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FORWARD AND BACKWARD PLANE WAVES IN A GENERALIZED ISOTROPIC MEDIUM

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Analytical expressions are derived for the complex wave number and characteristic wave impedance of plane waves propagating in a generalized isotropic medium with electric and magnetic losses. This approach does not require the operation of taking the square root of a complex number.

Keywords: generalized isotropic medium, complex permittivity, complex permeability, electromagnetic plane waves, forward wave, backward wave, wave number, characteristic wave impedance, intrinsic impedance of a medium.

Electromagnetic waves are capable of propagating in the most diverse natural media and synthetic materials. Objects of the ambient medium, such as soil, mineral ores, water, ice, snow, the tissues of living organisms, etc., are, as a rule, dissipative and possess the most diverse values of their effective material parameters. To describe the interaction of harmonic radio waves and light waves with many natural media and materials, it is sufficient to assign a complex permittivity and a positive permeability [1]. Contemporary technologies present heightened requirements on materials interacting with electromagnetic fields and waves, one consequence of which was the appearance of the concept of electromagnetic metamaterials – synthetic composites with values of the permittivity and permeability that are usually not encountered in nature [2]. Heightened interest has recently been manifested in metamaterials with both negative permittivity and negative permeability. They are associated with the phenomenon of negative refraction, which brings in its train the creation of flat lenses, surmounting of the diffraction limit, the creation of anti-reflection coatings, etc. In this regard, it is useful to consider the interrelationship between the characteristics of normal waves propagating in a generalized linear homogeneous isotropic dissipative medium and the complex material parameters (permittivity and permeability) of the medium. The real parts of the material parameters can take both positive and negative values in all possible combinations.

Interest in studying the properties of the wave numbers and wave resistances (impedances) of plane waves in a generalized medium is also due to the need to address the paradoxical concept of a *negative refractive index*. This new term has obtained widespread use in present-day macroscopic electrodynamics in application to metamaterials and has entered into monographs and textbooks (see, for example, [3–5]). At the same time, the wave number (or its real part), proportional to the refractive index with a positive coefficient equal to the wave number for vacuum, should always be a positive quantity since it is the length of the wave vector (see, for example, [6]). The ambiguity in the choice of sign on the refractive index together with the need to apply the non-unique mathematical operation of extraction of the square root of a complex number creates a nontrivial problem that to this day has attracted undiverted attention [7, 8]. In this work we adopt the canonical definition of the real part of the wave number, and with equal standing employ both existing definitions of the characteristic wave impedance, which makes it possible to obtain an unambiguous solution of the problem without using the square root of a complex number.

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We consider homogeneous plane waves with angular frequency ω and wave vector $\boldsymbol{k}$, which are characterized by the factor $\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$, where $\boldsymbol{r}$ is the radius vector, t is time, and $\mathrm{i}=\sqrt{-1}$. In a generalized homogeneous linear medium the directions of propagation of the phase front and the energy flux of a plane wave can either coincide (*forward* waves) or be opposite to one another (*backward* waves). We characterize the positive direction of the wavefront by the unit vector $\hat{\boldsymbol{k}}$, and the direction of the energy flux (the positive direction of the ray) by the unit vector $\hat{\boldsymbol{s}}$. Their scalar product $a=\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{s}}$ is the sign function: $a=+1$ for the forward wave and $a=-1$ for the backward wave. The complex wave vector $\boldsymbol{k}$ must be represented in general form so that the wave will decay in the direction of energy transfer, that is, so that

$$\lim_{\rho\to+\infty}\left|\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\right|=\lim_{\rho\to+\infty}\left\{\exp\left[-ak''\left(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}\right)\right]\right\}=\lim_{\rho\to+\infty}\left\{\exp[-k''(\hat{\boldsymbol{s}}\cdot\boldsymbol{r})]\right\}=0\,, \tag{1}$$

where $\rho=\hat{\boldsymbol{s}}\cdot\boldsymbol{r}>0$ and $k=k'+\mathrm{i}\,ak''$ is the wave number. The extinction factor in formula (1) is a positive quantity $(k''>0)$. The phase factor $k'=|\mathrm{Re}\,\boldsymbol{k}|>0$ is a positive quantity by definition, being defined as the length of the real part of the wave vector $\boldsymbol{k}$. The generalized form of writing down the wave number allows us to unify and simplify further calculations. Normalizing to the wave number for vacuum $k_0=\omega\sqrt{\mu_0\varepsilon_0}$ (where ε_0 and μ_0 are the vacuum permittivity and permeability, respectively), we obtain a positive refractive index $n=k'/k_0$ and positive decay coefficient $\kappa=k''/k_0$.

The homogeneous Maxwell equations, written in terms of the complex amplitudes of plane waves of indicated type (1), transform to a system of algebraic vector equations

$$\boldsymbol{k}\times\boldsymbol{E}=\omega\boldsymbol{B}\,,\ \boldsymbol{k}\times\boldsymbol{H}=-\omega\boldsymbol{D}\,,\ \boldsymbol{k}\cdot\boldsymbol{B}=0\,,\ \boldsymbol{k}\cdot\boldsymbol{D}=0\,, \tag{2}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the field strengths, and $\boldsymbol{D}$ and $\boldsymbol{B}$ are the inductions of the electric and magnetic fields. An isotropic magnetodielectric is described by the constitutive relations $\boldsymbol{D}=\varepsilon\boldsymbol{E}$ and $\boldsymbol{B}=\mu\boldsymbol{H}$ with two material scalars, where $\varepsilon=\varepsilon'+\mathrm{i}\,\varepsilon''$ is the complex permittivity and $\mu=\mu'+\mathrm{i}\,\mu''$ is the complex permeability. The medium is found in a state of thermodynamic equilibrium and is characterized by thermal losses; therefore, $\varepsilon''>0$ and $\mu''>0$, whereas the real parts of the permittivity and permeability of a generalized medium ε' and μ' can take both positive and negative values.

Let the triple of unit vectors $\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{e}}_2$, $\hat{\boldsymbol{e}}_3$ form a right-handed orthogonal basis in which the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are assigned by the expressions

$$\boldsymbol{E}=E\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\hat{\boldsymbol{e}}_1\,,\ \boldsymbol{B}=B\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\hat{\boldsymbol{e}}_2 \tag{3}$$

with complex scalar amplitudes E and B, respectively. As a consequence of formulas (2) and (3) we have $\hat{\boldsymbol{k}}=\hat{\boldsymbol{e}}_3=\hat{\boldsymbol{e}}_1\times\hat{\boldsymbol{e}}_2$. The curl equations amongst Eqs. (2) take the form

$$kE\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}}_1=\omega B\hat{\boldsymbol{e}}_2\,,\ kH\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}}_2=-\omega D\hat{\boldsymbol{e}}_1\,.$$

These relations allow us to introduce a characteristic complex wave impedance η by way of the formulas

$$\eta=\frac{E}{H}=\frac{k}{\omega\varepsilon}=\frac{\omega\mu}{k}\,. \tag{4}$$

It is important to distinguish it in the general case from the quantity $Z=\sqrt{\mu/\varepsilon}$, which is usually called either the wave impedance or the intrinsic (internal) impedance of the medium [9]. The moduli of the impedances η and Z coincide;

therefore, for plane waves in a medium with positive values of ε and μ there is no difference between the quantities η and Z. In the transition to complex quantities, the sign of Z becomes indeterminate since it depends on the way in which the cut on a bifolia Riemann surface is made and on the choice of the branch of the square root. In the literature, the difference between these quantities is usually not distinguished and η is simply replaced by Z [7, 10–13]. In [8] they derive it but do not use the relation for dimensionless quantities analogous to formula (4). In [1] in explicit form, and in [14, 15] implicitly, only the second part of formula (4) is derived and used.

The direction of energy transfer of the wave is given by the Umov–Poynting vector

$$\boldsymbol{S} = (1/2)\,\mathrm{Re}\,\boldsymbol{E}\times\boldsymbol{H}^* = \left|(1/2)\,\mathrm{Re}\,\boldsymbol{E}\times\boldsymbol{H}^*\right|\hat{\boldsymbol{s}}\,,$$

where the $*$ symbol denotes the complex conjugate. For plane waves of the form given by Eq. (3) we find

$$\boldsymbol{S} = |E|^2\, a\,\mathrm{Re}\,\eta \exp(-2k''\rho)\,\hat{\boldsymbol{s}}\,.$$

It follows from this formula that the inequality $a\,\mathrm{Re}\,\eta > 0$ holds in a generalized medium. Consequently, forward waves always correspond to positive values of $\eta' = \mathrm{Re}\,\eta$, and backward waves, to negative values of η'.

A comparison of the two representations in relations (4) for the characteristic impedance yields the relations

$$k^2 = \omega^2\mu\varepsilon\,,\ \eta^2 = \mu/\varepsilon\,, \tag{5}$$

which are used below to calculate the wave number k and impedance η. From relations (4) two additional relations also follow

$$k\eta = \omega\mu\,,\ k/\eta = \omega\varepsilon\,, \tag{6}$$

which together with relations (5) form a closed system of polynomial equations, the solution of which ensures an unambiguous determination of k and η in terms of the prescribed values of ε and μ. The method adopted here for finding k and η employing relations (6), after separating out the real and imaginary parts, allows us to get around the problem of choosing the branch of the square root of a complex number.

To simplify further calculations, it is useful to introduce normalized quantities, denoting them by capital letters of the Latin and Greek alphabets:

$$K = \frac{k}{|k|}\,,\ \mathrm{H} = \frac{\eta}{|\eta|}\,,\ \mathrm{E} = \frac{\varepsilon}{|\varepsilon|}\,,\ \mathrm{M} = \frac{\mu}{|\mu|}\,,$$

after which the investigated system of equations takes the form

$$K^2 = \mathrm{ME}\,,\ \mathrm{H}^2 = \mathrm{M/E}\,,\ K\mathrm{H} = \mathrm{M}\,,\ K/\mathrm{H} = \mathrm{E}\,, \tag{7}$$

where the complex numbers now look like this:

$$K = K' + \mathrm{i}\,aK''\,,\ \mathrm{H} = \mathrm{H}' + \mathrm{i}\,\mathrm{H}''\,,\ \mathrm{E} = \mathrm{E}' + \mathrm{i}\,\mathrm{E}''\,,\ \mathrm{M} = \mathrm{M}' + \mathrm{i}\,\mathrm{M}''\,,$$

and they are unimodular: $|K| = |\mathrm{H}| = |\mathrm{E}| = |\mathrm{M}| = 1$. System of four complex equations (7) is equivalent to the following system of eight equations for the corresponding real quantities:

$$K'^2 - K''^2 = \mathrm{M}'\mathrm{E}' - \mathrm{M}''\mathrm{E}''\,,\ 2aK'K'' = \mathrm{M}'\mathrm{E}'' + \mathrm{M}''\mathrm{E}'\,, \tag{8}$$

$$\mathrm{H}'^2 - \mathrm{H}''^2 = \mathrm{M}'\mathrm{E}' + \mathrm{M}''\mathrm{E}'', \ 2\mathrm{H}'\mathrm{H}'' = \mathrm{M}''\mathrm{E}' - \mathrm{M}''\mathrm{E}'', \tag{9}$$

$$K'\mathrm{H}' - aK''\mathrm{H}'' = \mathrm{M}', \ K'\mathrm{H}'' + aK''\mathrm{H}' = \mathrm{M}'', \tag{10}$$

$$K'\mathrm{H}' + aK''\mathrm{H}'' = \mathrm{E}', \ -K'\mathrm{H}'' + aK''\mathrm{H}' = \mathrm{E}''. \tag{11}$$

Combining the equations in pairs (10) and (11), we obtain the relations

$$2K'\mathrm{H}' = \mathrm{M}' + \mathrm{E}', \ 2aK''\mathrm{H}' = \mathrm{M}'' + \mathrm{E}'', \ -2aK''\mathrm{H}'' = \mathrm{M}' - \mathrm{E}', \ 2K'\mathrm{H}'' = \mathrm{M}'' - \mathrm{E}''. \tag{12}$$

Equations (12) together with Eqs. (8) and (9) make it possible for us to find the squares of the quantities of interest:

$$K'^2 = \frac{(\mathrm{M}'+\mathrm{E}')(\mathrm{M}''-\mathrm{E}'')}{2(\mathrm{M}''\mathrm{E}'-\mathrm{M}'\mathrm{E}'')}, \ K''^2 = -\frac{(\mathrm{M}'-\mathrm{E}')(\mathrm{M}''+\mathrm{E}'')}{2(\mathrm{M}''\mathrm{E}'-\mathrm{M}'\mathrm{E}'')}, \tag{13}$$

$$\mathrm{H}'^2 = \frac{(\mathrm{M}'+\mathrm{E}')(\mathrm{M}''+\mathrm{E}'')}{2(\mathrm{M}''\mathrm{E}'+\mathrm{M}'\mathrm{E}'')}, \ \mathrm{H}''^2 = -\frac{(\mathrm{M}'-\mathrm{E}')(\mathrm{M}''-\mathrm{E}'')}{2(\mathrm{M}''\mathrm{E}'+\mathrm{M}'\mathrm{E}'')}. \tag{14}$$

In order to eliminate uncertainty in formulas (13) and (14) of the form $0/0$ that arise for some values of the material parameters, it is advantageous to make a substitution of the form $\mathrm{Re}(z_\nu)/|z_\nu| = \cos\varphi_\nu$, $\mathrm{Im}(z_\nu)/|z_\nu| = \sin\varphi_\nu$ (the number $z = \mathrm{M}$ or E, and the subscript on it takes the values $\nu = \mu$ or ε), after which perfect squares are formed on the right-hand sides of these formulas. The quantities K' and K'' and the absolute values of the quantities H' and H'' take the form

$$K' = \left|\cos\frac{\varphi_\mu + \varphi_\varepsilon}{2}\right|, \ K'' = \left|\sin\frac{\varphi_\mu + \varphi_\varepsilon}{2}\right|, \ |\mathrm{H}'| = \left|\cos\frac{\varphi_\mu - \varphi_\varepsilon}{2}\right|, \ |\mathrm{H}''| = \left|\sin\frac{\varphi_\mu - \varphi_\varepsilon}{2}\right|. \tag{15}$$

We find the signs of H' and H'' starting out from additional relations (6) and noting that $K' > 0$, $K'' > 0$, $\mathrm{M}'' > 0$, and $\mathrm{E}'' > 0$. It follows from Eqs. (12) that

$$a\mathrm{H}' > 0, \ \mathrm{sgn}\,\mathrm{H}' = \mathrm{sgn}(\mathrm{M}' + \mathrm{E}'). \tag{16}$$

Hence it is clear that fulfillment of the inequality $\mathrm{M}' + \mathrm{E}' < 0$ is the condition for the existence of a backward wave, which is in complete agreement with the analogous condition in [15]. Multiplying the expressions in Eqs. (12) together pairwise, we find

$$\mathrm{sgn}\,\mathrm{H}'' = \mathrm{sgn}(\mathrm{M}'' - \mathrm{E}''), \ \mathrm{sgn}\,\mathrm{H}'' = \mathrm{sgn}\left(\mathrm{E}'^2 - \mathrm{M}'^2\right), \ \left(\mathrm{E}'^2 - \mathrm{M}'^2\right)(\mathrm{M}'' - \mathrm{E}'') > 0. \tag{17}$$

We obtain the final form of the expressions for the real and imaginary parts of the wave number by applying trigonometric identities that interrelate functions of integer and half-integer arguments to Eqs. (15) and taking into account the formulas for the moduli

$$|k| = \sqrt{k'^2 + k''^2} = \omega\sqrt{|\mu'\varepsilon'|}\sqrt[4]{\left(1+\delta_\mu^2\right)\left(1+\delta_\varepsilon^2\right)}, \ |\eta| = \sqrt{\left|\frac{\mu}{\varepsilon}\right|} = \sqrt{\left|\frac{\mu'}{\varepsilon'}\right|}\sqrt[4]{\frac{1+\delta_\mu^2}{1+\delta_\varepsilon^2}},$$

where the quantities $\delta_\mu = \mu''/\mu' = \tan\varphi_\mu$ and $\delta_\varepsilon = \varepsilon''/\varepsilon' = \tan\varphi_\varepsilon$ are the tangents of the magnetic and electric losses, respectively:

$$k' = \frac{\omega}{\sqrt{2}}\sqrt{|\mu'\varepsilon'|}\sqrt{\sqrt{\left(1+\delta_\mu^2\right)\left(1+\delta_\varepsilon^2\right)} + \left(1-\delta_\mu\delta_\varepsilon\right)\operatorname{sgn}\left(\mu'\varepsilon'\right)}\,, \tag{18}$$

$$k'' = \frac{\omega}{\sqrt{2}}\sqrt{|\mu'\varepsilon'|}\sqrt{\sqrt{\left(1+\delta_\mu^2\right)\left(1+\delta_\varepsilon^2\right)} - \left(1-\delta_\mu\delta_\varepsilon\right)\operatorname{sgn}\left(\mu'\varepsilon'\right)}\,. \tag{19}$$

In the analogous formulas for the characteristic impedance we have taken relations (16) and (17) into account:

$$\eta' = \frac{1}{\sqrt{2}}\sqrt{\left|\frac{\mu'}{\varepsilon'}\right|}\sqrt{\frac{\sqrt{\left(1+\delta_\mu^2\right)\left(1+\delta_\varepsilon^2\right)} + \left(1+\delta_\mu\delta_\varepsilon\right)\operatorname{sgn}\left(\mu'\varepsilon'\right)}{1+\delta_\varepsilon^2}}\operatorname{sgn}\left(\frac{\mu'}{|\mu|} + \frac{\varepsilon'}{|\varepsilon|}\right), \tag{20}$$

$$\eta'' = \frac{1}{\sqrt{2}}\sqrt{\left|\frac{\mu'}{\varepsilon'}\right|}\sqrt{\frac{\sqrt{\left(1+\delta_\mu^2\right)\left(1+\delta_\varepsilon^2\right)} - \left(1+\delta_\mu\delta_\varepsilon\right)\operatorname{sgn}\left(\mu'\varepsilon'\right)}{1+\delta_\varepsilon^2}}\operatorname{sgn}\left[\left(\frac{\varepsilon'}{|\varepsilon|}\right)^2 - \left(\frac{\mu'}{|\mu|}\right)^2\right]\operatorname{sgn}\left(\frac{\mu''}{|\mu|} - \frac{\varepsilon''}{|\varepsilon|}\right). \tag{21}$$

Let us consider the plane of material parameters in the coordinates $x = \varepsilon'/|\varepsilon|$ (the horizontal axis) and $y = \mu'/|\mu|$ (the vertical axis). The possible values of the parameters of a generalized medium are located within the limits of a square with vertices $(+1,+1)$ (transparent magnetodielectric: $\varepsilon > 0$, $\mu > 0$), $(-1,-1)$ (Veselago medium: $\varepsilon < 0$, $\mu < 0$), $(-1,+1)$ (opaque plasma: $\varepsilon < 0$, $\mu > 0$), $(+1,-1)$ (opaque magnet: $\varepsilon > 0$, $\mu < 0$). The condition $\left|\delta_\mu\delta_\varepsilon\right| = 1$ is satisfied on the circle of unit radius with center at the origin $\left|\delta_\mu\delta_\varepsilon\right| = 1$. The diagonal of the square $\mu'/|\mu| + \varepsilon'/|\varepsilon| = 0$, as can be seen from formula (20), demarcates the regions of the forward and backward waves. The forward wave zone is located to the right of and above this diagonal ($-y < x < 1$), and the backward wave zone, to the left and below it ($-1 < x < -y$). Thus, in a dissipative medium backward waves are possible not only in the case of simultaneous negative values of ε' and μ', but also in cases with mixed signs.

The imaginary part of the characteristic impedance, as follows from formula (21), provided that $\mu''/|\mu| - \varepsilon''/|\varepsilon| > 0$, is positive in the regions $|y| < |x|$ subtending the horizontal axis (the x axis), and negative in the mixed regions $|x| < |y|$ subtending the vertical axis (the y axis).

Formulas (18)–(21) allow one to calculate the wave parameters for arbitrary values of ε and μ. In particular, near the zero point of the parameter plane (such a hypothetical state of the medium is referred to in the English-language literature as the *nihility* (*nothingness*), where $\varepsilon \approx \mathrm{i}\,\varepsilon'' \to 0$, $\mu \approx \mathrm{i}\,\mu'' \to 0$, $|\delta_\varepsilon| \gg 1$, and $\left|\delta_\mu\right| \gg 1$) we have $k' = 0$, $k'' \approx \omega\sqrt{\mu''\varepsilon''}$, $\eta' = \sqrt{\mu''/\varepsilon''}\operatorname{sgn}\left(\delta_\mu^{-1} + \delta_\varepsilon^{-1}\right)$, and $\eta'' = 0$.

In conclusion we point to the connection between the two definitions of the wave number and the refractive index. We introduce the number $\tilde{k} = \tilde{k}' + \mathrm{i}\,\tilde{k}'' = k_0\left(\tilde{n} + \mathrm{i}\,\tilde{\kappa}\right)$, which is associated with the wave number k by the formula $\tilde{k} = ak = ak' + \mathrm{i}\,k'' = k_0\left(an + \mathrm{i}\,\kappa\right)$. It follows from a comparison of the real and imaginary parts that the decay coefficients are identical ($\tilde{\kappa} = \kappa > 0$), but the refractive indices are interrelated by the formula

$$\tilde{n} = an = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{s}} n\,, \tag{22}$$

i.e., they differ in sign in the case of the backward wave. The refractive index $\tilde{n}$ defined by formula (22) is known in crystal optics as the ray refractive index [16]. Therefore, in application to a Veselago medium it is more valid to use the term *ray refractive index* instead of the term *negative refractive index*.

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