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Investigation of influence of internal structure of heterogeneous materials on plastic flow and fracture

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Abstract

A combined elastic–viscous-plastic model is proposed for simulating deformation of heterogeneous media. The calculations are carried out by quasi-static and dynamic methods in 2D (both for plane strain and plane stress) and 3D formulations. The results of simulation of plastic strain localisation and fracture in meso-volumes of ceramics with pseudo-plastic matrix and polycrystalline materials are presented and discussed. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The complex hierarchical internal structure of materials brings into existence different physical phenomena which take place at different scale levels. In many cases examination at the meso-scale level is of great importance. Within the framework of continuum mechanics, models of different types can be made for different material organisation [1]. For the polycrystalline material, grains with different crystallographic orientations may be considered. Differences in elastic moduli of fibres (or inclusions) and matrix may be used for the composites, while those in yield and strength limits need to be specified for welded joints and hardened surfaces. Analysis may require a combination of the aforementioned parameters. Heterogeneity of meso-structure could lead to stress concentration and strain localisation at the meso-level. The heterogeneous character of the stress–strain state has been observed experimentally by the deformation of weld joints, hardened surfaces, polycrystals and other heterogeneous material at the micro- and meso-scale levels [2]. This work is concerned with a heterogeneous medium and makes use of the meso-volumes of materials which contain different elements that may be grains, matrix and hard inclusions, different phases of certain substance, etc. The development of deformation at different scales of space and time needs the use of quasi-static and dynamic approaches.

2. Model of medium and numerical technique

The calculations based on combined elastic–viscous-plastic model of medium were carried out.

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Viscoelastic properties of material are described by the equations of linear viscoelasticity [3]. Plastic flow of solids is simulated on the basis of the model of the medium with hardening and microdamage accumulation [4]. For modelling slow relaxation processes and creeping a quasi-static method based on the Lagrange variational equation of incremental theory of plasticity [5,6] is constructed. The mass, momentum and energy conservation laws in differential form are used for modelling deformation of a solid in the dynamic approach. The numerical method is akin to the one described in Ref. [7] for solving dynamic problems. This method is developed to solve the fracture problem with explicit opening of meso- and macro-cracks along the cell boundaries. For this purpose the algorithm of splitting of the calculation grid nodes is used [8].

3. Discussion of numerical results

Two- and three-dimensional computer programs are available for analysing the dynamic and quasi-static deformation of heterogeneous solids. Validity of the numerical codes was verified by comparing the results obtained from different numerical methods for different problems, particularly, those obtained by both 2D (quasi-static and dynamic) and 3D simulations of material with complex internal structure under small rate loading are in a good agreement.

3.1. Viscoelastic material

Let us consider emergence of stress concentrators and stress relaxation in a plate made of a polyurethane matrix with an inclusion in a state of plane stress under stretching. The structure of the model sample under uniaxial loading is shown in Fig. 1(a), while Fig. 1(b) displays the applied strain as a function of time. Mechanical properties of the structure elements are presented in Table 1. Viscoelastic properties of the matrix material are taken from [3]. In this model example it is assumed that the instantaneous and long-time moduli of the matrix and inclusion are the same but their relaxation times are different and the Poisson's ratio is time independent.

Fig. 2 shows the distribution of the second invariant of the stress deviator. Due to deceleration of relaxation processes in the inclusion there is certain stress concentration in it. At the initial stage of loading stress in the inclusion increases and then relaxes both in the matrix and the inclusion but at different rates. Fig. 3 shows a macroscopic curve of shear and volume stress relaxation. Here

$$J_2(D_\sigma) = \sqrt{\sum_{\Delta V_i} [\Delta V_i (3/2) s_{ij} s_{ij}] \Big/ V}$$

and

$$\sigma_V = \sqrt{\sum_{\Delta V_i} \left[\Delta V_i (\sigma_{ii}/3)^2\right] \Big/ V}$$

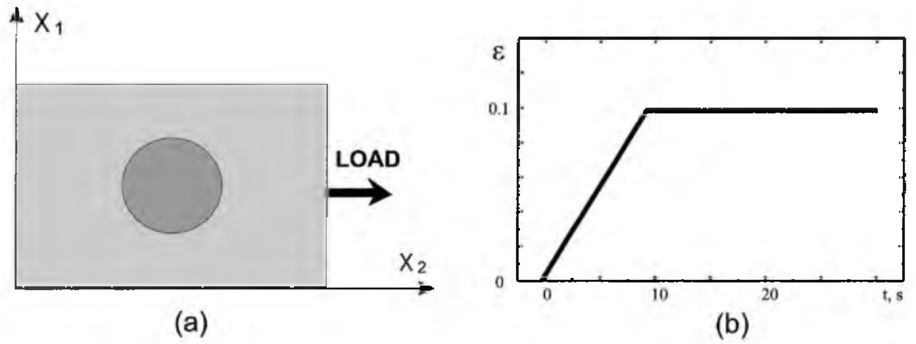

Fig. 1. The structure of viscous-elastic model sample (a) and time-dependent stretching (b).

Table 1
Coefficient and relaxation time for shear stress function

κ	G_1^{κ} (GPa)	$t_{G_1^{\kappa}}$ (s)	
		Matrix	Inclusion
0	7	–	–
1	14	1.5×10^{-5}	1.5×10^{-2}
2	7.5	$1.5\times 10^{\;4}$	$1.5\times 10^{\;1}$
3	7	1.5×10^{-3}	1.5
4	5.5	$1.5\times 10^{\;2}$	1.5×10^{1}
5	4.3	1.5×10^{-1}	1.5×10^{2}
6	2.1	1.5	1.5×10^{3}
7	1.6	1.5×10^{1}	1.5×10^{4}
8	0.3	1.5×10^{2}	1.5×10^{5}

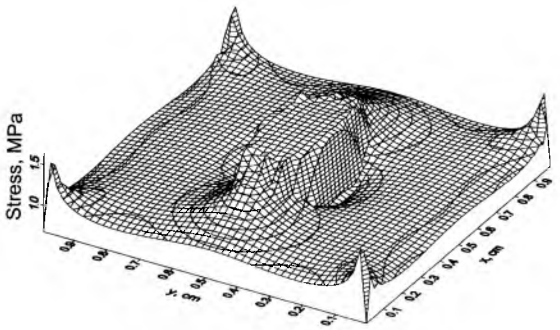

Fig. 2. Distribution of shear stresses in the heterogeneous viscous-elastic sample.

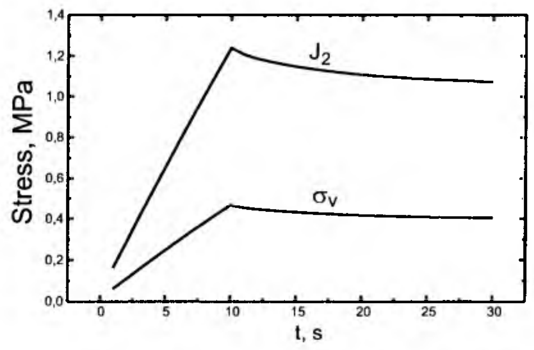

Fig. 3. Shear and cubic stress relaxation in the heterogeneous viscous-elastic sample.

are the averaged second stress deviator invariant and the volumetric stress, respectively, which are calculated for every loading step to examine the transition of material properties from the meso- to the macro-scale level.

3.2. Ceramic composite

Calculation of local non-elastic strain is made for a ceramic composite under stretching with ZrO_2 as the binding material, while the hard inclusions are Al_2O_3. Their properties are given in Table 2. Both elastic perfectly-plastic and elastic–plastic models are used for a matrix. Inclusion is supposed to be elastic only until fracture.

Fig. 4 shows the internal structure of a composite sample for 2D (plain strain) and 3D calculations with the scheme of loading. Fig. 5 displays results of 2D simulation for the elastic perfectly-plastic matrix. Effective strains $I_2(D_\varepsilon) = \sqrt{(3/2e_{ij})e_{ij}}$ reaches a maximum value in the vicinity of sectors of the matrix–inclusion interface which are orthogonal to the axis of tension. When plastic flow occurs, localised shear bands are formed from those regions extending toward the directions of maximum shear stress, as Fig. 5 shows. The results of 3D modelling of a cubic sample with a similar structure stretched uniformly are shown in Figs. 6 and 7. Distribution of the effective shear strains at the faces of the sample is shown in Fig. 6. Localised strain pattern is like the one in 2D modelling. Fig. 7 shows a deformed surface of the sample. Note that the correspondence of localised strain regions and relief of the deformed surface is not evident.

The next step in the description of the material behaviour is to take into account, nucleation and growth of cracks during loading. These are expected in the regions of the highest stress due to the fracture model of maximum tensile stress used. Elastic perfectly-plastic matrix restricts the increase of stress. For the geometry given the maximum acting stress is limited by $\sigma^0 \approx 1.27\sigma_{0,2}$. That is why, if $\sigma^* \geqslant \sigma^0$, fracture of elastic perfectly-plastic material can not be described on the basis of the stress criterion. Hence, to simulate cracking it is necessary to take into consideration matrix hardening.

Fig. 8(a) shows that for the case of hardening matrix, the distribution of effective strains is more uniform, though strains near the matrix–inclusion

Table 2
Material properties of ZrO_2 and Al_2O_3

Material	ρ (g/cm^3)	K (GPa)	μ (GPa)	$\sigma_{0.2}$ (GPa)	σ^* (GPa)
Inclusion Al_2O_3	3.99	257	152	1.9	2.6
Matrix ZrO_2	5.6	152	65.5	1	1.5
ZrO_2/Al_2O_3 interface	–	–	–	–	0.8

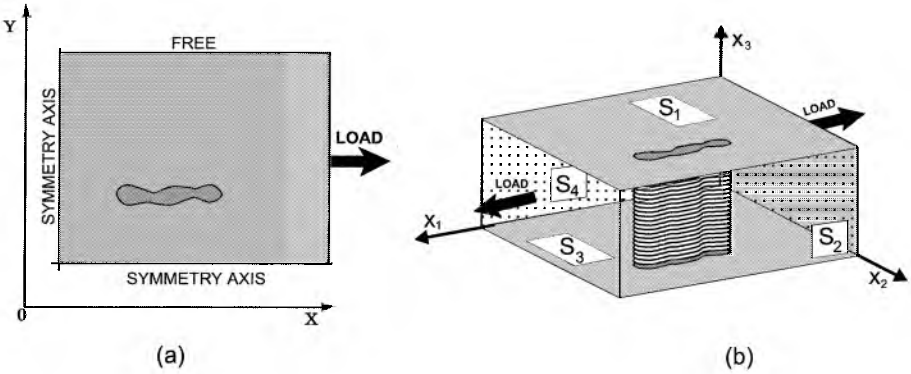

Fig. 4. Structure of the composite sample and load condition for 2D- (a) and 3D- (b) calculation.

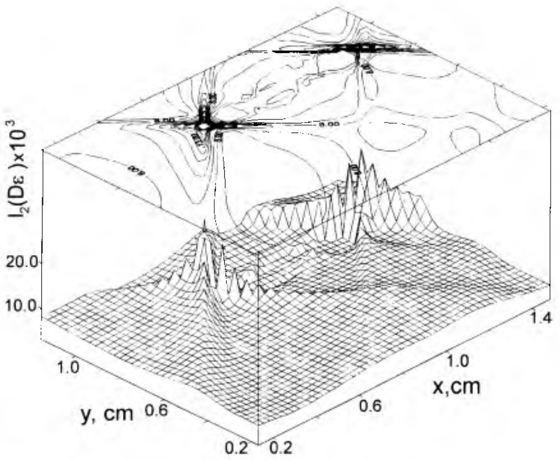

Fig. 5. Effective strains in ceramic sample with elastic perfectly-plastic matrix.

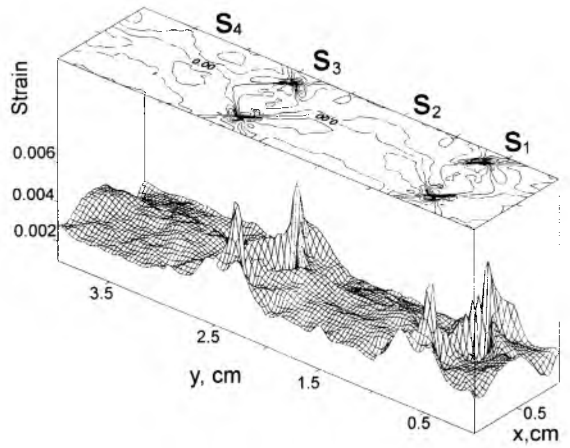

Fig. 6. Strain localisation in the ceramic sample under stretching at $\varepsilon_{11} = 0.2\%$ (3D simulation).

interface are higher. One can see in Fig. 8(b) that in the thin part of the hard inclusion and in the sectors of matrix–inclusions interface orthogonal to the axis of tension there acts the highest tensile stress. The primary crack nucleates just in the thinnest zone of the inclusion and spreads throughout its whole section. Plastic matrix inhibits growth of cracks. Immediately after that the secondary crack nucleates in the matrix–inclusion interface. Fig. 9 displays the distribution of effec-

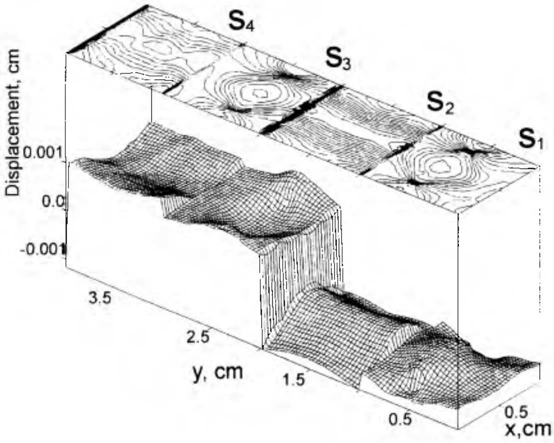

Fig. 7. Deformed surface of faces of cubic sample (3D simulation).

tive strains and stresses after nucleation of cracks. In the vicinity of crack tips the strain growth takes place causing the stress to grow. The crack tips become new and more powerful stress concentrators. The stress redistribution from the hard inclusion to the matrix happens after cracking, which is shown in Fig. 9(b). Nucleation and growth of a crack is accompanied by release of elastic energy, generation and propagation of elastic waves. Fig. 10 displays the velocity fields after crack nucleation. The particle velocities at the front of elastic waves and behind preponderate the ones in stationary zones of flow. A vortex flow can be seen in the vicinity of the crack tips in Fig. 10.

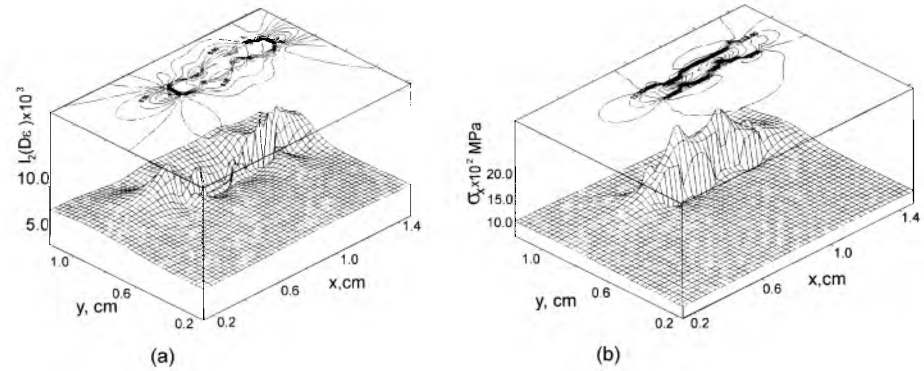

Fig. 8. Distribution of effective strains (a) and stresses (b) in the ceramic sample before cracking.

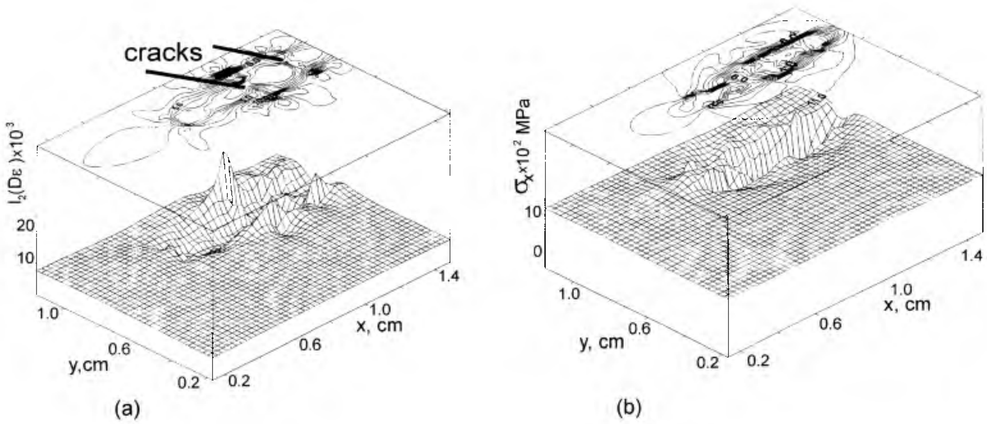

Fig. 9. Distribution of effective strains (a) and stresses (b) in the ceramic sample after cracking.

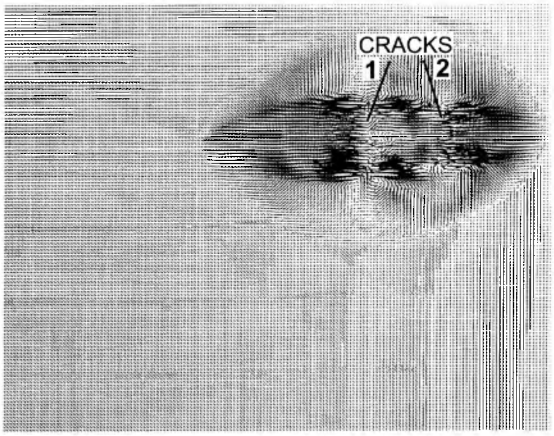

Fig. 10. Velocity field in the ceramic sample after cracking.

3.3. Polycrystalline materials

For a polycrystalline case we investigated a meso-volume of steel. The elastic perfectly-plastic model for grains was adopted with flow stress varying from 200 to 300 MPa for different grains. Elastic properties are assumed to be the same for all the grains. The meso-volume was loaded in tension in vertical direction both up and down. For the lateral sides the boundary conditions of prohibition to move in horizontal direction was set, and only sliding in vertical direction was permitted in the simulation. So, there is a restrained deformation condition in this case. In Fig. 11(a) one can see plastic flow which starts in the weakest grains. It is non-uniform, there are small bands of localised plastic strain in separate grains. It is interesting that at the same moment we observe an elastic stage in the stress–strain curve. Later, the other grains undergo plastic deformation, Fig. 11(b). There are no large bands of localised plastic strain as it was in the condition with free surfaces described in Ref. [9]. There are only small ones inside the grains. Distribution of the effective plastic strains in Fig. 11(b) reflects the polycrystalline structure of the meso-volume as well. Another interesting result can be revealed from the analysis of velocity field. It is quasi-uniform in the meso-volume under uniaxial stretching, but has a fine structure that can be revealed if one subtracts the average values of velocity in rows from the velocity in each node of the calculation grid. In such a relative flow shown in Fig. 12 there appear vortexes. A similar situation was observed in modelling plane shock

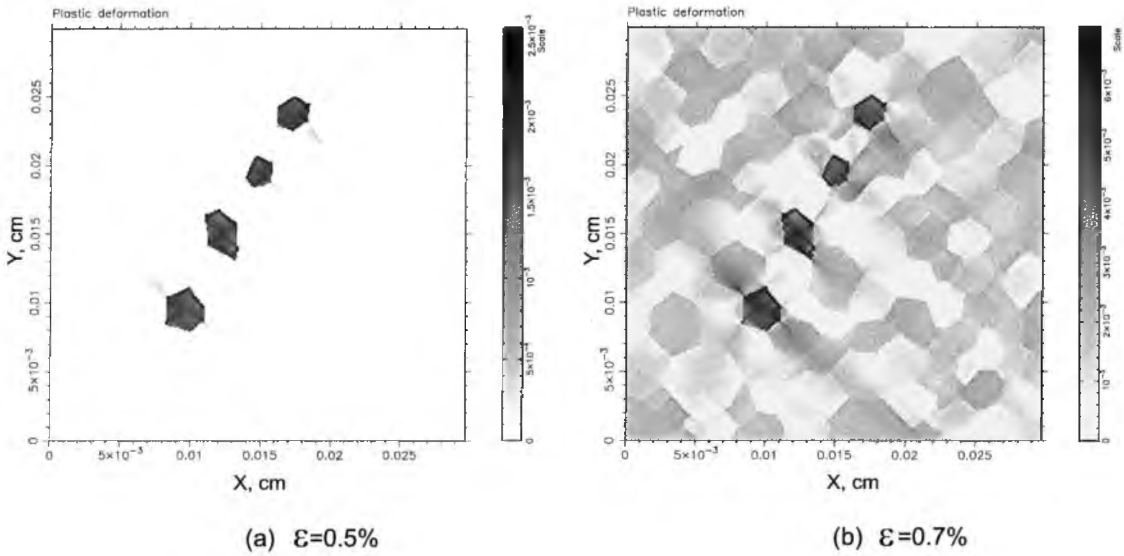

Fig. 11. Distribution of effective plastic strains in the mesovolume of polycrystalline steel.

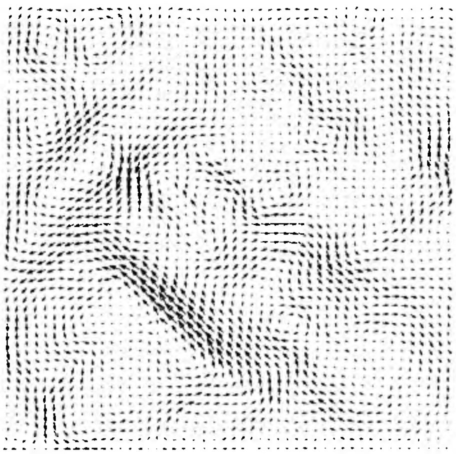

Fig. 12. Velocity fields in the mesovolume of polycrystalline steel.

wave propagation in meso-volumes of polycrystalline material in Ref. [10].

4. Conclusion

Based on a combined model of elastic–viscous-plastic medium, 2D and 3D computer programs have been developed for both quasi-static and dynamic problems to examine deformation of solids at the meso- and macro-scale levels. Numerical results have been obtained for the investigation of the influence of stress relaxation, hardening and fracture in the elements of mesostructure on the macro-properties of the material. Effects of strain localisation and cracking for heterogeneous medium have been simulated and discussed.

Acknowledgements

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