Control Strategies for Discrete Delayed Systems with Unknown Input Using Nonparametric Algorithms

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***Abstract*—The paper deals with the control algorithms for discrete delayed systems with unknown inputs (disturbances). Control algorithm is based on local criterion with using Kalman filtering and nonparametric estimator. Examples are given to illustrate the proposed approach.**

***Keywords*—algorithm of control with delay; nonparametric estimator; system with unknown input**

I. Introduction

Locally optimal discrete control systems are a special type of the discrete model predictive control [1, 2] (MPC) with one step forecast. The main advantage of the method of locally optimal control is a significant simplification on the synthesis of the procedure. Last years, the MPC procedures have been applied to technical systems [2], chemical processes [3], inventory control [4], production-inventory system[5-7], and portfolio optimization [8].

In this paper, for the control-delayed systems with unknown input, we propose estimates of the unknown input, obtained by making use of the least mean squares (LMS) method [9-12] and nonparametric algorithms [13-18]. The suggested approach allows one to improve the estimation accuracy of state vector and unknown input. There is presented an example illustrating the effectiveness of the proposed control strategies, using filtering algorithm with nonparametric estimators in comparison with the known algorithms.

II. Model of Discrete System

Consider the model of object, which is described by the discrete equation

$$x(k+1) = Ax(k) + Bu(k-h) + Fs(k),$$

$$x(0) = x_0,\ u(j) = \psi(j),\ j = -h, -h+1, \ldots, -1, \quad (1)$$

where $x(k) \in R^n$ is a state vector, $u(k-h) \in R^m$ is a control vector, h is a value of time delay, $s(k) \in R^n$ is a disturbances vector, x_0 and $\psi(j)$ ($j = -h, -h+1, \ldots, -1,$) are the known vectors. Matrices A, B, F are given constant matrices .

The local criterion has the form

$$I(k) = (x(k+1) - z)^{\mathrm{T}} C(x(k+1) - z)$$
$$+ u^{\mathrm{T}}(k-h) Du(k-h), \quad (2)$$

where $C > 0, D \ge 0$ are weight matrices, z is a vector, which is selected by additional criterion. Transform criterion (2):

$$I(k) = u^{\mathrm{T}}(k-h)(B^{\mathrm{T}}CB + D)u(k-h) + u^{\mathrm{T}}(k-h)B^{\mathrm{T}}C$$
$$\times (Ax(k) - s(k) - z) + (Ax(k) - s(k) - z)^{\mathrm{T}} CBu(k-h).$$

Now, obtain the optimal control from the equation

$$\frac{dI(k)}{du(k-h)} = 0\,. \quad (3)$$

From (3), we have

$$(B^{\mathrm{T}}CB + D)u(k-h) + B^{\mathrm{T}}C(Ax(k) - s(k) - z) = 0\,. \quad (4)$$

Then, from (4)

$$u(k-h) = -(B^{\mathrm{T}}CB + D)^{-1} B^{\mathrm{T}}C(Ax(k) - s(k) - z). \quad (5)$$

According to (1), we get the following equalities:

$$x(k) = Ax(k-1) + Bu(k-h-1) - s(k-1)\,,$$
$$x(k-1) = Ax(k-2) + Bu(k-h-2) - s(k-2)\,,$$
$$\vdots$$
$$x(k-h+1) = Ax(k-h) + Bu(k-2h) - s(k-h)\,. \quad (6)$$

Now, using (6), the locally optimal control (5) is represented as follows:

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$$u(k-h)=-(B^{\mathrm{T}}CB+D)^{-1}B^{\mathrm{T}}C(A^{h+1}x(k-h)$$

$$+\sum_{i=1}^{h}A^{i}Bu(k-h-i)-\sum_{i=0}^{h}A^{i}s(k-i)-z). \qquad (7)$$

Note that the control (7), formed at the moment $(k-h)$, demands the knowledge of $x(k-h)$, $s(k-h)$, past values of controls $u(k-h-i)$ and forecasts for a vector of disturbances at the moments k, $k-1$,..., $k-h+1$.

III. Control Using Indirect Observations

Model of disturbances is defined by the following difference equation:

$$s(k+1)=Rs(k)+r(k)+q(k) \quad s(0) = s_0, \qquad (8)$$

where R is ($n\times n$)-matrix, $r(k)$ is a vector of unknown input, $q(k)$ is a random vector. There are indirect observations

$$\omega(k)=\Phi s(k)+\tau(k), \qquad (9)$$

where $\omega(k)$ is a vector of observations, Φ is a matrix, $\tau(k)$ is a random vector of errors, $q(k),\tau(k)$ are sequences of the Gaussian random vectors with such characteristics:

$$M\{q(k)\}=0, M\{\tau(k)\}=0,$$

$$M\{q(k)q^{T}(j)\}=Q\delta_{kj},\ M\{\tau(k)\tau^{T}(j)\}=T\delta_{kj},$$

$$M\{q(k)\tau^{T}(j)\}=0, \qquad (10)$$

where $M\{\}$ denotes the mathematical expectation, δ_{kj} is the Kronecker symbol.

Introduce the local criterion

$$I(k)=\mathrm{M}\{(w(k+1)-z(k))^{\mathrm{T}}C(w(k+1)-z(k))+$$

$$+u^{\mathrm{T}}(k-h)Du(k-h)/X_0^k\}, \qquad (11)$$

where $C > 0$, $D \geq 0$ are weight matrices, $z(k)$ is specified tracked vector, $X_0^k=\{x(0),x(1),...,x(k)\}$.

Basing on the principle of separation, find control with making use of filtering estimates and forecast estimates for components of vector s. As a result, we obtain the following control strategy for the current time (k - h):

$$u(k-h)=\bar{D}(HA^{h+1}x(k-h)+\sum_{i=1}^{h}HA^{i}Bu(k-h-i)$$

$$+HA^{h}F\hat{s}_f(k-h)+\sum_{i=0}^{h-1}HA^{i}F\hat{s}_p(k-i)-z(k)), \qquad (12)$$

where $\bar{D}=-(B^{\mathrm{T}}H^{\mathrm{T}}CHB+D)^{-1}B^{\mathrm{T}}H^{\mathrm{T}}C$, $\hat{s}_f(k-h)$ and $\hat{s}_p(k-i)$ are filtering estimates and forecast estimates, which are based on the optimal Kalman filtering algorithms using vector of estimates of the unknown input $\hat{r}(\cdot)$:

$$\hat{s}_f(k-h)=R\hat{s}_f(k-h-1)+\hat{r}(k-h-1)+K_f(k-h)$$

$$\times[w(k-h)-H(R\hat{s}_f(k-h-1)+\hat{r}(k-h-1))],$$

$$\hat{s}_f(0)=\bar{s}_0, \qquad (13)$$

$$K_f(k-h)=P(k-h/k-h-1)H^{\mathrm{T}}$$

$$\times(HP(k-h/k-h-1)H^{\mathrm{T}}+\mathrm{T})^{-1}, \qquad (14)$$

$$P(k-h/k-h-1)=RP(k-h-1)R^{\mathrm{T}}+Q, \qquad (15)$$

$$P(k-h)=(E_{n_1}-K_f(k-h)H)P(k-h/k-h-1),$$

$$P(0)=P_0. \qquad (16)$$

To construct the forecast estimates, we have to use the extrapolator, which allows one to calculate the estimate of the forecasts of disturbances by 1 step:

$$\hat{s}_p(k-h+1)=R\hat{s}_p(k-h)+f+\hat{r}(k-h)$$

$$+K_p(k-h)(\omega(k-h)-H\hat{s}_p(k-h)),\ \hat{s}_p(0)=\bar{s}_0, \qquad (17)$$

$$K_p(k-h)=RP_{pr}(k-h)H^{\mathrm{T}}(HP_{pr}(k-h)\Phi^{\mathrm{T}}+\mathrm{T})^{-1}, \qquad (18)$$

$$P_{pr}(k-h+1)=(R-K_p(k-h)H)P_{pr}(k-h)$$

$$\times(R-K_p(k-h)H)^{\mathrm{T}}+Q+K_p(k-h)TK_p^{\mathrm{T}}(k-h),$$

$$P_{pr}(0)=P_0. \qquad (19)$$

The forecasts for the next steps $j=2,...,h-1$ are determined as follows:

$$\hat{s}_p(k-h+j)=R\hat{s}_p(k-h+j-1)+f+\hat{r}(k-h+j-1). \qquad (20)$$

Estimates of the vector $\hat{r}(\cdot)$, obtained by using the LMS method [9-12] and nonparametric algorithms [13-18], are based on the minimization criterion

$$J=\sum_{i=1}^{k-h-1}\left\{\|\chi(i)\|_V^2+\|r(i-1)\|_W^2\right\}, \qquad (21)$$

where $\chi(i)=\omega(i)-\Phi R$ [illegible] ([illegible] $R\hat{s}_f(k-2)$ $+\hat{r}(k-2)$), $V > 0$, $W \geq 0$ are weight matrices. So,

$$\hat{r}(k-h)=[\Phi^{\mathrm{T}}V\Phi+W]^{-1}\Phi^{\mathrm{T}}V\Omega(k-h). \qquad (22)$$

We take the j-th component of the vector Ω in the form of the following analog of the known Nadaraya-Watson nonparametric regression estimate [19,20]:

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$$\Omega_j(p) = \frac{\sum_{i=1}^{p} \chi(i)_j(i) K\left(\frac{p-i+1}{h_j}\right)}{\sum_{i=1}^{p} K\left(\frac{p-i+1}{h_j}\right)}. \quad (23)$$

Here $K(\cdot)$ is a kernel function, h_j is a bandwidth parameter.

IV. An Illusrative Example

Simulations are performed by the following conditions:

$$A = \begin{pmatrix} 0.997 & 0 \\ 0 & 0.8 \end{pmatrix}, \; R = \begin{pmatrix} 0 & 1 \\ 0.1 & 0.5 \end{pmatrix},$$

$$Q = \mathrm{diag}\{0.05 \quad 0.02\},$$

$$T = \mathrm{diag}\{0.05 \;\; 0.05\}, \; D = W = 0,$$

$$B = H = F = C = V = P_0 = E_2;$$

$$r_1(k) = \begin{cases} 2.6 & \text{if } 0 \le k < 50, \\ 2 & \text{if } 50 \le k < 100, \\ 3 & \text{if } 100 \le k \le 150, \end{cases}$$

$$r_2(k) = \begin{cases} 2.5 & \text{if } 0 \le k < 50, \\ 2 & \text{if } 50 \le k < 100, \\ 3 & \text{if } 100 \le k \le 150. \end{cases}$$

The control and filtering algorithms are compared with the algorithms using the LMS estimates from [9, 10]. These comparisons are given in Figs. 1–10:

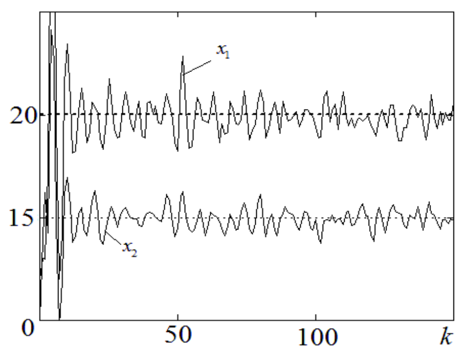

Fig. 1. The tracking of components z_1=20 and z_2=15 by x_1 and x_2 with use of the LSM estimates.

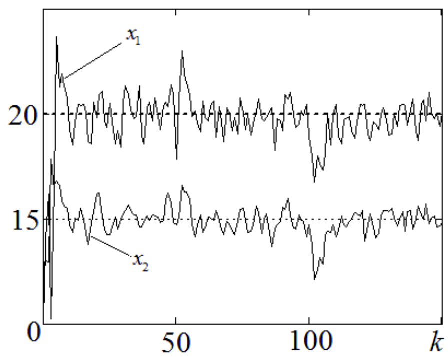

Fig. 2. The tracking of components z_1=20 and z_2=15 by x_1 and x_2 with use of nonparametric estimates.

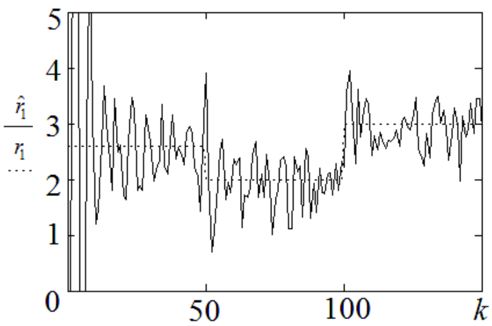

Fig. 3. The evaluation of unknown inputs r_1 with use of the LSM-estimates.

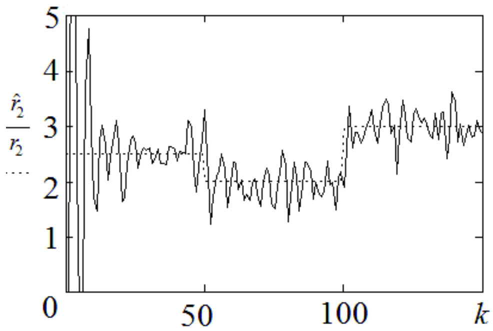

Fig. 4. The evaluation of unknown inputs r_2 with use of the LSM estimates.

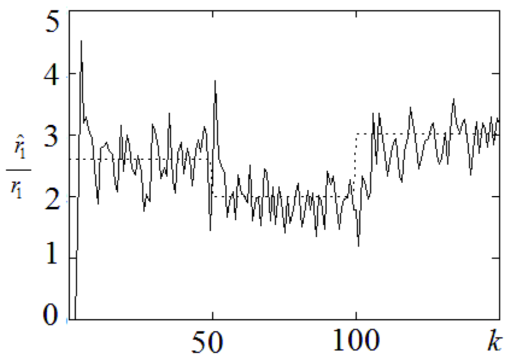

Fig. 5. The evaluation of unknown inputs r_1 with use of nonparametric estimates.

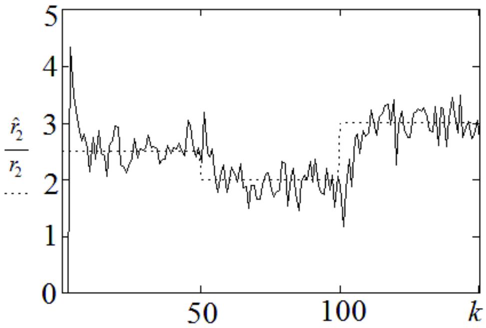

Fig. 6. The evaluation of unknown inputs r_2 with use of nonparametric estimates.

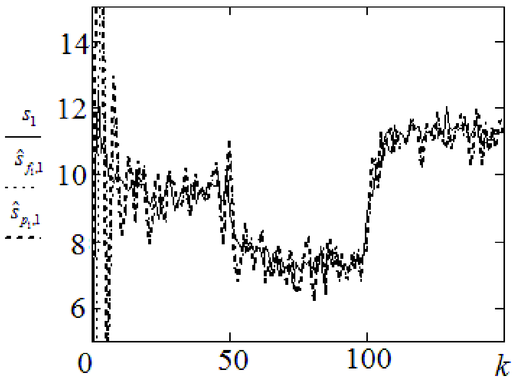

Fig. 7. The first component of perturbation vector, its estimate of filtration and estimate of forecast $s_1, \hat{s}_{f_1,1}, \hat{s}_{p_1,1}$ using the LSM estimates.

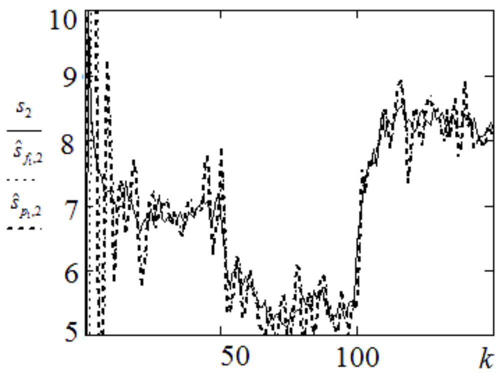

Fig. 8. The second component of perturbation vector, its estimate of filtration and estimate of forecast $s_2, \hat{s}_{f_1,2}, \hat{s}_{p_1,2}$ using the LSM estimates.

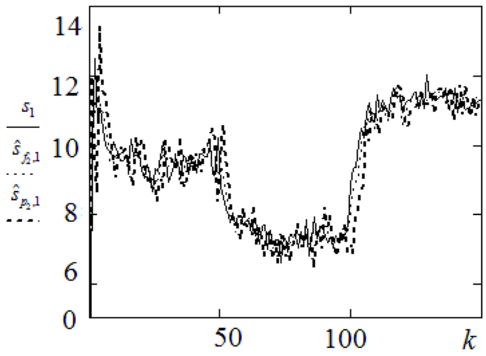

Fig. 9. The first component of perturbation vector, its estimate of filtration and estimate of forecast $s_1, \hat{s}_{f_2,1}, \hat{s}_{p_2,1}$ using nonparametric estimates.

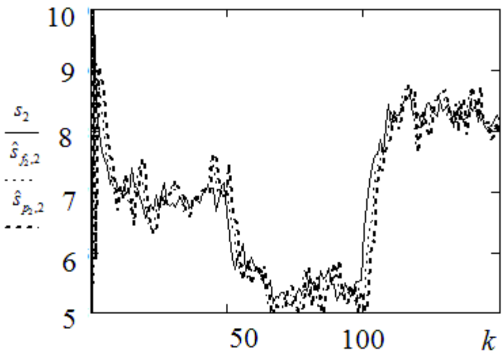

Fig. 10. The second component of perturbation vector, its estimate of filtration and estimate of forecast $s_2, \hat{s}_{f_2,2}, \hat{s}_{p_2,2}$ using nonparametric estimates.

Below, in Tables 1–4 empirical mean square errors (MSE) are given for $N = 150$ and by averaging 20 realizations:

$$\sigma_{x,i} = \sqrt{\frac{\sum_{k=1}^{N}(x_i(k) - z_i(k))^2}{N-1}},$$

$$\sigma_{sf,i} = \sqrt{\frac{\sum_{k=1}^{N}(s_i(k) - \hat{s}_{f,i}(k))^2}{N-1}},$$

$$\sigma_{sp,i} = \sqrt{\frac{\sum_{k=1}^{N}(s_i(k) - \hat{s}_{p,i}(k))^2}{N-1}},$$

$$\sigma_{r,i} = \sqrt{\frac{\sum_{k=1}^{N}(r_i(k) - \hat{r}_i(k))^2}{N-1}}, \quad i = \overline{1,2}.$$

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TABLE I. EMPIRICAL MSE OF STATE VECTOR $\sigma_{x,i}$

Components	LMS	Nonparametric
1	2.06	1.316
2	1.226	0.805

TABLE II. EMPIRICAL MSE OF FILTERING $\sigma_{sf,i}$

Components	LMS	Nonparametric
1	1.531	0.332
2	1.146	0.832

TABLE III. EMPIRICAL MSE OF FORECASTING $\sigma_{sp,i}$

Components	LMS	Nonparametric
1	1.593	0.798
2	1.084	0.508

TABLE IV. EMPIRICAL MSE OF ESTIMATING OF UNKNOWN INPUT $\sigma_{r,i}$

Components	LMS	Nonparametric
1	1.315	0.569
2	0.969	0.486

These results show that the estimation algorithms for unknown input and parameters using nonparametric procedures provide rather high accuracy of control and filtering for systems with unknown input and parameters.

V. Conclusion

In this paper, the algorithms of the Kalman filtering and control for discrete delayed systems with unknown input is developed. The proposed method has been verified by simulations. Figures and Tables show that the procedures with nonparametric estimators have the advantages in the accuracy compared to the known algorithms using the LMS estimates. The presented filtering algorithms with nonparametric technique may be used in solving the control problems for object with time-delay.

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