Noncommutative $\mathbb{R}^d$ via closed star product

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ABSTRACT: We consider linear star products on $\mathbb{R}^d$ of Lie algebra type. First we derive the closed formula for the polydifferential representation of the corresponding Lie algebra generators. Using this representation we define the Weyl star product on the dual of the Lie algebra. Then we construct a gauge operator relating the Weyl star product with the one which is closed with respect to some trace functional, $\text{Tr}(f \star g) = \text{Tr}(f \cdot g)$. We introduce the derivative operator on the algebra of the closed star product and show that the corresponding Leibniz rule holds true up to a total derivative. As a particular example we study the space $\mathbb{R}^3_\theta$ with $\mathfrak{su}(2)$ type noncommutativity and show that in this case the closed star product is the one obtained from the Duflo quantization map. As a result a Laplacian can be defined such that its commutative limit reproduces the ordinary commutative one. The deformed Leibniz rule is applied to scalar field theory to derive conservation laws and the corresponding noncommutative currents.

KEYWORDS: Non-Commutative Geometry, Space-Time Symmetries

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