

A mathematical model of metallized solid propellant combustion under the changing pressure

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Abstract. This paper presents the mathematical model describing a non-stationary combustion of metallized solid propellant. The model takes into account the heat transfer, the oxidizer decomposition and gasification of the solid propellant, two-phase, dual-velocity, two-temperature reactant flow over the propellant surface. The conditions on the surface perform the conservation of energy and mass fluxes. The model is based on the research works [1, 2]. Our research provides data of the non-stationary burning rate depending on the Al powder dispersion and the pressure drop value.

1 Mathematical model

The mathematical formulation of the problem in the coordinate system associated with the propellant surface has the following form:

The heat transfer and burn-out equations for condensed phase:

$$\rho_1 c_1 \left(\frac{\partial T_1}{\partial t} + u \frac{\partial T_1}{\partial x} \right) = \lambda_1 \frac{\partial^2 T_1}{\partial x^2} + Q_1 k_1 \rho_1 (1 - \eta) e^{-\frac{E_1}{RT_1}}, \quad (1)$$

$$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} = k_1 (1 - \eta) e^{-\frac{E_1}{RT_1}}. \quad (2)$$

The system of equations for reactant flow over propellant surface:

$$\rho_2 c_2 \left(\frac{\partial T_2}{\partial t} + v \frac{\partial T_2}{\partial x} \right) = \lambda_2 \frac{\partial^2 T_2}{\partial x^2} + \frac{dp}{dt} + Q_2 k_2 \rho_2 Y e^{-\frac{E_2}{RT_2}} + 4\pi\alpha r_3^2 n(T_3 - T_2), \quad (3)$$

$$\frac{\partial Y}{\partial t} + v \frac{\partial Y}{\partial x} = D \frac{\partial^2 Y}{\partial x^2} - k_2 Y e^{-\frac{E_2}{RT_2}}, \quad (4)$$

$$\frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2 v}{\partial x} = -G, \quad (5)$$

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$$p = \rho_2 RT / \mu, \quad (6)$$

$$c_3 \rho_3 \left(\frac{\partial T_3}{\partial t} + w \frac{\partial T_3}{\partial x} \right) = -4\pi \alpha r_3^2 n (T_3 - T_2) + 2\mu_{Al} / 3\mu_O G Q_{Al}, \quad (7)$$

$$\frac{\partial \rho_3}{\partial t} + \frac{\partial (\rho_3 w)}{\partial x} = G, \quad (8)$$

$$\frac{\partial w}{\partial t} + w \frac{\partial w}{\partial x} = -\tau_{fr}, \quad (9)$$

$$\frac{\partial n}{\partial t} + \frac{\partial (nw)}{\partial x} = 0, \quad (10)$$

The boundary conditions are:

$$\lambda_1 \frac{\partial T_1(x_s, t)}{\partial x} = \lambda_2 \frac{\partial T_2(x_s, t)}{\partial x}, T_1(x_s, t) = T_2(x_s, t), \frac{\partial T_1(-\infty, t)}{\partial x} = 0, \frac{\partial T_2(\infty, t)}{\partial x} = 0, \\ \frac{\partial Y(\infty, t)}{\partial x} = 0, \alpha_{Al} \rho_1 u = \rho_3 w(x_s, t), \rho_2(x_s, t) = P / RT_2(x_s, t) \\ n(x_s, t) = \rho_3(x_s, t) / \left(\frac{4}{3} \pi r_{Al,0}^3 \rho_k \right), \quad (11)$$

$$(1 - \alpha_{Al}) \rho_1 u = (\rho_2 v Y) |_{(x_s, t)} - D \rho_2(x_s, t) \frac{\partial Y(x_s, t)}{\partial x}, (1 - \alpha_{Al}) \rho_1 u = \rho_2(x_s, t) v(x_s, t).$$

The initial conditions are:

$$-\infty < x < x_s : T_1(x, 0) = T_0, \eta(x, 0) = 0, \quad (12)$$

$$x_s < x < \infty : T(x, 0) = T_{ig}, Y(x, 0) = 0, v(x, 0) = 0, p(0) = p_0 \quad (13)$$

$$\rho_2(x, 0) = p_0 \mu / RT_{ig}, \rho_3(x, 0) = 0, w(x, 0) = 0, n(x, 0) = 0.$$

The particle interaction force with the gas is defined by the formula:

$$\tau_{fr} = F_{fr} / \left(\frac{4}{3} \pi r_3^3 \rho_k \right), F_{fr} = C_R S_m \rho_2 (w - u) |u - w| / 2. \quad (14)$$

The drag coefficient is defined by the empirical formula [3]:

$$C_R = 24 / \text{Re} (1 + 0,15 \text{Re}^{0,682}), \text{Re} = 2r_k \rho_2 |u - w| / \eta. \quad (15)$$

The heat transfer coefficient is determined as:

$$\alpha = Nu \lambda_2 / 2r_k, Nu = 2 + \sqrt{Nu_l^2 + Nu_t^2}, \quad (16)$$

where $Nu_l = 0,664 \text{Re}^{0,5}$, $Nu_t = 0,037 \text{Re}^{0,8}$.

The mass-change rate of particles during combustion:

$$G = \frac{3\mu_O}{2\mu_{Al}} n \rho_k 4\pi k_{Al} a^{0,9} r_{Al}^{1,5}, k_{Al} = 2,22 \cdot 10^{-5} \text{ m}^{1,5} / \text{s}. \quad (17)$$

The radius of unburnt aluminum particle r_{Al} and the radius of a whole particle:

$$r_{Al} = \left[\frac{2\mu_{Al}}{3\mu_O} \left(\left(1 + \frac{3\mu_O}{2\mu_{Al}} r_{Al,0}^3 - \frac{\rho_3}{(4/3)\pi n \rho_k} \right) \right)^{1/3} \right]^{1/3}, r_3 = \left[r_{Al}^3 + \frac{\mu_{Al} + 3/2 \mu_O}{\mu_{Al}} (r_{Al,0}^3 - r_{Al}^3) \right]^{1/3} \quad (18)$$

The pressure drop value ranges from p_0 (at $t_{p,n}$ - start drop time) to p_k at the end of the drop ($t_{p,k}$). The linear combustion rate u is the rate of conversion level $\eta = 0,99$ motion.

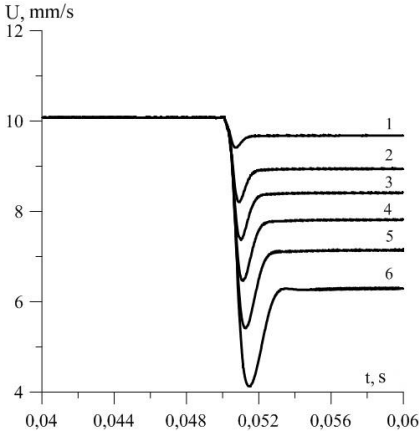


Fig. 1. Burning rate of N powder with micron sized aluminum particles in time. Pressure drop ranges from 100 atm to: 1 - 80, 2 - 60, 3 - 50, 4 - 40, 5 - 30, 6 - 20 atm. Pressure drop rate $10^{10} Pa / s$, $r_{Al} = 5 mkm$

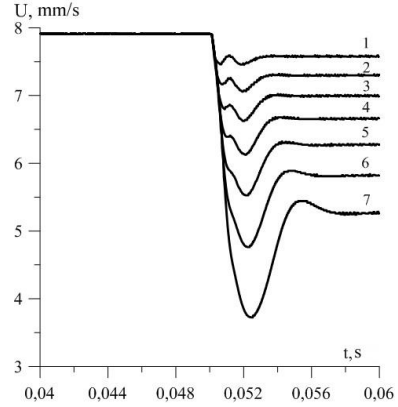


Fig. 2. Burning rate of N powder with micron sized aluminum particles in time. Pressure drop ranges from 100 atm to: 1 - 80, 2 - 70, 3 - 60, 4 - 50, 5 - 40, 6 - 30, 7 - 20 atm. Pressure drop rate $10^{10} Pa / s$, $r_{Al} = 15 mkm$

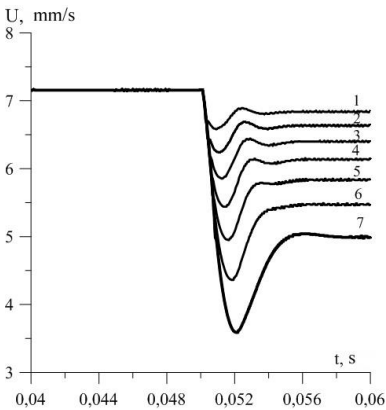


Fig. 3. Burning rate of N powder with micron sized aluminum particles in time. From 100 atm to: 1 - 80, 2 - 70, 3 - 60, 4 - 50, 5 - 40, 6 - 30, 7 - 20 atm. Pressure drop rate $10^{10} Pa / s$, $r_{Al} = 30 mkm$

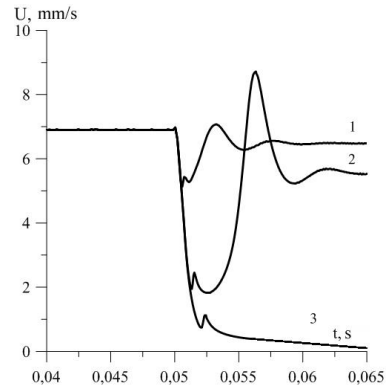


Fig. 4. Burning rate of N powder in time. Pressure drop ranges from 100 to: 1 - 80, 2 - 40, 3 - 20 atm. Pressure drop rate $10^{10} Pa / s$

Here: t is the time, x is the coordinate, x_s is the coordinate of combustion surface, ρ_1 , ρ_2 , ρ_3 , ρ_{Al} , ρ_k is the density of solid propellant, gas, reduced density of particles, aluminum, particle in the flow of combustion product, T_1 , T_2 , T_3 is the temperature of solid propellant, gas phase, aluminum particles in gas phase, T_0 , T_{ig} is the initial temperature of solid and gas phases, η is the conversion degree of the condensed-phase material, u is the linear combustion rate, v , w is the gas velocity and particle velocity, n is the number of particles in a unit volume, r_{Al} , r_3 is the radius of unburnt particle of aluminum and the whole radius of a particle, c_1 , c_2 , c_3 is the specific heat of solid propellant, gas and

particles at a constant pressure, λ is the thermal conductivity, Q_1 is the thermal effect of reaction in the solid phase, Q_2 — thermal effect of reaction in the gas phase, Q_{Al} — heat of aluminum combustion, k is the pre-exponential factor in the Arrhenius law, E is the energy of activation, R is the universal gas constant, α_{Al} is the aluminum mass fraction in solid propellant, p is the gas phase pressure, p_0 , p_k is the initial and final pressure in gas phase, $t_{p,n}$, $t_{p,k}$ is the start and end time of pressure drop, Y is the oxidant concentration in the gas phase, D is the diffusion coefficient, μ , μ_{Al} , μ_O is the molar masses of gas phase, aluminum particles and oxygen, G is the mass change rate of particles during combustion, τ_{fr} is the particle interaction force with the gas, α is the heat transfer coefficient. The quantities related to the condensed phase are denoted by index 1, to the gas is the 2, to particles is the 3, to the initial conditions is the 0.

We have solved the system of equation numerically using the methods described in the papers [1,2]. The calculations of N powder combustion of with micron sized aluminum particles were carried out at the following values of the thermophysical and kinetic quantities from [1,2]

We have conducted the calculation of the non-stationary burning rate during the pressure drop over the propellant surface. The composition combusts stationary under the pressure of 100 atm, at the time of 40 ms the pressure over the surface begins to drop in linear fashion at the rate of 10^{10} Pa/s. The values of the final pressure are 80, 70, 60, 50, 40, 30 and 20 atm. The size of Al particles varies from 5 to 30 μm (Pic.1, Pic.2, Pic. 3).

The combustion rate during the pressure drop is less than under the final pressure. Pic. 4 shows the combustion rate of non-metallized N powder. One can see in the picture that the combustion attenuates, when the pressure drops to 20 atm. In comparison to the pattern for metallized N powder, the addition of Al particles increases the combustion stability and the particle size growth reduces the time of transient interval.

Conclusion

We developed the mathematical model of metallized solid propellant non-stationary combustion. The statement of the problem includes the heat transfer, the oxidizer decomposition and gasification of the solid propellant, two-phase, dual-velocity, two-temperature reactant flow over the propellant surface. The model takes into account the diffusion and the exothermal reaction in gas phase, heating and combustion of Al particles in the flow, the particle velocity lag relative to the gas. The conditions on the surface perform the conservation of energy and mass fluxes. The paper provides data of non-stationary combustion under decreasing pressure. The work investigates transient-state conditions depending on the pressure change and the size of Al particles in solid propellant.

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References

1. A.Y. Krainov, D.A. Krainov, V.A. Poryazov, J. Eng. Phys. Thermophys. **89**, 458 (2016)
2. A.Y. Krainov, V.A. Poryazov, Combust., Explosion and Shock Waves **51**, 664 (2015)

3. Spravochnik po teploobmennikam. V dvukh tomakh. **1**. M .: Energoatomizdat, 1987. [in Russian]