

Study of melting of a pure gallium under influence of magnetic field in a square cavity with a local heat source

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2015 IOP Conf. Ser.: Mater. Sci. Eng. 93 012004

(<http://iopscience.iop.org/1757-899X/93/1/012004>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 37.21.156.253

This content was downloaded on 18/10/2015 at 12:39

Please note that [terms and conditions apply](#).

Study of melting of a pure gallium under influence of magnetic field in a square cavity with a local heat source

N S Bondareva¹ and M A Sheremet²

Tomsk State University, 36, Lenin ave., Tomsk, 634050, Russia

E-mail: ¹r0dniki@mail.ru, ²sheremet@math.tsu.ru

Abstract. Numerical analysis of the unsteady natural convection with phase transitions inside the pure gallium under the influence of a uniform magnetic field with a heat source of constant temperature has been conducted in the presence of magnetic field. The vertical walls of the cavity are kept at low constant temperatures whereas the top and bottom horizontal walls are adiabatic with the exception of the heat source of high constant temperature. The mathematical model formulated in dimensionless stream function, vorticity and temperature variables is solved using the implicit finite difference schemes of the second order. The governing parameters are the Rayleigh and Hartmann numbers, and the dimensionless time. The effects of these parameters on the streamlines and isotherms are analyzed.

1. Introduction

Numerous studies of magnetic natural convection with solid-liquid phase change occupy one of the central stages of the heat and mass transfer problems. The interaction of magnetic field with natural convection in the melting process can have a very strong effect on the resulting flow structure and performance of the material. Because of the high electrical conductivity in metals, magnetic induction significantly suppresses natural convection and consequently smoothes the temperature differences. During solidification using imposed magnetic field produced semi-conductors with suitable quality.

Melting and solidification coupled with natural convection under the influence of magnetic field is a topic of many experimental and numerical studies. Wittig and Nikrityuk [1] presented flow visualization of two- and three-dimensional pure gallium melting. The authors compared the obtained results of 2D and 3D simulations and noted an essential influence of the third coordinate. Bouabdallah and Bessaih [2] performed numerical simulations of solidification with magnetic field in cubical enclosure. Authors showed that the liquid phase is suppressed by the Lorentz force and in this connection the phase front becomes regular.

Present numerical study is devoted to natural convection with melting under the influence of the magnetic field in square cavity with a local heat source. The heat source of a constant temperature is located on the bottom wall of the considered enclosure and has a square shape. Vertical walls of the cavity are cooled while the horizontal walls are adiabatic. Mathematical simulation of unsteady melting process is carried out on the basis of dimensionless variables such as stream function, vorticity and temperature. The special smoothing function is introduced to simplify the solving process. Governing equations are solved using the finite difference method.



2. Mathematical model and numerical method

Natural convection with phase transition and uniform magnetic field in a square cavity with an internal heat source is considered numerically. Schematic illustration of physical model is shown in Figure 1. Square heat source is located at the centre of the bottom wall and it has a constant temperature that is greater than the temperature of melting. Vertical walls of the cavity are cooled and have constant temperature that is less than the melting temperature. Horizontal walls are supposed to be adiabatic. Initially material has the solid state and its temperature is equal to the fusion temperature. For the description of this problem the following assumptions have been introduced in the model: the Boussinesq approximation is valid, the melt is Newtonian and incompressible, the flow is laminar, physical properties of material is not depend on the temperature.

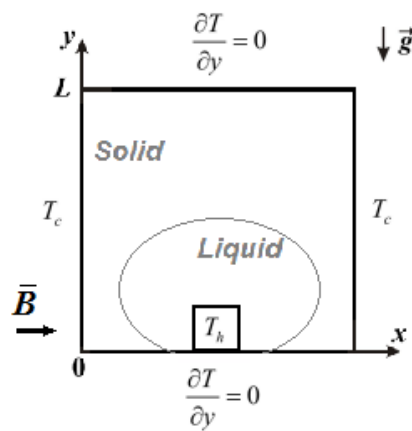


Figure 1. Schematic illustration of the cavity.

The governing equations in this statement can be written using the following conservation laws:

– conservation of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

– conservation of momentum in liquid phase:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u + (\vec{j} \times \vec{B})_x, \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v + (\vec{j} \times \vec{B})_y + g\beta(T - T_F), \quad (3)$$

– conservation of energy:

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

where

$$h = \begin{cases} \rho_s c_s T, & T < T_F, \\ \rho_s c_s T_F + \rho_l L_F + \rho_l c_l (T - T_F), & T \geq T_F. \end{cases} \quad (5)$$

Here x, y are the Cartesian coordinates, respectively; t is the time; g is the gravity acceleration; ν is the viscosity; β is the thermal expansion coefficient of liquid; ρ is the density; u, v are the velocity components in x - and y -directions, respectively; p is the pressure; \vec{j} is the current density; \vec{B} is the magnetic intensity; T is the temperature; T_h is the heat source temperature; T_F is the melting temperature; h is the enthalpy, k is the thermal conductivity; L_F is the latent heat; c_s and c_l are the specific heats of solid and liquid phases; ρ_s and ρ_l are the densities of solid and liquid phases, respectively.

To remove discontinuous from the energy equation (4) at the phase front the following smoothing function is introduced:

$$\varphi = \begin{cases} 0, & T < T_F - \eta \\ \frac{T - (T_F - \eta)}{2\eta}, & T_F - \eta \leq T \leq T_F + \eta \\ 1, & T > T_F + \eta \end{cases} \quad (6)$$

New variables such as the stream function ψ and vorticity ω have been used to simplify solving of equations:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (7)$$

To reduce the governing equations to non-dimensional form we used the following variables:

$$X = x/L, \quad Y = y/L, \quad U = u/\sqrt{g\beta\Delta T L}, \quad V = v/\sqrt{g\beta\Delta T L}, \quad \tau = t\sqrt{g\beta\Delta T/L}, \\ \Theta = (T - T_F)/\Delta T, \quad \Psi = \psi/\sqrt{g\beta\Delta T L^3}, \quad \Omega = \omega L/\sqrt{g\beta\Delta T L}$$

where L is the cavity size; $\Delta T = T_h - T_F$; X, Y are the dimensionless coordinates corresponding to x and y coordinates, respectively; U, V are the dimensionless velocities corresponding to dimensional velocities u, v , respectively; τ is the dimensionless time, Θ is the dimensionless temperature, Ψ is the dimensionless stream function; Ω is the dimensionless vorticity.

Reduced governing equations (1)–(4) can be rewritten as follows:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega, \quad (8)$$

$$\frac{\partial \Omega}{\partial \tau} + U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \sqrt{\frac{\text{Pr}}{\text{Ra}}} \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + \text{Ha}^2 \sqrt{\frac{\text{Pr}}{\text{Ra}}} \times \left[\left(\frac{\partial U}{\partial X} - \frac{\partial V}{\partial Y} \right) \sin(\varphi) \cos(\varphi) - \right. \\ \left. - \frac{\partial V}{\partial X} \cos^2(\varphi) + \frac{\partial U}{\partial Y} \sin^2(\varphi) \right] + \frac{\partial \Theta}{\partial X} \quad (9)$$

$$\beta(\varphi) \left[\frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} \right] + \text{Ste} \left[\frac{\partial \varphi}{\partial \tau} + U \frac{\partial \varphi}{\partial X} + V \frac{\partial \varphi}{\partial Y} \right] = \frac{\xi(\varphi)}{\sqrt{\text{Ra} \cdot \text{Pr}}} \left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right) \quad (10)$$

where $\beta(\varphi) = \frac{\rho_s c_s}{\rho_l c_l} + \varphi \left(1 - \frac{\rho_s c_s}{\rho_l c_l} \right)$, $\xi(\varphi) = \frac{k_s}{k_l} + \varphi \left(1 - \frac{k_s}{k_l} \right)$.

One can see that magnetic natural convection with melting is governed by six dimensionless parameters such as the Rayleigh, Prandtl, Hartmann and Stefan numbers, ratio of volumetric heat capacities and thermal conductivity ratio.

The following boundary conditions for the stream function are set on the solid walls and the phase front $\Psi = 0$, $\partial\Psi/\partial\bar{n} = 0$. Vorticity values on the walls are found from the Poisson equation (8). Stream function and vorticity inside the solid phase are equal to zero.

The governing equations (8)–(10) with corresponding initial and boundary conditions are solved using the finite difference method [3, 4]. Validation of the created algorithm is based on the benchmark problem proposed by Gau and Viskanta [5]. It should be noted, that obtained solid-liquid interphases are in good agreement with experimental data of Gau and Viskanta [5].

3. Results and discussion

Physical parameters of pure gallium used in this investigation are: $\rho_l = 6093 \text{ kg/m}^3$, $\rho_s = 6095 \text{ kg/m}^3$, $\beta = 1.2 \cdot 10^{-4} \text{ K}^{-1}$, $k = 32 \text{ W/(m} \cdot \text{K)}$, $T_f = 29.78 \text{ }^\circ\text{C}$, $L_f = 8.016 \cdot 10^4 \text{ J/kg}$, $c = 381.5 \text{ J/kg}$, $\mu = 1.81 \cdot 10^{-3} \text{ kg/(m} \cdot \text{s)}$. Numerical studies have been carried out at the following values of the governing parameters: $7.17 \cdot 10^5 \leq Ra \leq 3.59 \cdot 10^6$, $0 \leq Ha \leq 100$, $2.1 \leq Ste \leq 10.51$, $\rho_s c_s / (\rho_l c_l) = 1$ and $k_s / k_l = 1$. Prandtl number is set 0.0216. Dimensionless parameters selected correspond to properties of the pure gallium. Size of the enclosure is $L = 5 \text{ cm}$. Dimensionless temperature of the heat source is $\Theta_h = 1$ and on the vertical walls we have $\Theta_h = -0.3$.

The effect of the magnetic field on heat transfer is illustrated in Figure 2. This figure shows isotherms for Rayleigh number $Ra = 3.59 \cdot 10^6$ at several times. It can be seen that strong magnetic field suppresses natural convection and as a result of the velocity decreases the melting process slows down and the phase front becomes smooth and symmetric. During the melting process without magnetic field few convection cells are formed inside the melt zone. The multi-cellular mode is a time-dependent because the temperature difference leads to more complicated flow patterns. With imposed magnetic field the inhibition of fluid occurs, that is presented in Figure 3. Also this Figure 3 shows the comparison of streamlines for two cases.

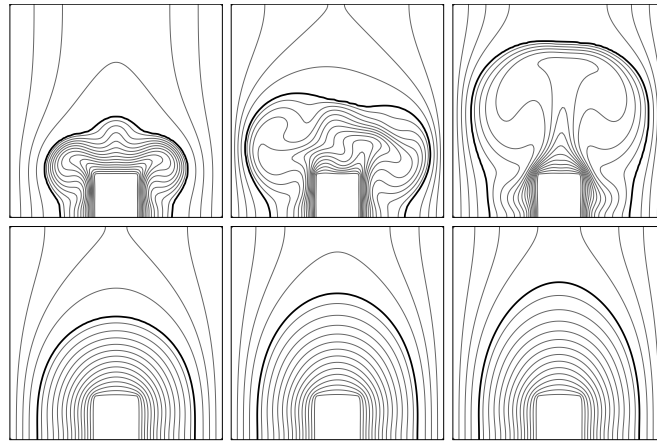


Figure 2. Isotherms for $Ha = 0$ (top row) and $Ha = 100$ (bottom row) for the time moments $t = 12, 30$ and 60 seconds (from left to right).

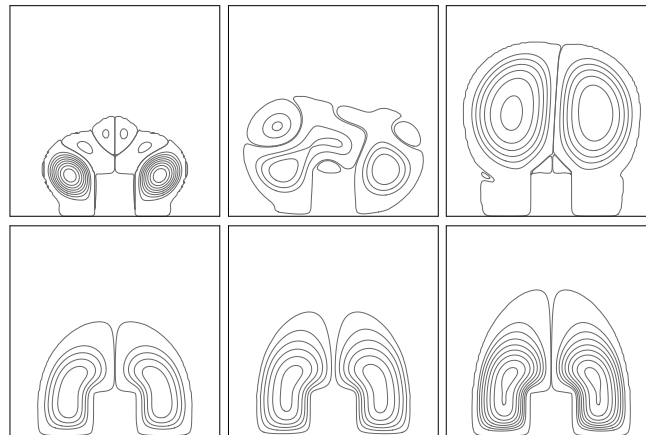


Figure 3. Streamlines at $Ha = 0$ (top row) and $Ha = 100$ (bottom row) for the time moments $t = 12, 30$ and 60 seconds (from left to right).

4. Conclusions

Natural convection of pure gallium under the influence of a uniform magnetic field with phase change inside a square cavity with a local heat source has been studied numerically. Governing equations have been formulated using the special enthalpy formulation with the dimensionless stream function and vorticity variables. The local characteristics of melting process have been obtained in wide ranges of the Rayleigh and Hartmann numbers. The effect of the magnetic field on the fluid flow and heat transfer has been analyzed. It should be noted that increasing of magnetic intensity leads to an attenuation of convective flow and slowing of melting.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research, Grant No. 14-08-31137 mol_a and by the Grants Council (under the President of the Russian Federation), Grant No. MD-6942.2015.8.

References

- [1] Wittig K and Nikrityuk P A 2011 *Materials Sci. Engineering: IOP Conf. Series* **27** 012054.
- [2] Bouabdallah S and Bessaïh R 2012 *Int. J. Heat Fluid Flow* **37** 154–166.
- [3] Bondareva N S, Volokitin O G, Morozova O O and Sheremet M A 2013 *Thermophysics and Aeromechanics* **20** 621–629.
- [4] Sheremet M A 2010 *J. Engineering Thermophysics* **19** 119–127.
- [5] Gau C and Viskanta R 1986 *J. Heat Transfer* **108** 174–181.