# Theoretical study of the blood stream in a tube in the presence of a steady-state magnetic field

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## ABSTRACT

The paper presents theoretical study of biomagnetic fluid (such as blood) flow through a tube under magnetic external field. In this work, we consider blood as a conducting and magnetic fluid that is Newtonian and incompressible. The motion of blood in a tube is described by Navier Stokes and continuity equations. The magnetic field effect on a limit region from the tube, where behavior of blood stream is changed. In dependence of the distance from the field localization, the concentration of magnetic cells of blood is changed, and their velocity shape is different from the original one (parabolic form).

This work is very important for many biomedical applications and bioengineering such as magnetic resonance imaging (MRI), magnetic drug delivery and targeting, magnetic separation and hyperthermic treatments.

Keywords: Magnetic field, blood, Newtonian fluid, permanent magnet, magnetic resonance imaging (MRI), magnetic drug delivery, Magneto hydrodynamic (MHD), ferrohydrodynamics (FHD).

## 1. INTRODUCTION

Dynamics of biological fluids (such as blood or lymph) in the presence of magnetic field may play a significant role in numerous bioengineering and biomedical applications.[1-3] the governing equations for incompressible fluid flow are similar to those used in ferrohydrodynamics (FHD). In papers[3,4], the viscous, steady, two-dimensional, incompressible, laminar biomagnetic fluid (blood) flow between two parallel flat plates (channel) with the fluid considered as homogeneous was studied. Moreover, magnetic fluids are used in medicine, especially in high-gradient magnetic separation (HGMS), magnetic drug targeting (MDT) and ferro-fluid sealing.[5] Blood behaves as a magnetic fluid due to complex interaction of the intercellular proteins, cell membrane and hemoglobin with magnetic field. The magnetic property of the blood is determined by the state of oxygenation of hemoglobin (a form of iron oxide presenting in red blood cells (RBCs)).[6-10]

When blood is pumped through the tube, the magnetic particles (RBCs or special magnetic drug delivery containers) are directed toward the wall of the tube and accumulated near the wall at the high magnetic effect of the external magnetic field. In this region, where deoxygenated blood is flowing, blood magnetization can be considered as for a paramagnetic material. When the magnet is removed, the effect of magnetic field vanishes and blood behaves as a homogenous fluid and the velocity profile of it is parabolic. Blood in normal conditions is electric conducting fluid so the principle of magnetohydrodynamics (MHD)[11-13]which deals with the interaction of electrically conducting fluids with magnetic fields. Therefore, the physical laws governing such phenomena are described by a combination of both Maxwell's equations of electromagnetism and Navier-Stokes equations dealing with conservation of mass and momentum, so effect of Lorentz force should be taken into consideration.

In addition to effect of magnetization (FHD)[14], coupling between MHD and FHD is used in numerous applications, especially bioengineering and biomedicine, such as magnetic drug delivery and targeting, magnetic particle separation, and hyperthermia.

In the present work, we have obtained the exact solution for viscous, Newtonian and incompressible fluid (blood)

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flowing through a rectangular tube in the presence of the external static magnetic field which produced by a permanent magnet with rectangular cross-section that is localized outside at the midway from the tube.

## 2. FORMULATION OF THE PROBLEM

Biomagnetic fluid (blood) flow in a tube (microchannel) with the rectangular cross-section with a width ($w_c$), a height ($h_c$), and a length ($l_c$).is analyzed. In this model, blood flow considered as a laminar steady flow of viscous and incompressible fluid. The motion of blood is considered in two dimensions (x, y) with the corresponding velocity components in Cartesian coordinates (u, v).

The magnetic field which produced by a permanent magnet is located outside from the channel, and this field is perpendicular to the direction of flow, see Fig.1.

Theoretically, blood is a conducting fluid, thus due to generated electric current we take the Lorentz force for consideration, also we consider the magnetization effect because blood is a magnetic fluid so the gradient of magnetic field should be accounted for. When the fluid (blood) enters and leaves the locally applied magnetic field, where the gradient of the magnetic field strength is high, the force due to magnetization as well as the Lorentz force arise. In the region inside of which the magnetic field is uniform, the Lorentz force prevails and the magnetization force becomes zero.

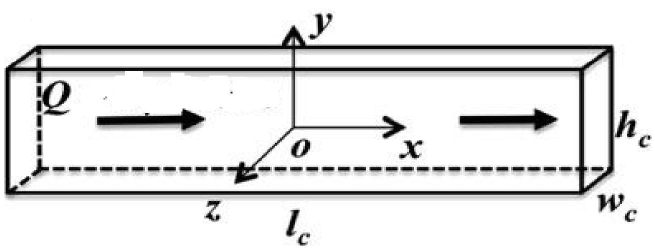



Figure 1: Dimensions of a microchannel (listed in table 1) with length $l_c$, height $h_c$ and width $w_c$. A laminar flow with flow rate (Q) is introduced into the channel, the Lorentz force prevails and the magnetization force becomes zero.

## 3. DYNAMIC EQUATION OF MOTION

The motion of blood in a tube is governed by the conservation laws of mass and momentum in the following equations.[15]

$$\nabla.\overline{v} = 0 \tag{1}$$

$$\rho(\overline{v}.\nabla\overline{v}) = -\nabla p + \eta\nabla^2\overline{v} + \overline{f}\ , \tag{2}$$

where $\overline{v}$ is the velocity vector; ρ is the density; $\nabla p$ is the pressure gradient; η is the viscosity of fluid; $\overline{f}$ is the external force per unit volume, it includes any external forces such as gravity or electromagnetic forces. In this work, we consider the motion in rectangular tube, so Eqs. (1) and (2) in rectangular coordinate in two dimensions ($x$, $y$) can be rewritten as the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+F_{mx}$$
$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+F_{my} , \quad (4)$$

where $F_{mx}$ and $F_{my}$ are the volume magnetic forces in *x*- and y-directions, while $\rho$ and $\eta$ are the density and the viscosity of fluid (blood), respectively. The total magnetic force due to the electric conductivity and magnetization is given by:

$$\overline{F}_m = \overline{F}_L + \overline{F}_M$$
$$\overline{F}_m = \overline{J}\times\overline{B}+\mu\left(\overline{M}\bullet\nabla\right)\overline{H}, \quad (5)$$

where magnetization vector $\overline{M}$ and magnetic field vector $\overline{H}$ are determined from Maxwell's equations:

$$\nabla\times\overline{H}=\overline{J}=\sigma\left(\overline{V}\times\overline{B}\right),$$
$$\nabla\bullet\overline{B}=0, \quad (6)$$
$$\overline{B}=\mu_0\left(\overline{H}+\overline{M}\right).$$

For free space $\overline{B} = \mu_0 \overline{H}$ , $\overline{M} = \chi \overline{H}$ , where $\mu_0$ permeability of free space and $\chi$ represents magnetic susceptibility.

The first term ( $\overline{J}\times\overline{B}$ ) in equation (5) represents the Lorentz force due to the electrical conductivity of blood and this term due to the effect of magnetohydrdynamics (MHD).The term $\mu\left(\overline{M}\bullet\nabla\right)\overline{H}$ represents magnetization force due to the effect of ferrohydrodynamics ( FHD). The total magnetic force in (*x*, *y*) direction is given by $\overline{F}_m(x,y)$ as below:

$$\overline{F}_m(x,y)=\overline{F}_{mx}(x,y)\hat{x}+\overline{F}_{my}(x,y)\hat{y},$$
$$\overline{H}=\overline{H}_x(x,y)\hat{x}+\overline{H}_y(x,y)\hat{y},$$
$$\nabla H^2=H_x\frac{\partial H_x}{\partial y}+H_y\frac{\partial H_y}{\partial x}, \quad (7)$$
$$\left(\nabla\times\overline{H}\right)\times\overline{H}=-H_y\left(\frac{\partial H_x}{\partial y}-\frac{\partial H_y}{\partial x}\right)\hat{x}-H_x\left(\frac{\partial H_x}{\partial y}-\frac{\partial H_y}{\partial x}\right)\hat{y}$$

Using equations for the intensity of magnetic field $\overline{H}$ , and the strength of magnetic field $\overline{B}$ , and the relation between them in Eqs. (5) - (7), the total magnetic force $\overline{F}_m(x,y)$ can be given as:

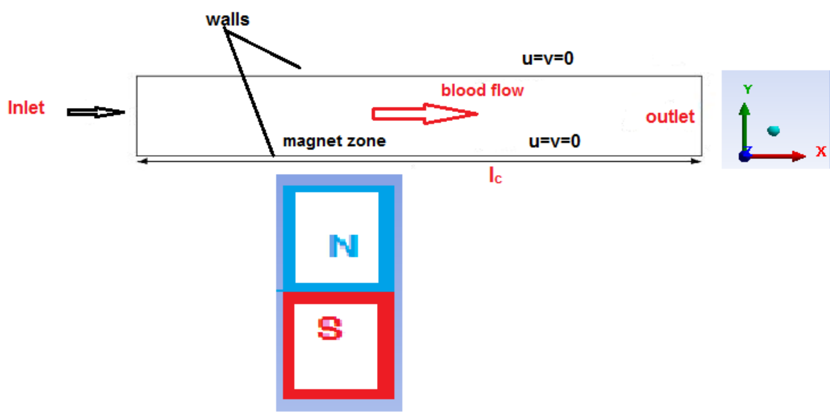
$$\overline{F}_m=\mu\left(\nabla\times\overline{H}\right)\times\overline{H}+\mu\chi\left(\overline{H}.\nabla\right)\overline{H},$$
$$\overline{F}_m=-\mu\left(\overline{H}_y\left(\left(\frac{\partial\overline{H}_x}{\partial y}-\frac{\partial\overline{H}_y}{\partial x}\right)+\chi\left(\overline{H}_y-\overline{H}_x\right)\right)\hat{x}+\overline{H}_x\left(\left(\frac{\partial\overline{H}_x}{\partial y}-\frac{\partial\overline{H}_y}{\partial x}\right)+\chi\left(\overline{H}_y-\overline{H}_x\right)\right)\hat{y}\right) \quad (8)$$

The total value of magnetic force in Eq. (8) is computed by using the formula for magnetic field ( $\overline{H}$ ) for rectangular permanent magnet taken as E. P. Furlani.[16]

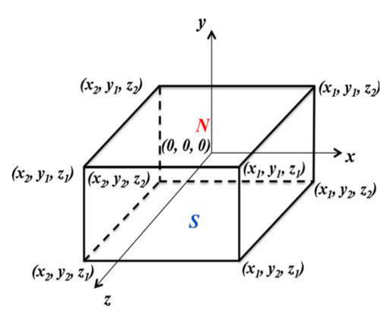
## 4. BOUNDARY CONDITIONS

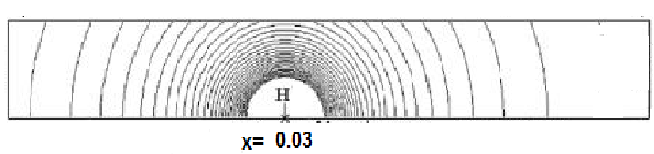
We considered the motion is steady so $\upsilon_x = \upsilon_y = 0$ and also the pressure is constant and the inlet velocity is defined by a parabolic profile [$\upsilon_x = 2\upsilon_{av} (1 - (x/R)^2)$], $\upsilon_{av}$ is the average of velocity.

Figure 2 (a) Represent geometric domain of the channel with length $l_c$ a uniform flow is specified as the inlet velocity profile and no slip conditions is applied at the lower and upper walls i.e. stationary walls. The magnetic field applied on a limit distance from the channel. This field changes from the pattern of the flow. (b) Represent domain and the contour of magnetic field which explain the strength of magnetic field is stronger near the walls and this leads to the accumulation of blood cells in this region is high, but the strength of magnetic field weak far from the wall of tube and accumulation of blood decreases in these regions. The equations of motion (3) and (4) were solved numerically by software Ansys Fluent by using a define function for adding MHD (magnetohydrodynamic principle) and velocity profile.




**a)**




**b)**

Figure 2: Geometry of the cross-section of tube (a) and representation of the dimensions of permanent rectangular magnet in Cartesian coordinate and contour for the intensity of magnetic field at a limit distance x=0.03 (b).

## 5. RESULTS AND DISCUSSION

When blood pumped in the tube, the behavior of velocity profile is parabolic in the inlet part from the tube to which the magnetic field is not applied. When blood enters the magnetic part of the tube, the magnetic particles deviate towards the wall and accumulate near the tube wall, where the intensity of magnetic field is high and this deviation changes velocity profile from parabolic to bluntness' form.

Figure 3 explains the behavior of the component of the velocity in two-dimension microchannel. In the absence of magnetic field, the velocity profile not deviates from parabolic shape.

Figure 4 explains contour for velocity component under localized magnetic field at x =0.03 at this point perturbation in the flow pattern and the fluctuations occurs in velocity profile and this fluctuation increases by increasing the strength of magnetic field. The shape of velocity profile deviates from parabolic, the accumulation of blood leads especially to aggregation of RBCs and increase of blood viscosity in this region. In the regions which magnetic field absence i.e. B=0, the velocity profile has a parabolic shape.

Figure 5 explains solving Newtonian flow of blood under localized magnetic field outside the tube by using the effect of MHD and FHD the numerical solution of governing equations was converging to a small threshold value of $10^{-7}$ within 450 total numbers of iterations.

This study helps us to understand magnetorheological (MR) fluid behavior and also may help in biomedical application such as magnetic drug delivery which used trapping and capture of magnetic nanoparticles to the to the target.[17-21]

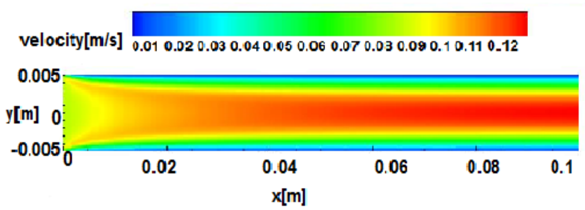



Figure 3: Contours of the velocity component v in two- dimensions microchannel with a Newtonian fluid with a uniform inlet velocity distribution.

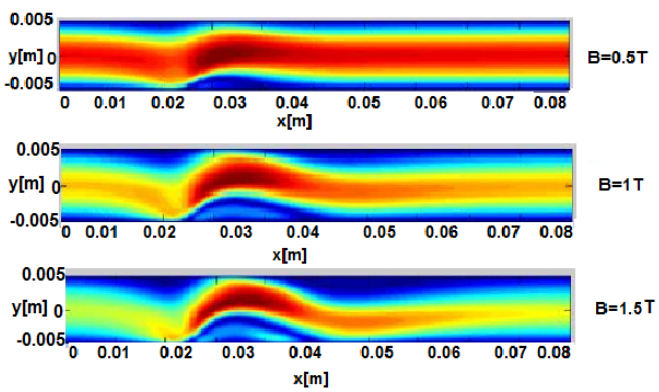



Figure 4: Contours of velocity components for different magnetic fields B values.

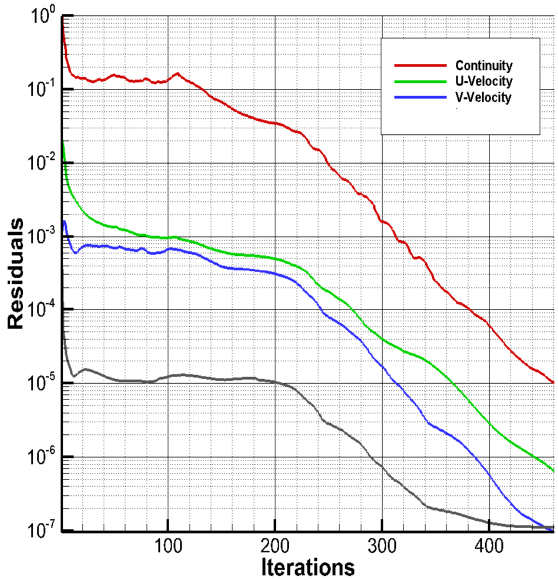



Figure 5: Residual history of simulations: Newtonian fluid flow

Table 1. The physical and geometric parameters used in simulations.

| **Symbol** | **Name** | **Initial value** |
|---|---|---|
| | **Permanent magnet** | |
| $M$ | Magnetization of the magnet | $3.189\times10^{5}$ A/m |
| $l_m$ | Length of the magnet | 3 mm |
| $h_m$ | Height of the magnet | 5 mm |
| $w_m$ | Width of the magnet | 1 mm |
| | **Microchannel** | |
| $l_c$ | Length of the channel | 100 mm |
| $h_c$ | Height of the channel | 20 mm |
| $w_c$ | Width of the channel | 0.26 mm |

## CONCLUSION

In the paper, theoretical study of biomagnetic fluid flow through a tube under magnetic external field as a model of blood flow in a vessel uploaded by a local static magnetic field is presented. Blood was considered as a conducting and magnetic fluid, Newtonian and incompressible. The motion of blood in a model vessel is described by Navier Stokes and continuity equations.

Blood in normal conditions is electric conducting fluid so the principle of magnetohydrodynamic (MHD) which deals with the interaction of electrically conducting fluids with magnetic fields is applicable. Therefore, the physical laws governing such phenomena are described by a combination of both Maxwell's equations of electromagnetism and Navier-Stokes equations dealing with conservation of mass and momentum. In this model, the combination of MHD and ferrohydrodynamics (FHD) is taken for consideration. The equations of motion are solved numerically by simulations, which are used in computational fluid dynamics (CFD). Software for simulation is Ansys Fluent. From the results, it was found that perturbation in the flow pattern and the fluctuations occurs in velocity profile and this fluctuation increases by increasing the strength of magnetic field. The shape of velocity profile deviates from parabolic, the accumulation of blood leads especially to aggregation of RBCs and increase of blood viscosity in this region. Results of this study can be used in numerous applications, especially in bioengineering and biomedicine, such as capture and trapping of magnetic nanoparticles, for magnetic particle separation, and hyperthermic treatments.

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