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SYSTEM ANALYSIS AND OPERATIONS RESEARCH

Comparative Analysis of the Performance of Selective and Group Repeat Transmission Modes in a Transport Protocol

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Abstract—We propose a model of a virtual connection controlled by a transport protocol in the selective and group failure modes as a Markov chain with discrete time that accounts for the influence of protocol parameters of window size and timeout duration for waiting for acknowledgements, probabilities of distorting segments in individual links of the data transmission path on the throughput of a transport connection. We have analyzed how the throughput of the control procedure depends on protocol parameters, level of errors in communication channels, and round-trip delay. We have proposed a method for choosing protocol parameters.

Keywords: transport protocol, data transmission path, Markov chain, virtual connection performance, window size, timeout duration.

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1. INTRODUCTION

One of the most important parameters for the quality of interaction between network applications and the software and hardware of computer networks is the throughput of transport connections. This operational characteristic is to a significant extent determined by the transport protocol and its parameters: window size and timeout duration [1, 2]. Modeling user connections and analysis of its potential capabilities have been carried out, e.g., in [2–6]. However, results have been obtained either only for a single-link data transmission path [2–5] or under significant restrictions on protocol parameters [6]. Modern transport protocols offer a wide variety of congestion control mechanisms [7]. A wide spectrum of studies [7–21] in the field of control over transport protocol parameters has been conducted in order to prevent and alleviate congestions, oriented towards constructing congestion diagnostics models with various indicators [7] and adapting protocol parameters to changing network load and connectivity, level of losses, activity of interacting users and so on. Here implementation of controlling mechanisms to alleviate congestions is based on the available bandwidth for transport connections under current values of protocol parameters and predictions of changed values. However, potential capabilities of the transport protocol have still not been studied; there are no analytic results on how protocol parameters, repeat modes for lost segments, characteristics of the data transmission path, and competition between user communication instances over the bandwidth of shareable network channels influence the resulting operational characteristics of the transport connection. It has not been studied how relations between round-trip delay and protocol parameters influence the throughput of a data transmission path controlled by a transport protocol. Besides, data transmission processes in computer networks are of a significantly discrete nature [22] caused by the pipeline transfer mechanism in the network segments of bounded size and applying algorithms with decision feedback on various levels of the network architecture; however, most results [2–4, 7–20] are based on models with continuous time which reduces their applicability.

In this work, we propose a mathematical model for the data transmission process in the information transport phase in the form of a discrete time Markov chain (Section 2), analytically compute stationary distributions of state probabilities for selective and group failure modes [2] (Sections 3 and 4), obtain analytic relations for the throughput and compare based on these relations the potential capabilities of transport connections in various failure modes (Section 5), and find expressions for reasonable values of protocol parameters, namely window size and timeout duration (Section 6).

2. THE TRANSPORT CONNECTION MODEL

Let us consider the process of transferring data between users of a transport protocol based on an algorithm with decision feedback [2] which operates in either selective or group failure mode. A sample family of such reliable protocols is given by the TCP protocol which dominates modern computer networks [1]. In the selective failure mode, only segments that have not been successfully received are subject to repeat sending; in the group failure mode, all packets starting from the first one that has not been received are resent [2]. We assume that interacting users have an unbounded flow of data for transmission, and exchange is done with protocol blocks of data defined in the transport protocol of identical size (segments). The receiver's acknowledgements that confirm correct reception of the data are carried over in segments of the opposite flow. We assume that re-reception links along the data transmission path have identical performance in both directions, and duration of the transmission loop for a segment in an individual link is t. In the general case, the length of path from source to destination that carries the information flow and the length of the reverse path that carries acknowledgements for received segments may be different. We assume that the length of the data transmission path expressed as the number of re-reception links in the forward direction equals $D_{\mathrm{f}} \geqslant 1$. The reverse path, which carries acknowledgements for the sender that a sequence of data segments has been correctly received, has length $D_{\mathrm{r}} \geqslant 1$. We also know the probabilities of distorting a segment in communication channels $R_{\mathrm{f}}(d)$, $d = \overline{1, D_{\mathrm{f}}}$, for the forward transmission direction and $R_{\mathrm{r}}(d)$, $d = \overline{1, D_{\mathrm{r}}}$, for the reverse direction, for each re-reception segment. Then the fidelity of transmitting a segment of data along the path from sender to receiver and back are $F_{\mathrm{f}} = \prod_{d=1}^{D_{\mathrm{f}}}(1 - R_{\mathrm{f}}(d))$ and $F_{\mathrm{r}} = \prod_{d=1}^{D_{\mathrm{r}}}(1 - R_{\mathrm{r}}(d))$ respectively. We assume that there are no losses of segments due to the lack of buffer memory at the nodes of the path. Control over the data flow is implemented with a sliding window mechanism [1, 2] with window size $W \geqslant 1$ (a protocol parameter). The information transfer process in the virtual connection controlled by the transport protocol can be described with a discrete time Markov process (with tick duration t) since the time between reception of acknowledgements has geometric distribution with parameter F_{r}. This model generalizes the formalizations of the data transmission process proposed in [3–6] for the case of a transport connection of arbitrary length. The space of possible states of the Markov chain is determined by the timeout duration S for waiting for the acknowledgement expressed in terms of the number of ticks of time t. The timeout duration is related to the length of the path and window size by inequalities $S > W$, $S \geqslant D_{\mathrm{f}} + D_{\mathrm{r}}$. Obviously, the total length of the direct and reverse paths can be interpreted as the round-trip delay $D = D_{\mathrm{f}} + D_{\mathrm{r}}$ expressed in cycles t (disregarding losses of protocol blocks when transmitting along the path). States of the Markov chain $i = \overline{0, W}$ correspond to the size of the queue of transmitted but not yet confirmed segments in the flow source, and states $i = \overline{W + 1, S - 1}$ correspond to the time during which the sender is not active and is waiting for acknowledgement of correct reception of the sent sequence of W segments. From the zero state to the $(D - 1)$th, the source moves with each tick t with the probability of a deterministic event. In states $i \geqslant D - 1$, after another discrete cycle t is over, the sender begins to receive acknowledgements, and depending on delivery results the sender transmits new segments (after positive acknowledgement) or repeats distorted ones. When the loop in state $D-1$ ends, this corresponds to the time when the first segment reaches the receiver, and an acknowledgement for it arrives. The state number grows further with the probability of distorting an acknowledgement $1-F_{\mathrm{r}}$ in the reverse path. In states $i \geqslant D-1$, in the selective failure mode getting an acknowledgement generates a transition to the $(D-1)$th state for $W \geqslant D$ or to state $D+W-2-i$ for $W \leqslant D$. In the group failure mode, for the original states $i \geqslant D-1$ we return to states $D-1$ (for $W \geqslant D$) or $D+W-2-i$ (for $W \leqslant D$) when an acknowledgement is received only in case when the receiver successfully receives the data arrived until moment $i-D+1$, otherwise the system returns to zero state, since the queue of transmitted but not yet confirmed segments at this moment is emptied. Since in states $i \geqslant W$ the source temporarily stops sending segments, receiving acknowledgements in states $i=\overline{W, D+W-3}$ leads to a transition to states $D+W-2-i$, and from states $i=\overline{D+W-2, S-2}$ to the zero state. This holds for selective failures, and in the group failure mode the said changes in the states occur when positive acknowledgements arrive. In state $S-1$ the timeout of waiting for acknowledgement from the receiver regarding the correctness of received segments ends, and the system unconditionally transitions into the zero state in all failure modes.

3. STATE PROBABILITIES FOR THE SELECTIVE FAILURE MODE

Transition probabilities π_{ij} from the original state i to the resulting state j in the Markov chain that describes the process of transmitting an information flow in the selective failure mode have the form

$$\pi_{ij}=\begin{cases}1, & i=\overline{0, D-2}; \quad j=i+1 \\ 1-F_{\mathrm{r}}, & i=\overline{D-1, S-2}; \quad j=i+1 \\ F_{\mathrm{r}}, & i=\overline{D-1, W-1}; \quad W \geqslant D; \quad j=D-1 \\ F_{\mathrm{r}}, & i=\overline{D-1, D+W-3}; \quad W \leqslant D; \quad j=D+W-2-i \\ F_{\mathrm{r}}, & i=\overline{W, D+W-3}; \quad W \geqslant D; \quad j=D+W-2-i \\ F_{\mathrm{r}}, & i=\overline{D+W-2, S-2}; \quad j=0 \\ 1, & i=S-1; \quad j=0.\end{cases}$$

The diversity of the forms of solutions for the system of equilibrium equations for state probabilities in the Markov chain is defined by the relations between protocol parameters W, S and the total path length D. Since the timeout duration must exceed window size and must not be shorter than the round-trip delay ($S \geqslant D$), there are four possible solutions for different domains of changing protocol parameters.

For protocol parameters related to the total path length by inequalities of the form

$$W \geqslant D, \quad S \geqslant D+W-1, \tag{1}$$

the system of equilibrium equations can be written as follows:

$$P_0=P_{S-1}+F_{\mathrm{r}} \sum_{i=D+W-2}^{S-2} P_i, \tag{2}$$

$$P_i=P_{i-1}+F_{\mathrm{r}} P_{D+W-2-i}, \quad i=\overline{1, D-2}, \tag{3}$$

$$P_{D-1}=P_{D-2}+F_{\mathrm{r}} \sum_{i=D-1}^{W-1} P_i, \tag{4}$$

$$P_i=P_{i-1}(1-F_{\mathrm{r}}), \quad i=\overline{D, S-1}. \tag{5}$$

Due to the normalization condition for the solution of this system, we define the following relations:

$$\begin{aligned} &P_i = P_0(1-F_\mathrm{r})^{-i}, \quad i=\overline{0,D-2},\\ &P_i = P_0(1-F_\mathrm{r})^{i-D-W+2}, \quad i=\overline{D-1,S-1},\\ &P_0 = \frac{F_\mathrm{r}(1-F_\mathrm{r})^{W-1}}{1-(1-F_\mathrm{r})^{W}+(1-F_\mathrm{r})^{W-D+1}\left[1-(1-F_\mathrm{r})^{S-W}\right]}. \end{aligned} \tag{6}$$

If the window size W exceeds the total data transmission path length, and the domain of timeout duration values S has interval constraints

$$W \geqslant D, \quad W+1 \leqslant S \leqslant D+W-1, \tag{7}$$

Eqs. (2) and (3) for states $i=\overline{0,D-2}$ can be transformed to

$$\begin{aligned} &P_0 = P_{S-1},\\ &P_i = P_{i-1}, \quad i=\overline{1,D+W-S-1},\\ &P_i = P_{i-1}+F_\mathrm{r}P_{D+W-2-i}, \quad i=\overline{D+W-S,D-2}, \end{aligned} \tag{8}$$

and probabilities of Markov chain states take the form

$$\begin{aligned} &P_i = P_0, \quad i=\overline{0,D+W-S-1},\\ &P_i = P_0(1-F_\mathrm{r})^{D+W-1-S-i}, \quad i=\overline{D+W-S,D-2},\\ &P_i = P_0(1-F_\mathrm{r})^{i+1-S}, \quad i=\overline{D-1,S-1},\\ &P_0 = \frac{F_\mathrm{r}(1-F_\mathrm{r})^{S}}{(1-F_\mathrm{r})^{D}+(1-F_\mathrm{r})^{W+1}+(1-F_\mathrm{r})^{S}\left[F_\mathrm{r}(D+W-S+1)-2\right]}. \end{aligned} \tag{9}$$

Under constraints

$$1 \leqslant W \leqslant D, \quad S \geqslant D+W-1 \tag{10}$$

equilibrium Eqs. (3) and (4) can be rewritten as

$$\begin{aligned} &P_i = P_{i-1}+F_\mathrm{r}P_{D+W-2-i}, \quad i=\overline{1,W-1},\\ &P_i = P_{i-1}, \quad i=\overline{W,D-1}. \end{aligned}$$

State probabilities here have a subset ($i=\overline{W,D-1}$) of values invariant to the state index:

$$\begin{aligned} &P_i = P_0(1-F_\mathrm{r})^{-i},\ i=\overline{0,W-1}, \qquad P_i = P_0(1-F_\mathrm{r})^{-W+1},\ i=\overline{W,D-1},\\ &P_i = P_0(1-F_\mathrm{r})^{i-D-W+2}, \quad i=\overline{D,S-1},\\ &P_0 = \frac{F_\mathrm{r}(1-F_\mathrm{r})^{W-1}}{2+F_\mathrm{r}(D-W-1)-(1-F_\mathrm{r})^{W}-(1-F_\mathrm{r})^{S-D+1}}. \end{aligned} \tag{11}$$

In case of interval constraints on both protocol parameters

$$1 \leqslant W \leqslant D, \quad \max\{W+1,D\} \leqslant S \leqslant D+W-1 \tag{12}$$

Eq. (2) takes the form (8), and Eqs. (3) and (4) are transformed to the following:

$$\begin{aligned} &P_i = P_{i-1}, \quad i=\overline{1,D+W-S-1},\overline{W,D-1},\\ &P_i = P_{i-1}+F_\mathrm{r}P_{D+W-2-i}, \quad i=\overline{D+W-S,W-1}. \end{aligned}$$

A solution of the system of local equilibrium equations will be determined by relations with two subsets ($i=\overline{0,D+W-S-1},\ \overline{W,D-1}$) of values of state probabilities independent of the state index:

$$
\begin{aligned}
&P_i = P_0, \quad i=\overline{0,D+W-1-S},\\
&P_i = P_0(1-F_{\mathrm{r}})^{D+W-1-S-i}, \quad i=\overline{D+W-S,W-1},\\
&P_i = P_0(1-F_{\mathrm{r}})^{D-S}, \quad i=\overline{W,D-1},\\
&P_i = P_0(1-F_{\mathrm{r}})^{i+1-S}, \quad i=\overline{D,S-1},\\
&P_0 = \frac{F_{\mathrm{r}}(1-F_{\mathrm{r}})^S}{(1-F_{\mathrm{r}})^D\left[F_{\mathrm{r}}(D-W-1)+2\right]+(1-F_{\mathrm{r}})^S\left[F_{\mathrm{r}}(D+W-S+1)-2\right]}.
\end{aligned}
\tag{13}
$$

Thus, stationary distribution of state probabilities in the Markov chain for different relations between window size W, timeout duration S, and total length of the data transmission path D in (1), (7), (10) and (12) is defined by relations (6), (9), (11) and (13) respectively. It is easy to see from the said solutions that they are joined on the boundaries of domains of protocol parameters: window size W and timeout duration S. For minimal timeout duration ($S=D$) the Markov chain states are equiprobable and invariant to window size: $P_i=\frac{1}{D}$, $i=\overline{0,D-1}$. If $S=D+1$, then state probabilities are determined by two sets with uniform distribution of values:

$$
\begin{aligned}
P_i &= \frac{1-F_{\mathrm{r}}}{1+D-F_{\mathrm{r}}W}, \quad i=\overline{0,W-2},D,\\
P_i &= \frac{1}{1+D-F_{\mathrm{r}}W}, \quad i=\overline{W-1,D-1}.
\end{aligned}
$$

For a perfectly reliable reverse data transmission path ($F_{\mathrm{r}}=1$), the entire probability mass is either uniformly distributed among the states $i=\overline{W-1,D-1}$ ($P_i=\frac{1}{1+D-W}$, $i=\overline{W-1,D-1}$; $1\leqslant W\leqslant D$) or concentrated in state $D-1$ ($P_{D-1}=1$, $W\geqslant D$).

4. STATE PROBABILITIES FOR THE GROUP FAILURE MODE

Let us consider the group failure mode. Transition probabilities of the Markov chain that defined the dynamics of the queue of segments in the source awaiting for acknowledgement are defined as follows:

$$
\pi_{ij} = \begin{cases}
1, & i=\overline{0,D-2}, \quad j=i+1\\
1-F_{\mathrm{r}}, & i=\overline{D-1,S-2}, \quad j=i+1\\
F_{\mathrm{r}}F_{\mathrm{f}}^{i-D+2}, & i=\overline{D-1,W-1}, \quad W\geqslant D, \quad j=D-1\\
F_{\mathrm{r}}(1-F_{\mathrm{f}}^{i-D+2}), & i=\overline{D-1,W-1}, \quad W\geqslant D, \quad j=0\\
F_{\mathrm{r}}F_{\mathrm{f}}^{i-D+2}, & i=\overline{D-1,D+W-3}, \quad W\leqslant D, \quad j=D+W-2-i\\
F_{\mathrm{r}}(1-F_{\mathrm{f}}^{i-D+2}), & i=\overline{D-1,D+W-3}, \quad W\leqslant D, \quad j=0\\
F_{\mathrm{r}}, & i=\overline{W,D+W-3}, \quad W\geqslant D, \quad j=D+W-2-i\\
F_{\mathrm{r}}, & i=\overline{D+W-2,S-2}, \quad j=0\\
1, & i=S-1, \quad j=0.
\end{cases}
$$

Similar to the case of selective failure mode, solution of the system of equilibrium equations in the group failure mode for different domains of admissible values of protocol parameters has four analytic versions.

For protocol parameters bounded only from below (1), the system of local equilibrium equations has the following form:

$$P_0 = P_{S-1} + F_{\mathrm{r}} \left\{ \sum_{i=D-1}^{D+W-3} P_i (1 - F_{\mathrm{f}}^{i-D+2}) + \sum_{i=D+W-2}^{S-2} P_i \right\}, \tag{14}$$

$$P_i = P_{i-1} + F_{\mathrm{r}} P_{D+W-2-i} F_{\mathrm{f}}^{D+W-2-i}, \quad i = \overline{1, D-2}, \tag{15}$$

$$P_{D-1} = P_{D-2} + F_{\mathrm{r}} \sum_{i=D-1}^{W-1} P_i F_{\mathrm{f}}^{i-D+2}, \tag{16}$$

$$P_i = P_{i-1}(1 - F_{\mathrm{r}}), \quad i = \overline{D, S-1}. \tag{17}$$

State probabilities found from this system are defined by the following relations:

$$\begin{aligned} P_i =& P_0 \frac{1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{W-1-i}}{1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{W-1}}, \quad \Phi = F_{\mathrm{f}}(1 - F_{\mathrm{r}}), \quad i = \overline{0, D-2}, \\ P_i =& P_0 \frac{(1 - \Phi)(1 - F_{\mathrm{r}})^{i-D+1}}{1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{W-1}}, \quad i = \overline{D-1, S-1}, \\ P_0 =& F_{\mathrm{r}}(1 - \Phi) \left[1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{W-1} \right] \Big/ \Big\{ (D-1)(1 - F_{\mathrm{f}}) F_{\mathrm{r}} (1 - \Phi) \\ & + F_{\mathrm{f}} F_{\mathrm{r}}^2 \left(\Phi^{W-D+1} - \Phi^W \right) + (1 - \Phi)^2 \left[1 - (1 - F_{\mathrm{r}})^{S-D+1} \right] \Big\}. \end{aligned} \tag{18}$$

Domains of values of protocol parameters (7) lead to the following change in Eqs. (14) and (15):

$$\begin{aligned} & P_0 = P_{S-1} + F_{\mathrm{r}} \sum_{i=D-1}^{S-2} P_i (1 - F_{\mathrm{f}}^{i-D+2}), \\ & P_i = P_{i-1}, \quad i = \overline{1, D+W-S-1}, \\ & P_i = P_{i-1} + F_{\mathrm{r}} P_{D+W-2-i} F_{\mathrm{f}}^{D+W-2-i}, \quad i = \overline{D+W-S, D-2}. \end{aligned} \tag{19}$$

A solution of the resulting system of equilibrium equations now has the following form:

$$\begin{aligned} P_i =& P_0, \quad i = \overline{0, D+W-S-1}, \\ P_i =& P_0 \frac{1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{W-1-i}}{1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{S-D}}, \quad i = \overline{D+W-S, D-2}, \\ P_i =& P_0 \frac{(1 - \Phi)(1 - F_{\mathrm{r}})^{i-D+1}}{1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{S-D}}, \quad i = \overline{D-1, S-1}, \\ P_0 =& F_{\mathrm{r}}(1 - \Phi) \left[1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{S-D} \right] \Big/ \Big\{ (D+W-S) F_{\mathrm{r}} (1 - \Phi) \\ & \times \left[1 - F_{\mathrm{f}} + F_{\mathrm{f}} F_{\mathrm{r}} \Phi^{S-D} \right] + (S-W-1) F_{\mathrm{r}} (1 - F_{\mathrm{f}})(1 - \Phi) \\ & + F_{\mathrm{f}} F_{\mathrm{r}}^2 \left(\Phi^{W-D+1} - \Phi^{S-D} \right) + (1 - \Phi)^2 \left[1 - (1 - F_{\mathrm{r}})^{S-D+1} \right] \Big\}. \end{aligned} \tag{20}$$

Under constraints of the form (10) equilibrium Eqs. (15) and (16) from the original system of equilibrium equations are transformed to the following form:

$$\begin{aligned} & P_i = P_{i-1} + F_{\mathrm{r}} P_{D+W-2-i} F_{\mathrm{r}}^{D+W-2-i}, \quad i = \overline{1, W-1}, \\ & P_i = P_{i-1}, \quad i = \overline{W, D-1}. \end{aligned}$$

A solution of this system is defined by the following relations:

$$
\begin{aligned}
&P_i = P_0\frac{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{W-1-i}}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{W-1}}, \quad i=\overline{0,W-1},\\
&P_i = P_0\frac{1-\Phi}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{W-1}}, \quad i=\overline{W,D-2},\\
&P_i = P_0\frac{(1-\Phi)(1-F_{\rm r})^{i-D+1}}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{W-1}}, \quad i=\overline{D-1,S-1},\\
&P_0 = F_{\rm r}(1-\Phi)\left[1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{W-1}\right]\Big/\Big\{(W-1)F_{\rm r}(1-F_{\rm f})(1-\Phi)\\
&\qquad + F_{\rm f}F_{\rm r}^2\Phi(1-\Phi^{W-1}) + (D-W)F_{\rm r}(1-\Phi)^2 + (1-\Phi)^2\left[1-(1-F_{\rm r})^{S-D+1}\right]\Big\}.
\end{aligned}
\tag{21}
$$

Values of protocol parameters that have interval constraints (12) change Eq. (14) into (19) and change Eqs. (15) and (16) into the following:

$$
\begin{aligned}
&P_i = P_{i-1}, \quad i=\overline{1,D+W-S-1},\overline{W,D-1},\\
&P_i = P_{i-1} + F_{\rm r}P_{D+W-2-i}F_{\rm f}^{D+W-2-i}, \quad i=\overline{D+W-S,W-1}.
\end{aligned}
$$

Then provide a solution in the following form:

$$
\begin{aligned}
&P_i = P_0, \quad i=\overline{0,D+W-S-1},\\
&P_i = P_0\frac{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{W-1-i}}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{S-D}}, \quad i=\overline{D+W-S,W-1},\\
&P_i = P_0\frac{1-\Phi}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{S-D}}, \quad i=\overline{W,D-2},\\
&P_i = P_0\frac{(1-\Phi)(1-F_{\rm r})^{i-D+1}}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{S-D}}, \quad i=\overline{D-1,S-1},\\
&P_0 = F_{\rm r}(1-\Phi)\left[1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{S-D}\right]\Big/\Big\{(D+W-S)F_{\rm r}(1-\Phi)\\
&\qquad \times\left[1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi^{S-D}\right] + (D-W)F_{\rm r}(1-\Phi)^2\\
&\qquad + (S-D)F_{\rm r}(1-F_{\rm f})(1-\Phi) + F_{\rm f}F_{\rm r}^2\left(1-\Phi^{S-D}\right)\\
&\qquad + (1-F_{\rm r})(1-\Phi)^2\left[1-(1-F_{\rm r})^{S-D}\right]\Big\}.
\end{aligned}
\tag{22}
$$

As a result, for four domains of compatible values of window size and timeout duration in (1), (7), (10), and (12) the stationary distribution of state probabilities in the Markov chain for the group failure mode is defined by relations (18), (20), (21), and (22) respectively. For $F_{\rm r}=1$, state probabilities in the Markov chain are transformed to the following form:

$$
\begin{aligned}
&P_i = P_0, \quad i=\overline{0,W-2},\\
&P_i = \frac{P_0}{1-F_{\rm f}}, \quad i=\overline{W-1,D-1},\\
&P_0 = \frac{1-F_{\rm f}}{D-F_{\rm f}(W-1)}.
\end{aligned}
$$

If in these relations for P_i we let $F_{\rm f}=1$, we immediately get the relationships for the selective failure mode. For timeout duration equal to the round-trip delay ($S=D$), Markov chain states, similar to the case of selective failure mode, are equiprobable and invariant to window size: $P_i=\frac{1}{D}$, $i=\overline{0,D-1}$. For $S=D+1$ state probabilities are determined by two uniformly distributed regions of values:

$$P_i = P_0, \quad i=\overline{0,W-2},$$
$$P_i = P_0\frac{1-\Phi}{1-F_{\rm f}+F_{\rm f}F_{\rm r}\Phi}, \quad i=\overline{W-1,D-1},$$
$$P_D = P_0\frac{(1-\Phi)(1-F_{\rm r})}{1-F_{\rm f}+F_{\rm f}F_{\rm f}\Phi}.$$

5. COMPARATIVE ANALYSIS OF THROUGHPUT FOR FAILURE MODES

The most important operational characteristic of a protocol is its throughput, defined by data transmission path parameters, overhead costs, and features of protocol procedures for control over transmission [1, 2]. Normalized performance of a transport connection is defined by the average number of non-distorted segments delivered to the receiver (with regard to the failure mode [2]) during the average time between two consecutive arrivals of acknowledgements [3–6]. Since the time between arrivals of acknowledgements has a geometric distribution with parameter $F_{\rm r}$, the average time between arrivals of acknowledgements over the duration of a cycle t will be $\bar{T}=1/F_{\rm r}$. Then for the selective failure procedure the throughput will be given by

$$Z_c(W,S) = F_{\rm r}\left\{\sum_{i=D-1}^{D+W-2}(i-D+2)F_{\rm f}P_i + WF_{\rm f}\sum_{i=D+W-1}^{S-1}P_i\right\}.$$

Taking into account the variability of expressions for state probabilities in a Markov chain, for different relations between protocol parameters and round-trip delay we obtain the following formulas for this parameter:

$$Z_c(W,S) = \begin{cases} \dfrac{F_{\rm f}\left\{(1-F_{\rm r})^D-(1-F_{\rm r})^{S+1}\left[1+(S-D+1)F_{\rm r}\right]\right\}}{(1-F_{\rm r})^D\left[F_{\rm r}(D-W-1)+2\right]+(1-F_{\rm r})^S\left[F_{\rm r}(D+W-S+1)-2\right]}, \\ \qquad W<D,\ D\leqslant S\leqslant D+W-1 \\ \dfrac{F_{\rm f}\left\{1-(1-F_{\rm r})^W-WF_{\rm r}(1-F_{\rm r})^{S-D+1}\right\}}{2+F_{\rm r}(D-W-1)-(1-F_{\rm r})^W-(1-F_{\rm r})^{S-D+1}}, \\ \qquad W<D,\ S\geqslant D+W-1 \\ \dfrac{F_{\rm f}\left\{(1-F_{\rm r})^D-(1-F_{\rm r})^{S+1}\left[1+(S-D+1)F_{\rm r}\right]\right\}}{(1-F_{\rm r})^D+(1-F_{\rm r})^{W+1}+(1-F_{\rm r})^S\left[F_{\rm r}(D+W-S+1)-2\right]}, \\ \qquad W\geqslant D,\ W+1\leqslant S\leqslant D+W-1 \\ \dfrac{F_{\rm f}\left\{1-(1-F_{\rm r})^W-WF_{\rm r}(1-F_{\rm r})^{S-D+1}\right\}}{1-(1-F_{\rm r})^W+(1-F_{\rm r})^{W-D+1}\left[1-(1-F_{\rm r})^{S-W}\right]}, \\ \qquad W\geqslant D,\ S\geqslant D+W-1. \end{cases} \quad (23)$$

The throughput of a transport connection with group failure mode, accounting for repeat transmission of all segments starting from the first that failed to be received [2], is given by relation

$$Z_g(W,S) = F_{\mathrm{f}}F_{\mathrm{r}}\left\{\sum_{i=D-1}^{D+W-2}\frac{1-F_{\mathrm{f}}^{i-D+2}}{1-F_{\mathrm{f}}}P_i + \frac{1-F_{\mathrm{f}}^W}{1-F_{\mathrm{f}}}\sum_{i=D+W-1}^{S-1}P_i\right\}.$$

Up to the factor P_0, we get that

$$Z_g(W,S) = \begin{cases} \dfrac{P_0F_{\mathrm{f}}\left\{1-F_{\mathrm{f}}-(1-\Phi)(1-F_{\mathrm{r}})^{S-D+1}+F_{\mathrm{f}}F_{\mathrm{r}}\Phi^{S-D+1}\right\}}{(1-F_{\mathrm{f}})\left[1-F_{\mathrm{f}}+F_{\mathrm{f}}F_{\mathrm{r}}\Phi^{S-D}\right]}, \\ \qquad \max\{W+1,D\}\leqslant S\leqslant D+W-1 \\ \dfrac{P_0F_{\mathrm{f}}\left\{(1-F_{\mathrm{f}})\left(1-\Phi^W\right)-(1-\Phi)\left(1-F_{\mathrm{f}}^W\right)(1-F_{\mathrm{r}})^{S-D+1}\right\}}{(1-F_{\mathrm{f}})\left[1-F_{\mathrm{f}}+F_{\mathrm{f}}F_{\mathrm{r}}\Phi^{W-1}\right]}, \\ \qquad S\geqslant D+W-1. \end{cases} \tag{24}$$

It is easy to check that for $F_{\mathrm{f}} = 1$ this relation, after resolving the 0/0 uncertainty, yields the relation for the selective failure mode. Unbounded growth of window size ($W \to \infty$), and consequently of timeout duration S as well, ensures a dependence of throughput in selective failure mode that is invariant to round-trip delay and data transmission fidelity in the reverse path that is defined only by the fidelity of transmitting segments in the forward direction $Z_c(\infty,\infty) = F_{\mathrm{f}}$. The same holds for the selective failure mode, a perfectly reliable reverse channel ($F_{\mathrm{r}} = 1$), and window size of at least the round-trip delay ($W \geqslant D$). For $F_{\mathrm{r}} = 1$ and $W \leqslant D$ the transport connection's throughput in the selective failure mode (23) is inversely proportional to the difference between total path length and window size reduced by one:

$$Z_c(W\leqslant D,S) = \frac{F_{\mathrm{f}}}{D-W+1}.$$

For $F_{\mathrm{r}} = 1$, for various window sizes the throughput in the group failure mode (24) is

$$Z_g(W,S) = \begin{cases} \dfrac{F_{\mathrm{f}}}{D-W+1+(W-1)(1-F_{\mathrm{f}})}, & W\leqslant D \\ \dfrac{F_{\mathrm{f}}}{1+(D-1)(1-F_{\mathrm{f}})}, & W\geqslant D. \end{cases}$$

For unit window size ($W = 1$) selective and group failure modes coincide, and the throughput is

$$Z(1,S) = \begin{cases} \dfrac{F_{\mathrm{f}}\left\{1-(1-F_{\mathrm{r}})^{S-D+1}\left[1+(S-D+1)F_{\mathrm{r}}\right]\right\}}{2+F_{\mathrm{r}}(D-2)+(1-F_{\mathrm{r}})^{S-D}\left[F_{\mathrm{r}}(D+2-S)-2\right]}, & D\leqslant S\leqslant D+W-1 \\ \dfrac{F_{\mathrm{f}}F_{\mathrm{r}}\left[1-(1-F_{\mathrm{r}})^{S-D+1}\right]}{1+F_{\mathrm{r}}(D-1)-(1-F_{\mathrm{r}})^{S-D+1}}, & S\geqslant D+W-1. \end{cases}$$

For minimal window sizes ($W = 1$) and timeouts ($S = D$) the protocol's throughput is

$$Z(1,D) = \frac{F_{\mathrm{f}}F_{\mathrm{r}}}{D}.$$

The same relation holds in case of $W < D$ and $S = D$ for selective and group failure modes, which implies that a timeout with minimal duration ensures that the protocol is invariant to failure

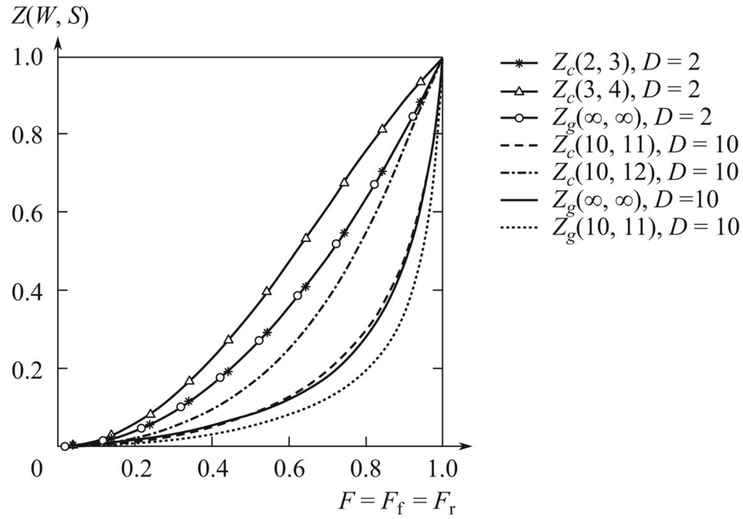

Fig. 1. Comparative relations of transport connection throughput in various repeat modes on the data transmission fidelity.

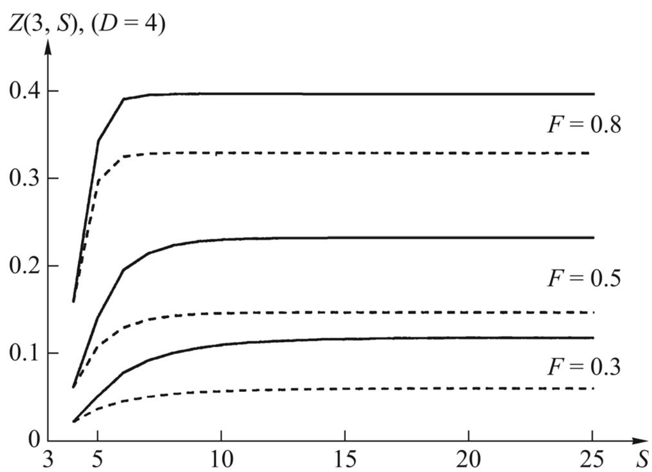

Fig. 2. Dependence of the transport connection throughput on the timeout duration for $F_{\mathrm{f}} = F_{\mathrm{r}} = F$. Solid lines correspond to the selective mode; dashed lines, to group mode.

mode, and the throughput is invariant to window size. For $W = 1$ and unbounded timeout duration ($S = \infty$) the throughput control procedure transforms into

$$Z(1,\infty) = \frac{F_{\mathrm{f}}F_{\mathrm{r}}}{1 + F_{\mathrm{r}}(D-1)}.$$

In the group failure mode, limit capabilities of the control procedure corresponding to unbounded window size ($W \to \infty$) are defined by the relation

$$Z_g(\infty,\infty) = \frac{F_{\mathrm{f}}F_{\mathrm{r}}}{1 - F_{\mathrm{f}}(1-F_{\mathrm{r}}) + F_{\mathrm{r}}(D-1)(1-F_{\mathrm{f}})}.$$

It is easy to see that this implies a significant dependence of the throughput on round-trip delay. For the same quality of direct and reverse data transmission paths ($F_{\mathrm{f}} = F_{\mathrm{r}} = F$), the potential

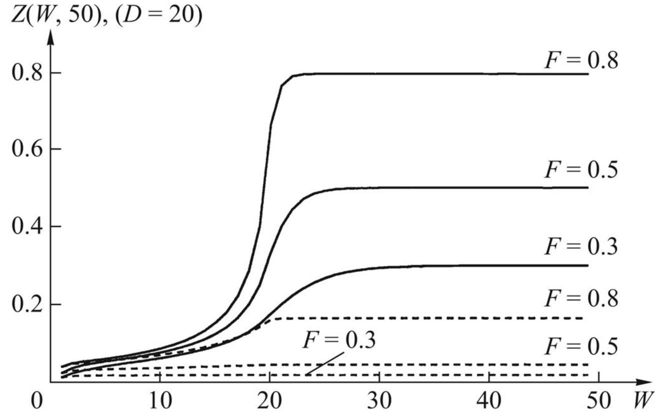

Fig. 3. Dependence of the transport connection throughput on window size for $F_{\rm f} = F_{\rm r} = F$. Solid lines correspond to the selective mode; dashed lines, to group mode.

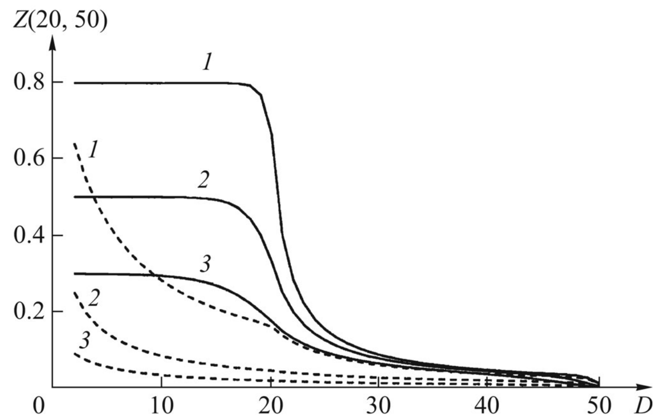

Fig. 4. Dependence of the transport connection throughput on round-trip delay for $F_{\rm f} = F_{\rm r} = F$. Solid lines correspond to the selective mode; dashed lines, to group mode. The plots *1* correspond to $F = 0.8$; plots *2*, to $F = 0.5$; plots *3*, to $F = 0.3$.

throughput in the group failure mode

$$Z_g(\infty, \infty) = \frac{F^2}{1 + F(D-2)(1-F)}$$

in the selective failure mode is virtually already achieved for protocol parameters $W = D$ and $S = D + 1$ (see Fig. 1):

$$Z_c(D, D+1) = \frac{F^2(3-2F)}{1 + D(1-F)}.$$

Here for single-link direct and reverse paths ($D = 2$) the throughput of both failure modes with the said protocol parameters coincides on the entire domain of data transmission fidelity F, and for a multi-link path ($D \geqslant 3$) the throughput equation holds at points $F = 0; 0.5; 1$. In the region $F \in (0; 0.5)$, the group failure mode is insignificantly better than selective failure with regard to throughput, and in the region $F \in (0.5; 1)$ selective failure is insignificantly better than group failure with respect to the same criterion (Fig. 1). As the timeout duration S grows, the point F of intermediate equality between throughputs of different failure modes moves to the left (towards the zero value), and the region where selective failure mode is better with respect to throughput increases together with the size of this advantage (see Fig. 1). We note, however, that for high levels of data transmission fidelity ($F \to 1$) the difference between throughputs in different failure modes quickly reduces. A characteristic dependence of the throughput of a transport connection on the timeout duration has the form of a curve with saturation (see Fig. 2) for any failure mode. The dependence of the throughput of a data transmission path on window size has a moderate growth segment for window size smaller than the round-trip delay ($W < D$), and as W grows further the throughput rapidly increases in the neighborhood of the round-trip delay up until saturation corresponding to a specific failure mode (see Fig. 3). As the data transmission path becomes longer (and consequently the round-trip delay increases as well), its throughput reduces, degrading significantly after the point when the round-trip delay exceeds window size ($D > W$). A characteristic dependence is shown on Fig. 4.

6. CHOOSING WINDOW SIZE AND TIMEOUT DURATION

Since the dependence of the throughput function (23) and (24) on protocol parameters has the form of curves with saturation, without singular points, rational values of window size W_{r} and timeout duration S_{r} can be chosen with a given value of throughput depending on the potential capabilities of a transport connection in various failure modes achieved for unbounded values of protocol parameters. Obviously, we have to look for a solution in the saturation zone of the throughput parameter corresponding to the domain of change for the protocol parameters with constraints from below (1). Here we have to specify the desired level of potential throughput in an arbitrary repeat mode $Z(\infty,\infty)$ for each coordinate of the throughput function. Since protocol parameters are related with inequality $S > W$, the choice procedure is divided into two stages. On the first stage, by a given level $y_W < 1$ we find a rational window size W_{r} by condition

$$Z(W_{\mathrm{r}},\infty) = y_W Z(\infty,\infty); \tag{25}$$

on the second stage, find a rational timeout duration S_{r} for the value W_{r} computed on the first stage with a given level $y_S < 1$ from equations

$$Z(W_{\mathrm{r}},S_{\mathrm{r}}) = y_S Z(W_{\mathrm{r}},\infty) = y_W y_S Z(\infty,\infty). \tag{26}$$

Here coefficients y_W and y_S that specify the levels of achieved throughput with respect to coordinates W and S according to condition (1) have the following constraints from below:

$$y_W \geqslant \frac{Z(D,\infty)}{Z(\infty,\infty)}, \quad y_S \geqslant \frac{Z(W_{\mathrm{r}}, D+W_{\mathrm{r}}-1)}{Z(W_{\mathrm{r}},\infty)}.$$

Equations (25) and (26) represent criteria for sequential choice of window size and timeout duration respectively. From these equations, for the selective failure mode due to (23) we get analytic relationships for rational values of protocol parameters:

$$W_{\mathrm{r}} = \left]\frac{1}{\ln(1-F_{\mathrm{r}})}\ln\frac{(1-y_W)(1-F_{\mathrm{r}})^{D-1}}{y_W+(1-y_W)(1-F_{\mathrm{r}})^{D-1}}\right[,$$

$$S_{\mathrm{r}} = D-1+\left]\frac{1}{\ln(1-F_{\mathrm{r}})}\ln\frac{(1-y)[1-(1-F_{\mathrm{r}})^{W_{\mathrm{r}}}]-y(1-F_{\mathrm{r}})^{W_{\mathrm{r}}-D+1}}{W_{\mathrm{r}}F_{\mathrm{r}}-y}\right[,$$

where $]\dots[$ denotes rounding to the next larger integer, $y = y_W y_S$. In case of group failure mode we compute parameters according to (25), (26) and (24), with the following relations:

$$W_{\rm r} = \left]\frac{1}{\ln\Phi}\,\ln\frac{(1-y_W)\Phi^{D-1}(1-\Phi)\big[1-\Phi+(D-1)(1-F_{\rm f})F_{\rm r}\big]}{\Phi^{D-1}(1-\Phi)\big[1-\Phi+(D-1)(1-F_{\rm f})F_{\rm r}\big]+y_W F_{\rm f}(1-F_{\rm r})^2(1-\Phi^{D-1})}\right[,$$

$$S_{\rm r} = D-1+\left]\frac{1}{\ln(1-F_{\rm r})}\ln(1-F_{\rm f})\right.$$
$$\left.\times\frac{(1-y-\Phi^{W_{\rm r}})(1-\Phi)\big[1-\Phi+(D-1)(1-F_{\rm f})F_{\rm r}\big]-yF_{\rm f}(1-F_{\rm r})^2(\Phi^{W_{\rm r}-D+1}-\Phi^{W_{\rm r}})}{(1-F_{\rm f}^{W_{\rm r}})(1-\Phi)\big[1-\Phi+(D-1)(1-F_{\rm f})F_{\rm r}\big]-y(1-F_{\rm f})(1-\Phi)^2}\right[.$$

7. CONCLUSION

In this work, we have proposed a model for the process of transferring data segments in a transport connection governed by a reliable transport protocol with acknowledgements for successfully received data, in the selective and group failure modes. The mathematical model is based on describing a queue of transmitted but not yet confirmed data segments with a Markov chain with finite number of states and discrete time. We obtain stationary distributions of Markov chain states for various intervals of window size and timeout duration. We have obtained analytic expressions for the throughput of transport connection in different failure modes. We have proposed a method for choosing the window size and timeout duration parameters. For a data transmission path with the same level of fidelity for delivering segments in the forward and reverse directions ($F_{\rm f} = F_{\rm r}$), by analyzing numerical results we have established that the potentially achievable throughput of the group failure mode, which is widely used in practice, corresponding to unbounded values of protocol parameters ($W = S = \infty$), in the selective failure mode is achieved in practice for window size coinciding with the round-trip delay duration ($W = D$) and minimal timeout duration ($S = W + 1$). We have found a significant dependence of the throughput in group failure mode of the total length of the transport connection D, which for a pipeline interpretation of the data transmission path can be easily explained with the need to restart the entire transport connection or a part of it in case of losing at least one segment in the sent sequence (repeat transmission of all segments starting from the first one lost). In general, throughput for any failure mode is to a significant extent determined by the relation between window size and round-trip delay duration. As a direction for further development of our studies we note the problem of analyzing the throughput of transport connections for users who have at least partly joint network routes and compete for available bandwidth of shareable re-reception segments. Besides, it appears important to look for a domain where it makes sense to use protocol procedures with an integrated forward error correction mechanism [23] based on transmitting a sequence of groups from informational and additional (redundant) segments that admit restoration on the receiving side even under distortion of a subset of segments in the group. In addition, it would be interesting to study the influence on the delays in multisegments user messages of the pipeline effect that arises in transmissions in multi-link transport connections.

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