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Theoretical simulation of the picosecond runaway-electron beam in coaxial diode filled with SF_6 at atmospheric pressure

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Abstract – This paper presents detailed results of gas discharge theoretical simulation and the explanation of probabilistic mechanism of fast-electrons generation. Within the framework of a hybrid mathematical model, the hydrodynamic and the kinetic approaches are used simultaneously in order to describe the dynamics of different components of a low-temperature discharge plasma. The breakdown of a coaxial diode occurs in the form of a dense plasma region expanding from the cathode. On this background there is a formation of runaway electrons that are initiated by the ensemble of plasma electrons generated in the region of locally enhanced electric field within the front of the dense plasma. It is shown that the power spectrum of fast electrons in the discharge contains the group of electrons with the so-called "anomalous" energies. Comparison of the calculation results with the existent experimental data gives a good agreement for all major process parameters.

Introduction. – The phenomenon of runaway-electrons generation in high-pressure gas discharges has been widely studied in recent years [1–4]. Mostly it is connected to the progress in the field of high-voltage pulse generation with a short rise time of the voltage amplitude and the development of experimental equipments with picosecond time resolution [3,4].

The main factor affecting the amount of fast electrons is the possibility of creating a strong overvoltage of a discharge gap at the initial stage of the current growth in the gas diode. Multiple overvoltage (compared to static breakdown voltage) is achieved for a short time during the application of a large-amplitude voltage pulse with a subnanosecond duration of the leading edge to the discharge gap. At present the fact of fast (runaway)-electrons detection can be viewed as firmly established at the initial stage of high-pressure gas breakdown in gaps with strongly non-uniform distribution of the electric field. At the same time, various researchers get fast-electron current pulses with largely spread parameters: amplitudes from 0.1 up to tens of amperes with durations from tens of picoseconds to nanosecond. As the number of runaway electrons is strongly dependent on several critical parameters (gas type, geometric enhancement of the electric field near the sharp edges of electrodes, scales of field-enhancement regions, time resolution of the experimental equipment), the experimental results of fast-electrons detection will also differ considerably.

The parameters of voltage pulse and detection methods are determined by the technical level of the experimental techniques that is very different for the various research groups. Then, the discharge gap geometries are often significantly different for different studies. In the case of a gap with smooth electrodes the electric field can be near-uniform (*e.g.* plane electrodes, sphere-to-plane geometry). The case where the cathode has small curvature radii (sharp-edged tube, thin wires, point-to-plane geometry) is more often used [1].

Inherently the transient and three-dimensional nature of the phenomena in real experiments represents a great challenge for theoretical modeling. Simple zero-dimensional and one-dimensional theoretical models [5,6] allow one to understand the mechanism of the runaway-electron beam formation, but do not provide some proper comparison with the existing experiments.

In addition, a flat one-dimensional model cannot explain the generation of runaway electrons with the so-called

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"anomalous" energies, corresponding to electrons kinetic energies above the maximum possible value eU_{max}, where U_{max} is the maximum amplitude of the applied voltage. One of the suggestions of how to explain these "anomalous" energies is that fast electrons gain the kinetic energy while moving synchronously within the local regions of enhanced electric-field movement [7,8].

The coaxial discharge gap model can be thought of as a compromise variant due to the fact that it combines mathematical one-dimensionality with the real spatial inhomogeneity of electric field and discharge plasma. In this model it is possible to adopt such important parameter from real experiments as degree of heterogeneity of electric field.

In this paper, we have applied a previously developed hybrid theoretical approach (see ref. [5]) to the coaxial geometry of the sub-nanosecond discharge gap in sulfur hexafluoride (SF_6) at atmospheric pressure. This gas has a molecule with high electron affinity that promotes a rapid attachment of free electrons to molecules forming stable negative ions. The attachment of free electrons leads to a significant increase of the static breakdown value of the reduced electric-field strength. At atmospheric pressure it equals 89 kV/cm, which is more than twice the level of the breakdown field in pure nitrogen (32 kV/cm). Moreover, the conversion of the free-electron conductivity of plasma into a ionic one will drastically reduce the conductivity of the plasma column allowing to maintain a relatively high electric-field strength. All of this improves the probability of the transition of electrons into a continuous-acceleration state, in spite of the fact that the complex molecule SF_6 has a relatively large elastic cross-section. So a high field strength allows an appreciable amount of fast-electrons generation [9].

This paper deals with detailed results of a theoretical modeling of the fast-electrons generation probabilistic mechanism during the breakdown in sulfur hexafluoride. The comparison of these results with the experimental data obtained for similar discharge conditions gives good agreement both qualitatively and quantitatively.

Hybrid model of gas discharge with runaway-electrons flow. – The main physical approximations of the discharge model and the corresponding computational method of the runaway-electrons distribution function have been previously described with respect to the one-dimensional planar discharge geometry in nitrogen, and methods for the numerical solution of the corresponding equations were also described in our paper [5]. Here, we have applied our simulation approach to the one-dimensional axisymmetric geometry of a discharge gap filled with electronegative gas.

Description of the discharge dynamics. Since in all calculations all significant processes occur at characteristic sub-nanosecond time scales, we have neglected the motion of ions in the mathematical description of current transporting (additional calculations show that taking into account ion motion will not lead to any significant changes in the discharge dynamics at nanosecond time scales).

In the discharge column production and attachment of electrons are described using the continuity equation for electrons density n_e in the drift-diffusion approximation:

$$\frac{\partial n_e}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left(w_e n_e - D_e\frac{\partial n_e}{\partial r}\right)\right\} + (\alpha-\eta)\,w_e n_e. \tag{1}$$

Here, $w_e(E)$ and D_e are electrons drift velocity and scalar diffusivity coefficient, respectively, $E(r)$ is the electric-field strenght, $\alpha(E)$ and $\eta(E)$ are the Townsend and attachment coefficients as functions of the local electric-field strength. All of these parameters for the electrons kinetics calculations in SF_6 were taken from paper [10].

The evolution of the electric-field strength was calculated from the total current $J(t)$ law of conservation in the 1D-axisymmetric case:

$$\varepsilon_0\frac{\partial E}{\partial t} = \frac{J(t)}{2\pi r} + e\left(w_e n_e - D_e\frac{\partial n_e}{\partial r}\right), \tag{2}$$

where e is the elementary charge and ε_0 is the vacuum absolute dielectric permittivity.

The running current $J(t)$ per unit length has been calculated from the Kirchhoff's equivalent equation for the discharge circuit including the source of voltage $U_0(t)$, ballast resistor R and coaxial gas-filled diode connected in series (diode length is L, the cathode radius is r_c, and the anode radius is $r_a > r_c$):

$$J(t) = \frac{1}{LR}\left(U_0(t) - \int\limits_{r_c}^{r_a} E(r,t)\mathrm{d}r\right). \tag{3}$$

The system of equations (1)–(3) describes the evolution of the initial distribution for given external pulsed-voltage source waveform $U_0(t)$. We set the initial low level of electrons density equal to $n_e(r,0) = n_0(r)$ and initial electric field to zero.

While ions are fixed in our gas breakdown model to control the spatial structure of the plasma discharge, we need to calculate their number density too. Here, n_p is the number densities of positive ions and n_n is the number density of negative ions of the SF_6 molecule. Both were calculated from Poisson's equation and the continuity equation, respectively:

$$\begin{aligned} n_p &= \tfrac{\varepsilon_0}{e}\tfrac{1}{r}\tfrac{\partial(rE)}{\partial r} + n_e + n_n, \\ \tfrac{\partial n_n}{\partial t} &= \eta(E) w_e n_e. \end{aligned} \tag{4}$$

Description of runaway electrons. The runaway-electron dynamics is described as a collisionless transport in the previously computed electric field $E(r,t)$, and any possible collisions lead to the loss of runaway electrons. The main points of the model are formulated as follows: 1) the amount of runaway electrons is small, and we can neglect their influence on the evolution of the gas discharge processes; 2) secondary plasma electrons have Maxwellian distribution function at the time of birth; 3) plasma electrons gain a directed velocity in a known electric field, and 4) the first collision of a runaway electron with the molecule leads to a loss.

For runaway electrons in atmospheric-pressure SF_6 the relativistic Boltzmann kinetic equation was used:

$$\gamma\left(\frac{\partial f}{\partial t}+\frac{p}{m\gamma}\cdot\frac{\partial f}{\partial r}-eE\cdot\frac{\partial f}{\partial p}\right)=G-V. \qquad (5)$$

Here, $f(r,p,t)$ is the spatial non-uniform momentum distribution function of runaway electrons; m is the electron mass, $p=mv\gamma$ is the electron momentum (while v is the runaway-electron velocity), $\gamma=\sqrt{1+(p/mc)^2}$ is a relativistic factor, c is the light velocity. In the right-hand side of (5) the electron rate expression G, and the loss term V of runaway electrons are given in the following forms:

$$\begin{aligned} G(r,p,t) &= \alpha(E)w_e(E)n_e(r,t)\cdot f_{0e}(p),\\ V(f,p) &= f(r,p,t)\cdot n_g v\sigma^*(p). \end{aligned} \qquad (6)$$

Here, $f_{0e}(p)$ is the initial distribution function supposed to be Maxwellian, n_g is the gas molecule number density, $\sigma^*(p)$ is the momentum-dependent transport electrons cross-section [11] with asymptotic extrapolation at high energies.

Simulation results. – Typical results of the discharge in atmospheric-pressure SF_6 simulation are given in figs. 1 and 2. The applied voltage pulse $U_0(t)$ from an external source has an amplitude of 200 kV with a duration of 1 ns at the pulse edge. We usually study the process of multi-electron initiation of gas breakdown, assuming an initial number density of electrons equal to $10^3\,\mathrm{cm}^{-3}$.

In order to simulate how do transmission line affects the discharge in experiments, we included the ballast resistor $R = 75\ \Omega$ in the equivalent circuit between diode and voltage source. Here, the total current in the discharge circuit (see expression (3)) is taken per unit length of the diode $L = 1$ cm. The coaxial diode has an external radius (anode) $r_a = 10$ mm, and an inner radius (cathode) $r_c = 0.5$ mm. Thus, we investigated the breakdown in the diode with high degree of initial electric-field inhomogeneity, $E(r_c)/E(r_a) = 20$.

Since in our model runaway electrons are the electrons that do not collide, we observe a wide energy range at the anode plane. Most of them are plasma electrons with low and middle energy. In order to calculate only the fast-electron spectrum and its current at the anode we added a foil filter that cuts off the low-energetic part (< 10 keV) of the electron beam. The attenuation factor of the filter corresponds to the thickness of the aluminum foil of 10 μm [12]. The current pulse of runaway electrons at the anode beyond the aluminum filter is also shown in fig. 1.

Figure 2 shows the spatio-temporal electric-field distributions at the breakdown stage. The instantaneous voltage drops at the diode are labeled at curves in accordance with the curve 2 in fig. 1.

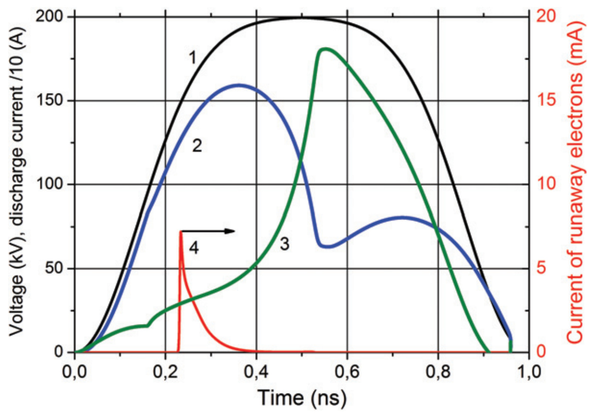

Fig. 1: (Colour online) Time profile of pulses: voltage at power source (1), diode voltage drop (2), total discharge current (3), and fast-electrons current (4).

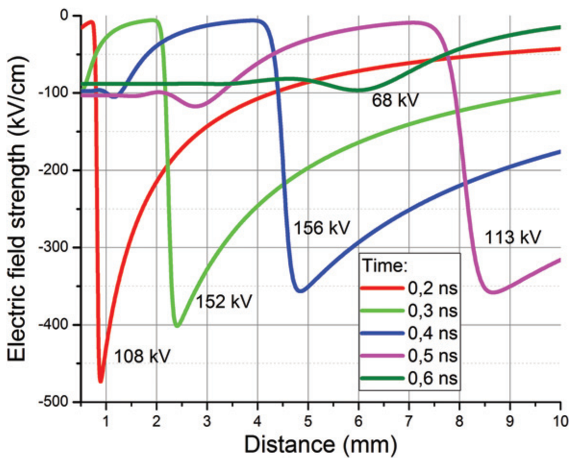

Fig. 2: (Colour online) The electric-field spatial distribution in the discharge gap for different time points after the arrival of the voltage pulse to the coaxial diode.

Usually, in the discharge gap the time interval from the instant of voltage pulse arrival to the beginning of the voltage decay at the diode is called the current switching time. The current switching in coaxial diode takes a longer time than in planar diodes at the same overvoltage level. Therefore, the voltage pulse at the coaxial diode has a smooth-bell shape, while the current-voltage pulse at the planar diode had a much more acute maximum at the very steep pulse trailing edge [5]. At breakdown the gap filling with plasma in the coaxial diode occurs spatially inhomogeneous. One can talk about the propagation of ionization waves during the gap breakdown in non-uniform geometry. It is clearly seen from fig. 2, whereas the movement of the wave front is reflected in the movement of the electric-field maximum.

The distribution of the electric-field strength ahead the ionization wave front is almost the same as in a vacuum

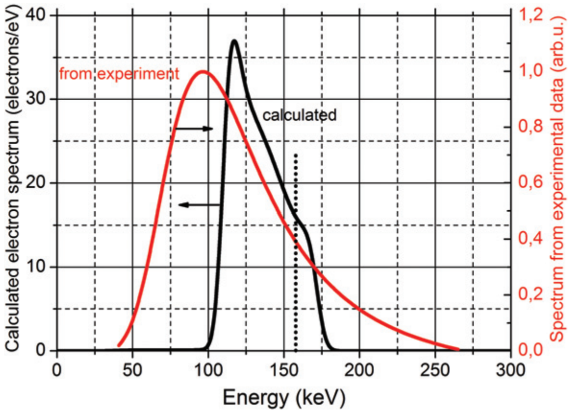

Fig. 3: (Colour online) Fast-electrons spectrum behind a 10 μm aluminum foil calculated and restored from experimental data. The vertical dotted line shows the amplitude level of the diode voltage U_{max}.

diode with appropriate electrodes radii. The current in this area is almost entirely capacitive. Directly behind the ionization wave front the electric-field strength falls almost to zero, as the electronic conductivity of plasma is very high. But closer to the cathode free electrons the number density greatly decreases due to the attachment to the electronegative gas molecules, and the conductivity becomes partially ionic. Therefore, the electric-field strength increases to approximately 100 kV/cm corresponding to a static breakdown voltage for SF_6 at atmospheric pressure. Such average conductivity decrease of the discharge plasma causes the appearance of a second peak at the voltage graph for the diode in fig. 1.

The most interesting results are related to runaway-electrons calculation. The runaway-electrons current pulse is also shown in fig. 1. As can be seen from figs. 1 and 2, the fast-electron current pulse starts and ends before the ionization front reaches the anode. As it was expected, reducing the field strength in the area of emissions by increasing the radius of the ionization wave front leads to a significant decrease of the fast-electrons number. Therefore one can conclude that the acute peak of fast-electrons current takes place at the initial stage of the breakdown due to electrons gain in the strong field near the cathode.

For possible applications of high-voltage discharges the power spectrum of fast electrons is the most important. The calculated integral (per pulse) of the fast-electron spectrum is shown in fig. 3 with a black line.

The first thing that attracts the attention is the existence of an electrons group whose energy exceeds the "amplitude" value $eU_{\max}$. Such electrons are known to be electrons with "anomalous" energy. In the test conditions the "anomalous" electrons fraction is equal to about 10%.

According to our calculations, the nature of "anomalous" energy, observed for some fraction of runaway electrons, is fully consistent with the mechanism of polarization acceleration, previously described in [7,8]. To assess the effect of this contribution in our situation let us consider the following example. The electron gains a typical kinetic energy equal to $W_0 = mc^2(\gamma_0 - 1)$ in the spatially limited region $0 < x < l$ with stationary electric field $E(x)$. Thus, $W_0 = e\int E(x)\mathrm{d}x$. If the localized region of longitudinally non-uniform electric field moves in space relative to the laboratory system of coordinates with constant speed $u < v_0 = c\sqrt{\gamma_0^2 - 1}/\gamma_0$ (for simplicity assume it $u \ll c$, c is light velocity), then the electron's kinetic energy in the laboratory system of coordinates W after acceleration will be equal to

$$W \approx W_0 \left(1 + \frac{u}{c}\sqrt{1 + \frac{2mc^2}{W_0}}\right). \qquad (7)$$

From this standpoint, we consider the acceleration of electrons with respect to the situation shown in fig. 2. As shown in fig. 1, the fast-electron current peak corresponds to the time point when the voltage drop at the diode is approximately 125 kV. From fig. 2 it is clearly seen that the velocity of the enhancement field region in discharge development changes from $\sim 1.5 \cdot 10^9$ to $\sim 3.8 \cdot 10^9$ cm/s, *i.e.* an average of 10% of the speed of light in vacuum. Therefore, according to formula (7) it turns out that the considered group of electrons should be accelerated to an energy of 163 keV. This energy is significantly higher than the instant applied voltage (125 kV) Later, for higher anode voltages the runaway electrons will gain truly "anomalous" energies corresponding to the spectrum in fig. 3.

Comparison with experimental data. – We understand that the exact comparison of one-dimensional simulation results with real three-dimensional experiments is difficult. Therefore, our experiments were performed in conditions that are most similar to the proposed model situation.

The RADAN-220 generator was able to create a voltage pulse of an amplitude up to 250 kV with 0.5 ns rise time in idle running mode. The diode breakdown occurred at the front-edge of the voltage pulse. The discharge gap with strongly non-uniform spatial distribution of the electric field was connected to the end of the transmission line with 100 Ω wave impedance. The discharge gap cathode was made of 100 μm stainless-steel foil as a hollow tube with a diameter of 6 mm. At a distance of 8 mm from the tube end a flat 10 μm thick aluminum foil anode was placed.

We placed the low-inductance collector of 20 mm diameter behind the foil anode, and its output signal was send to the DPO70604 real-time oscilloscope (6 GHz, 25 Gs/s). In our experiments we also measured the discharge current waveform and the voltage pulse using the capacitive voltage divider connected inside the transmission line near the discharge gap. This allowed obtaining the actual waveform of the voltage pulse across the diode. This experimental technique was described in detail in [9].

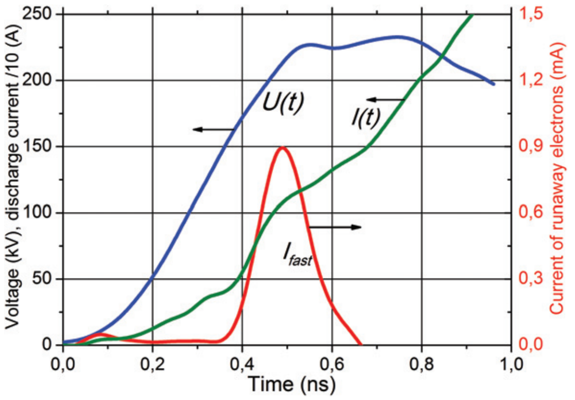

Fig. 4: (Colour online) The experimental dependencies *vs.* time for voltage across the diode, discharge current and fast-electron current.

Figure 4 shows the time dependencies of discharge current, diode voltage drop and current of the fast-electron beam recorded in experiments.

The oscilloscope bandwidth is equal to 6 GHz so the length of the recorded pulse at half-maximum slightly exceeds 80 ps. That is why the actual pulse can be considerably shorter than depicted in fig. 4. In this case, the total number of fast electrons in pulses is more illustrative for further comparison than current amplitudes In the experiment, the fast-electron current duration at half-maximum of 125 ps and the amplitude of 0.9 mA provide an estimate of $\sim 6 \cdot 10^5$ electrons total number. A similar method was applied to the fast-electron current pulse shown in fig. 1 (pulse half-maximum duration is 25 ps at 7 mA amplitude) providing an estimated of $\sim 11 \cdot 10^5$ electrons total number. Taking into account the essential difference in discharge gaps geometries we found the agreement to be perfect.

From the experimental beam attenuation data and [12] the power spectra of fast electrons were reconstructed using the same method described in [13]. The reconstructed result is shown in fig. 3 using arbitrary units. Taking into account the resolution features of the recovery method [13] we found the agreement between calculations and experiment to be satisfactory also.

Our result for the ionization front velocity also fits the experimental data. The methods for using the experimental data on the space-time picture of the light emission plasma to determine the ionization front velocity are described in detail in [14]. In the breakdown of the atmospheric-pressure mixture $SF_6 : N_2 = 39 : 1$ (without N_2 admixture, the SF_6 plasma glow was not enough for its accurate registration in the breakdown phase) the ionization wave velocities were determined to be $\sim 3.3 \cdot 10^9$ and $\sim 1.3 \cdot 10^{10}$ cm/s at wave propagation from the tube cathode to the middle of the gap and from the middle of the gap to the plane anode, respectively. Let us recall that in the simulation we obtained the ionization front velocities $\sim 1.5 \cdot 10^9$ near the cathode and $\sim 3.8 \cdot 10^9$ cm/s close to the anode.

Conclusions. – The following conclusions can be drawn from this work:

The numerical hybrid model has been successfully extended to one-dimensional axisymmetric discharge gap geometry. Despite the one-dimensionality, it is possible to take into account one of the main features of real discharges —their significant spatial heterogeneity. Therefore, such problem is sometimes referred to as a 1.5-dimensional one.

To simulate the plasma discharge components, we have used the hydrodynamic approximation, and used the kinetic Boltzmann equation for runaway-electrons simulation. We solved the relativistic Boltzmann kinetic equation in order to consider fast electrons with relativistic velocities. This allows simulating discharges for arbitrary values of applied voltages up to several hundreds of kilovolts.

As the demo, we have simulated the generation of runaway electrons in the high-voltage breakdown of sulfur hexafluoride at atmospheric pressure. For the discharge with three-component plasma kinetics we are able to efficiently compare simulation results with real experiments data.

For the first time in a theoretical model based on partial differential equations without any adjustable parameters the correct assessment of runaway-electrons number was obtained for the discharge in sulfur hexafluoride under atmospheric pressure. Other observable the discharge and fast-electrons beam characteristics (*e.g.* duration of the switching stage, power spectrum of fast electrons, ionization wave velocity, etc.) are also fits results of the theoretical simulation Hence, we can be sure that other effects occur.

Thus, in discharges with sharply non-uniform geometry the plasma fills the discharge gap likewise the ionization front expansion from local field enhancement regions. Then an appreciable part of the fast-electrons ensemble gains an energy excess value eU_{max}, *i.e.*, the electrons gain "anomalous" energies.

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