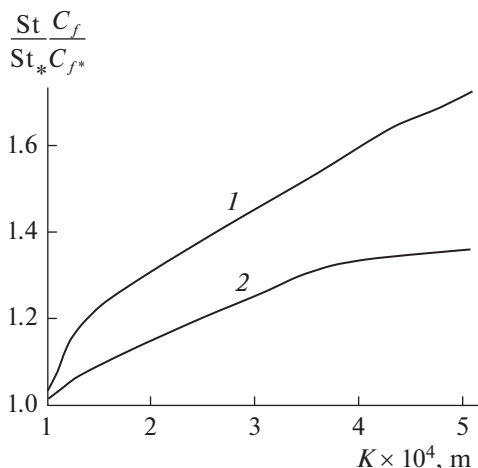


Fig. 2. ERH-dependence of (1) the friction coefficient and (2) the Stanton number at $t = t_p$.

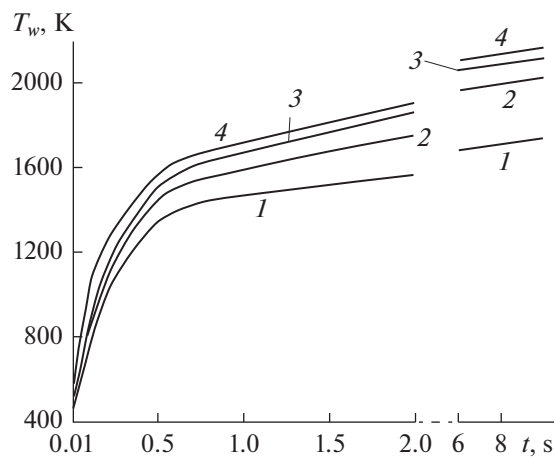


Fig. 3. Time-dependence of the surface temperature under total mass loss. Notations are the same as in Fig. 1.

From an analysis of the results, it follows that the Stanton number from (1) and, correspondingly, the convective heat flux from (3) and the TMP surface temperature increase with growing ERH (friction coefficient). Qualitatively, this result is in agreement with experimental data [2, 7].

Let us consider the TPM heating (see Fig. 3) in a combined destruction mode, to be precise, pyrolysis of the CFRP accounting for surface reactions (4), to obtain $(\rho v)_w = (\rho v)_{1w} + (\rho v)_{2w}$.

As expected, the TPM surface temperature increases as the ERH grows, which is qualitatively confirmed by experiments [20, 21]. In [20] the effect of roughness is insignificant for $K < 10^{-4}$ m; that is, the heat exchange is almost similar to that on a smooth surface. For $K > 5 \times 10^{-4}$ m, individual elements of developed roughness are broken off (chemico-mechanical chipping) under the action of a high-temperature gas flow and carried out into the latter. Since the CFRP components are not identically subject to mass loss, the BL can be destructed in individual places; consequently, the efficiency of thermal protection can reduce.

Of interest is to compare models in the combined destruction regime in the presence and absence of roughness. Figure 4 displays time-dependences of (b) surface temperature and (a) mass loss at $K = 0$ (curves 1) and $K = 1.27 \times 10^{-4}$ m (curves 2). The reduction of surface temperature for $K \neq 0$ at the initial step of CFRP heating is due to a higher blow-in (see Fig. 4a, curve 2) of gaseous thermochemical destruction products.

A decrease in surface temperature in the combined destruction regime under CFRP heating was observed in experiment [20]. This result of [20] is related to the growth of the contact surface due to roughness and to the corresponding growth in yield of volatile gaseous

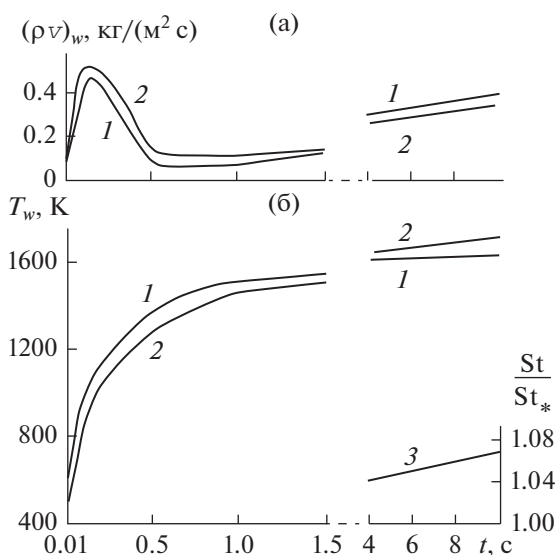


Fig. 4. Time-dependence of (a) the mass loss and (b) the surface temperature under total mass loss. Curves 1, 2, and 3 correspond to the smooth surface, rough surface, and Stanton number for the rough surface, respectively.

products formed under pyrolysis and chemical reactions of volatile components of the binder and filler.

The efficiency of pyrolysis as a heat barrier at the initial step of TPM destruction manifests itself through a considerable blocking of the convective heat flux from the gas phase. Thereafter, as the process of heat exchange is settled for $t > 0.5$ s, the hydrodynamical resistance due to the growth of the thickness of the CFRP coked layer [10, 18] and to the reduction of the gradient of gas pressure in pores [18] increases, and the value of mass loss stabilizes. As a result, the Stanton number from (1) starts growing for $t > 3$ s (see curve 3 in Fig. 4b), and, along with it, the value of the convec-

tive heat flux also increases in accordance with (3). This leads to an increase in heating and a rise in temperature of the CFRP rough surface as compared to a smooth one.

CONCLUSION

Relying on the proven results of [2, 5, 7, 10], a refined statement of the problem on the influence of surface roughness and mass loss on the heat exchange under CFRP heating is given. If the blow-in through rough surface is taken into account, the heat exchange in the TPM can both weaken and intensify. The calculation results qualitatively agree with experiments [7, 20, 21].

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REFERENCES

- Loitsyanskii, L.G., *Mekhanika zhidkosti i gaza* (Mechanics of Liquids and Gases), Moscow: Nauka, 1973.
- Schlichting, H., *Grenzschicht-Theorie* (Theory of Boundary Layer), Karlsruhe: Braun, 1951.
- Avduevskii, V.S., Galitseiskii, B.M., Glebov, G.A., et al., *Osnovy teploperedachi v aviatsionnoi i raketno-kosmicheskoi tekhnike* (Fundamentals of Heat Transfer in Aviation and Rocket and Space Technology), Koshkin, V.K., Ed., Moscow: Mashinostroenie, 1975.
- Polezhaev, Yu.V. and Yurevich, F.B., *Teplovaya zashchita* (Thermal Protection), Moscow: Energiya, 1976.
- Grishin, A.M., Golovanov, A.N., Zinchenko, V.I., Efimov, E.K., and Yakimov, A.S., *Matematicheskoe i fizicheskoe modelirovanie teplovoi zashchity* (Mathematical and Physical Simulation of Thermal Protection), Tomsk: Tomsk. Gos. Univ., 2011.
- Welsh, W.E., *AIAA J.*, 1970, no. 11, p. 1983.
- Nestler, D.E., *AIAA J.*, 1971, no. 9, p. 1799.
- Laganelli, A.L. and Nestler, D.E., *AIAA J.*, 1969, no. 7, p. 1319.
- Droblenkov, V.F., *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk*, 1955, no. 8, p. 17.
- Stepanova, E.V. and Yakimov, A.S., *High Temp.*, 2015, vol. 53, no. 2, p. 228.
- Gorskii, V.V. and Pugach, M.A., in *Nauka i tekhnologii. Mater. XXXIV Vseros. konf., posvyashchennoi 90-letiyu so dnya rozhdeniya akad. V.P. Makeeva* (Science and Technology: Proc. XXXIV All-Russian Conf. Dedicated to the 90th Anniversary of Acad. Makeev), Moscow: Ross. Akad. Nauk, 2014, p. 104.
- Grishin, A.M. and Fomin, V.M., *Sopryazhennye i nestatsionarnye zadachi mekhaniki reagiruyushchikh sred* (Conjugate and Nonstationary Problems of the Mechanics of Reacting Media), Novosibirsk: Nauka, 1984.
- Gorskii, V.V., Zabarko, D.A., and Olenicheva, A.A., *High Temp.*, 2012, vol. 50, no. 2, p. 286.
- Golovanov, A.N., *Combust., Explos. Shock Waves (Engl. Transl.)*, 1999, vol. 35, no. 3, p. 275.
- Vasilevskii, K.K. and Fedorov, O.G., in *Tplomassopereenos* (Heat and Mass Transfer), Minsk: Nauka i Tekhnika, 1968, vol. 2, p. 67.
- Samarskii, A.A., *Vvedenie v teoriyu raznostnykh skhem* (Introduction to the Theory of Difference Schemes), Moscow: Nauka, 1971.
- Buchnev, L.M., Smyslov, A.I., Dmitriev, I.A., et al., *Teplofiz. Vys. Temp.*, 1987, vol. 25, no. 6, p. 1120.
- Grishin, A.M., Parashin, A.D., and Yakimov, A.S., *Combust., Explos. Shock Waves (Engl. Transl.)*, 1993, vol. 29, no. 1, p. 82.
- Karapet'yants, M.Kh. and Karapet'yants, M.M., *Osnovnye termodinamicheskie konstanty neorganicheskikh i organicheskikh veshchestv* (Basic Thermodynamic Constants of Inorganic and Organic Substances), Moscow: Khimiya, 1968.
- Golovanov, A.N., Zima, V.P., and Stepanova, E.V., USSR Inventor's Certificate no. 2481239, *Byull. Izobret.*, 2013, no. 13.
- Zima, V.P. and Stepanova, E.V., *Abstracts of Papers, Vseros. shkoly-konf. s mezhdunarodnym uchastiem "Aktual'nye voprosy teplofiziki i fizicheskoi gidrogazodinamiki"* (All-Russian Conf. with Int. Participants on Actual Problems of Thermophysics and Physical Hydrodynamics), Novosibirsk: Inst. Teplofiz., Sib. Otd., Ross. Akad. Nauk, 2014.

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